ELEMENTS OF VECTORS

2.1 Explain the concept of Vectors

2.2 Define Scalar and Vector quantities

2.3 Give examples for scalar and vector quantities

The physical quantities are classified into two types:

i. Scalar physical quantities (or) Scalars
ii. Vector physical quantities (or) Vectors

Scalars: The physical quantities which have only magnitude but no direction are called Scalar quantities (or) Scalars.

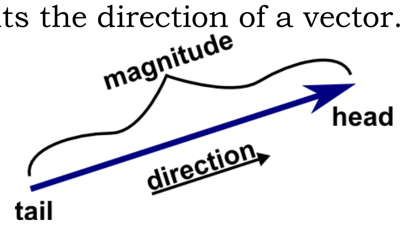
Ex: Distance, speed, work, power, energy...........

Vectors: The physical quantities which have both magnitude and direction are called Vector quantities (or) Vectors.

Ex: Displacement, velocity, force, momentum, torque...........

2.4 Represent vectors graphically

- The physical quantity which has both magnitude and direction is called Vector .So, vector is represented by an arrow.
- The length of the arrow represents the magnitude and the head of the arrow represents the direction of a vector.

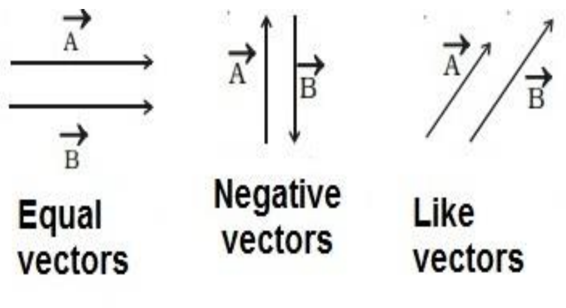


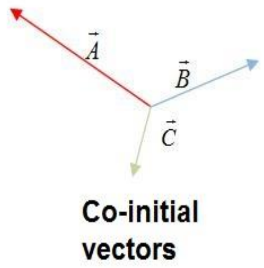
2.5 Classify the Vectors

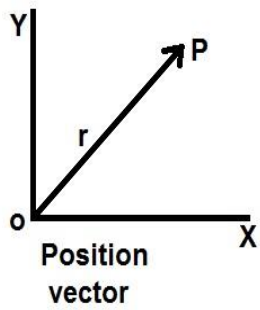
1. **Proper vector:** The vector whose magnitude is not equal to zero is called proper vector.
   If a vector $\vec{A}$ has $|\vec{A}| \neq 0$ then $\vec{a}$is a proper vector.
2. **Null vector:** The vector whose magnitude is equal to zero is called null vector.
   If a vector$\vec{A}$has $|\vec{A}| = 0$ then $\vec{a}$ is a null vector.
3. **Unit vector:** The vector whose magnitude is equal to one is called unit vector.
   If a vector $\vec{A}$has$|\vec{A}| = 1$ then $\vec{a}$ is a unit vector. It is denoted by a.
   $$unit\ vector = \frac{Vector}{it's\ magnitude} \Rightarrow \vec{A} = \frac{\vec{A}}{|\vec{A}|}$$
4. **Equal vectors:** The vectors having same magnitude and same direction are called equal vectors.

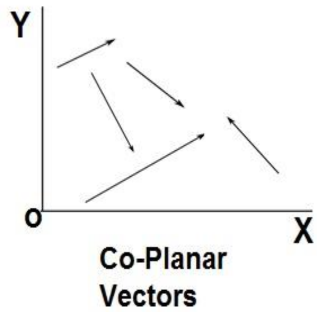
If $\vec{A}$ and $\vec{B}$ are equal vectors then$\vec{A} = \vec{B}$.

5. **Negative vectors:** The vectors having same magnitude but opposite in direction are called negative vectors. If $\vec{A}$ and $\vec{B}$ are negative vectors then $\vec{A} = -\vec{B}$
6. **Like vectors (or) Co – Directional vectors :** The vectors having same direction irrespective of their magnitudes are called like vectors (or) Co – directional vectors.
7. **Co – initial vectors :** The vectors having same initial point irrespective of their magnitudes and directionare called Co – initial vectors.
8. **Co – planar vectors :** The vectors lying in the same plane irrespective of their magnitudes ,directions and initial points are called Co – planar vectors.
9. **Position Vector :** The vector which represents the position of a point in space is called position vector.









<u>Addition and Subtraction of Vectors:</u>

Addition :

- The sum of two vectors$\vec{A}$ and $\vec{B}$ is obtained as follows.
- First draw a vector $\vec{A}$,from the head of $\vec{A}$draw the vector $\vec{B}$.
- Join the tail of $\vec{A}$ to the head of $\vec{B}$, this gives $\vec{A} + \vec{B}$.

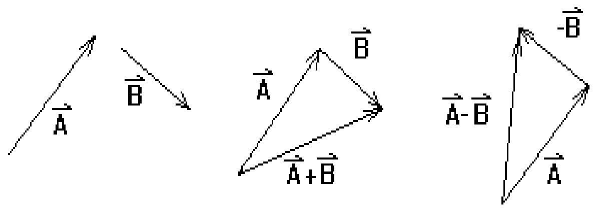


Figure 1. Addition and Subtraction of Vectors $\vec{A}$ and $\vec{B}$

Laws of vector addition :

i. Commutative law : If $\vec{A}$and $\vec{B}$ are two vectors,then $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

ii. Associative law : If $\vec{A}$,$\vec{B}$ and $\vec{C}$are three vectors,then $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

iii. Distributive law: If $\vec{A}$and $\vec{B}$ are two vectors and k is a scalar,then K ($\vec{A} + \vec{B}$) = $K\vec{A} + K\overline{B}$

2.14 Explain subtraction of vectors

The subtraction of two vectors$\vec{A}$ and $\vec{B}$ is obtained as follows.

1) First draw a vector$\vec{A}$,from the head of $\vec{A}$draw the vector - $\vec{B}$.
2) Join the tail of $\vec{A}$ to the head of - $\vec{B}$, this gives $\vec{A} - \vec{B}$.
3) The subtraction of vectors cannot obey the commutative and associative laws but obeys distributive law.

2.6 Resolve the vectors

1. The process of splitting a vector into various parts or components is called "RESOLUTION OF VECTOR"
2. These parts of a vector may act in different directions and are called "components of vector".
3. Generally there are three components of vector.
   - Component along X-axis called x-component
   - Component along Y-axis called Y-component
   - Component along Z-axis called Z-component
4. Here we will discuss only two components x-component & Y-component which are perpendicular to each other.
5. From $\Delta\, OPM$ weget,

   $sin\theta = \frac{A_y}{A} \Rightarrow A_y = Asin\theta$ and

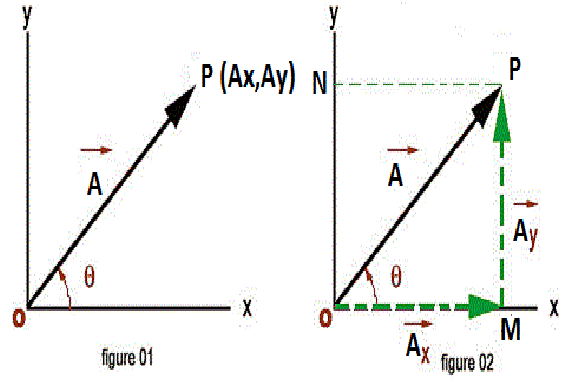
   $cos\theta = \frac{A_x}{A} \Rightarrow A_x = Acos\theta$
6. But $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$

   $\Rightarrow \overrightarrow{A} = (A\,cos\theta\,)\vec{\imath} + (A\,sin\theta\,)\vec{\jmath}$
7. The magnitude of the vector $\vec{A}$ is A.

$A = A_x^2 + A_y^2 = A^2 \Rightarrow \mathbf{A} = \sqrt{\boldsymbol{A_x^2} + \boldsymbol{A_y^2}}$

8. The direction of $\vec{A}$ is can be found from the angle $\theta \Rightarrow \frac{A_y}{A_x} = \frac{Asin\theta}{Acos\theta} = tan\theta$

   $\Rightarrow \boldsymbol{\theta} = \boldsymbol{tan^{-1}}\,(\frac{\boldsymbol{A_y}}{\boldsymbol{A_x}})$



2.7 Determine the Resultantof avectorbycomponentmethod

The vectors $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

1. If $\vec{A}$ and $\vec{B}$ are in same direction ,their resultant is$\vec{A}$ + $\vec{B}$.

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$+\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\,\hat{\imath} + (A_y + B_y)\,\hat{\jmath} + (A_z + B_z)\,\hat{k}$$

2. If $\vec{A}$ and $\vec{B}$ are in opposite direction ,their resultant is $\vec{A}$ - $\vec{B}$ or $\vec{B}$ - $\vec{A}$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$-\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\,\hat{\imath} + (A_y - B_y)\,\hat{\jmath} + (A_z - B_z)\,\hat{k}$$

(or)

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$
$$-\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} - \vec{A} = (B_x - A_x)\,\hat{\imath} + (B_y - A_y)\,\hat{\jmath} + (B_z - A_z)\,\hat{k}$$

2.8 Represent a vectorinspaceusingunitvectors$\hat{\imath}$,$\hat{\jmath}$, $\hat{k}$

1. Any vector can be resolved in to 3 components along X,Y,Z axes.The unit vectors along these axes are $\hat{\imath}$,$\hat{\jmath}$ and $\hat{k}$ respectively.
2. Consider a vector $\vec{A}$ and let $\overrightarrow{A_x}, \overrightarrow{A_y}, \overrightarrow{A_z}$ are the vector components of $\vec{A}$.
3. Wehave, $\hat{\imath} = \frac{\overrightarrow{A_x}}{A_x}$ and $\hat{\jmath} = \frac{\overrightarrow{A_y}}{A_y}$ and $\hat{k} = \frac{\overrightarrow{A_z}}{A_z}$
4. But $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y} + \overrightarrow{A_z} \Rightarrow \vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$
5. The magnitude of $\vec{A}$ is

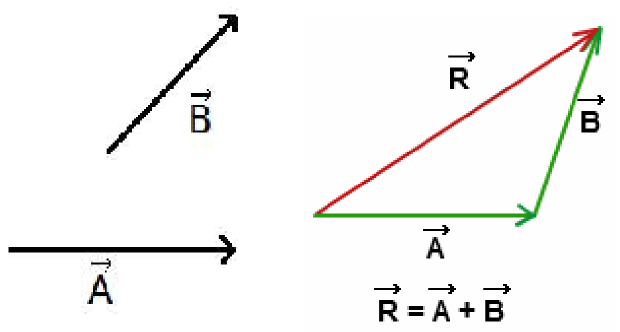
$|\vec{A}| = A\sqrt{A_x^2 + A_y^2 + A_z^2}$

2.9 State triangle law of addition of vectors

Statement: If two vectors be represented in magnitude and direction by the two sides of a triangle taken in order then their resultant is represented in magnitude and direction by the third side of the triangle taken in the reverse order.

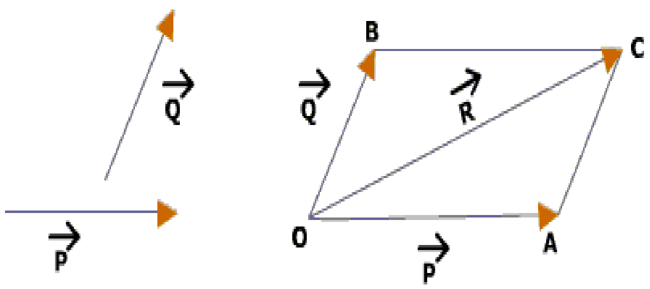
Explanation:

1. Let the two vectors $\vec{A}$ and $\vec{B}$. Bring the head of $\vec{A}$ to the tail of $\vec{B}$. Do this without changing the direction and length of the vector.
2. Join the tail of $\vec{A}$ and the head of $\vec{B}$ by a straight line with an arrow pointing towards the head of $\vec{B}$.



2.10 State parallelogram law of addition of vectors

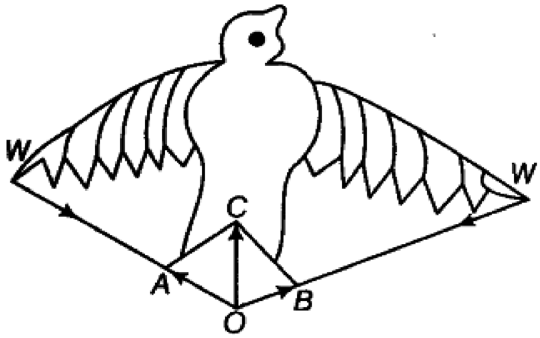
Statement : If two vectors acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal passing through the same point.



2.11 Illustrate parallelogram law of vectors in case of flying bird and sling.

Flying bird:

1. Flight of bird is an example of resultant of two vectors.
2. When the bird flies, it strikes the air with wings A and B towards O along vector AO and vector BO.
3. We know that action and reaction are equal and opposite.
4. Therefore, the air strikes the wings in opposite direction i.e. along vector OA and vector OB, resultant of which is in upward direction which balances the weight of the bird.

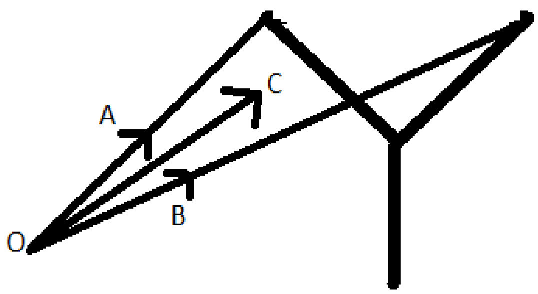


Working of sling:

1. It works on the principle of parallelogram law of vectors.
2. A rubber sling is attached to the two ends of a sling.

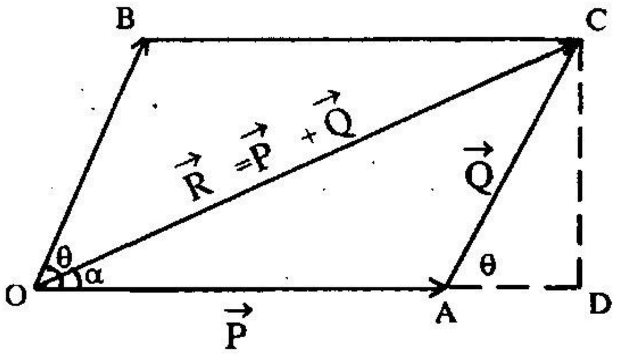
The stone to be thrown is held at the point O.

3. When the rubber string is pulled, The tensions are produced along OA and OB.
4. When the string is released, the stone moves under theeffect of resultant force OC.



2.12 Derive expression for magnitude and direction of resultant of two vectors

1) Let us consider two vectors $\vec{P}$ and $\vec{Q}$ which are inclined to each other at an angle θ.
2) Let the vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the two sides OA and OB of a parallelogram OACB.
3) The diagonal OC passing through the point O, gives the magnitude and direction of the resultant $\vec{R}$.
4) CD is drawn perpendicular to the extended OA, from C.
5) Let the angle made by $\vec{R}$ with $\vec{P}$ be α.



6) From the figure wehave,
OA = BC = P,OB = AC = Q, OC=R.
7) From the ΔACD we have,

$$\sin\theta = \frac{CD}{AC} = \frac{CD}{Q}$$

CD = Qsinθ

And $\cos\theta = \frac{AD}{AC} = \frac{AD}{Q}$

∴AD = Qcosθ

8) From right angle triangle OCD,
$OC^2 = OD^2 + CD^2$

$= (OA + AD)^2 + CD^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$= P^2 + Q^2\cos^2\theta^2 + 2.P.\ Q\cos\theta + Q^2\sin^2\theta$

$= P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2.P.\ Q\cos\theta$

$R^2 = P^2 + Q^2 + 2.P.\ Q\cos\theta$

$$\therefore \boldsymbol{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

This is the expression for the magnitude of the resultant vector.

9) From right angle triangle OCD wehave,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\Rightarrow \boldsymbol{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)}$$

This is the expression for the direction of the resultant vector.

Special Cases:

(i). When two vectors act in the same direction.
In this case, the angle between the two vectors
$\theta = 0°$

$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^0} =$

$\sqrt{P^2 + Q^2 + 2PQ} \Rightarrow R = P + Q$

$\tan\alpha = \left[\frac{Q\sin 0^0}{P+Q\cos 0^0}\right] = 0$

$\Rightarrow \alpha = \tan^{-1}(0) = 0^0$

$$\boldsymbol{R = P + Q \text{ and } \alpha = 0^0}$$

(ii). When two vectors act in the opposite direction. In this case, the angle between the two vectors
$\theta = 180°$

$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°}$

$= \sqrt{P^2 + Q^2 - 2PQ} \Rightarrow R = P - Q$

$\tan\alpha = \left[\frac{Q\sin 180°}{P+Q\cos 180°}\right] = 0$

$\Rightarrow \alpha = \tan^{-1}(0) = 0^0$

$$\boldsymbol{R = P - Q \text{ and } \alpha = 0^0}$$

(iii). When two vectors are at right angles to each other. In this case, $\theta = 90°$

$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$

$= \sqrt{P^2 + Q^2 + 0}$

$\Rightarrow R = \sqrt{P^2 + Q^2}$

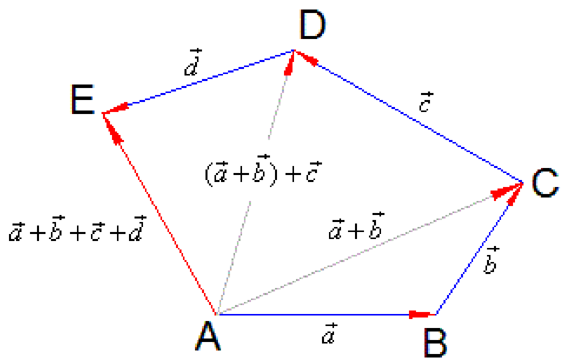
and $\tan\alpha = \left[\frac{Q\sin 90°}{P+Q\cos 90°}\right] = \frac{Q}{P}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{Q}{P}\right)$

$$\boldsymbol{R = \sqrt{P^2 + Q^2} \text{ and } \alpha = \tan^{-1}\left(\frac{Q}{P}\right)}$$

2.13 State polygon law of addition of vectors

Statement: If a number of vectors can be represented in magnitude and direction by the sides of polygon taken in the order, then their resultant is represented in magnitude and direction by the closing side of the polygon, taken in the opposite order.



Explanation:

1. Consider the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the 4 vectors acting at a point O.

2. These vectors be represented in magnitude and direction by the sides of a polygon taken in order.
3. Their resultant $\vec{R}$ is represented by the closing side $\overrightarrow{AE}$ taken in opposite order.
4. From ΔABC weget, $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b} \Rightarrow \overrightarrow{AC} = \vec{a} + \vec{b}$
5. From ΔACD weget, $\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD} = \vec{a} + \vec{b} + \vec{c} \Rightarrow \overrightarrow{AD} = \vec{a} + \vec{b} + \vec{c}$
6. From ΔADE weget, $\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE} = \vec{a} + \vec{b} + \vec{c} + \vec{d} \Rightarrow \overrightarrow{AE} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

$$\therefore \vec{R} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

2.15 Define Dot product of two vectors with examples (Work done, Power)

Definition : The dot product of two vectors $\vec{A}$ and $\vec{B}$ is the product of their magnitudes and the cosine of the angle between them.

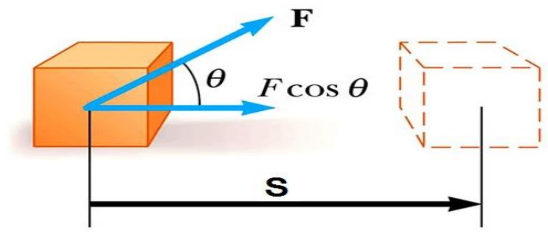
$$\therefore \vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta = AB\cos\theta$$

Where A - Magnitude of $\vec{A}$,
B - Magnitude of $\vec{B}$
θ – Angle between $\vec{A}$ and $\vec{B}$

Examples :

1.Work Done:

1. If a force F acting on a body making an angle θ with the horizontal,displaces through a distance S.
2. The components of the force are Fcosθ and Fsinθ.
3. Only the component of the force along the direction of motion of the object Fcosθ does the work.



∴W =Fcosθ X S = FScosθ ⇒ $W = \vec{F}.\vec{S}$
Hence, Work is the scalar product of the force and displacement vectors.

2.Power:

Power is the rate of doing work.

$$\therefore P = \frac{W}{t} = \frac{\vec{F}.\vec{S}}{t} = \vec{F}.\frac{\vec{S}}{t} = \vec{F}.\vec{V}$$

Hence, power is the scalar product of the force and velocity vectors.

2.16 Mention the properties of Dot product

If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are the 3 vectors then

1. It gives a scalar quantity.
2. It obeys commutative law. $\Rightarrow \vec{A}.\vec{B} = \vec{B}.\vec{A}$
3. It obeys distributive law. $\Rightarrow \vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$
4. If two vectors are parallel their dot product is maximum and positive.$\Rightarrow \vec{A}.\vec{B}$ = AB.
5. If two vectors are perpendicular their dot product is zero.$\Rightarrow \vec{A}.\vec{B} = 0$.
6. If two vectors are anti-parallel their dot product is minimum and negative.$\Rightarrow \vec{A}.\vec{B}$ = -AB
7. If $\hat{\imath},\hat{\jmath},\hat{k}$ are unit vector then $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$ and $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$.
8. If $\vec{A} = Ax\hat{\imath} + Ay\hat{\jmath} + Az\hat{k}$ and $\vec{B} = Bx\hat{\imath} + By\hat{\jmath} + Bz\hat{K}$ then
$$\vec{A}.\vec{B} = AxBx + AyBy + AzBz$$
9. If θ is the angle between $\vec{A}$ and $\vec{B}$, then $\cos\theta = \frac{\vec{A}.\vec{B}}{AB}$

2.17 Define Cross products of two vectors with examples (Torque, Linear velocity)

Definition: The cross product of two vectors $\vec{A}$ and $\vec{B}$ is a vector whose magnitude is ABsinθ and whose directions is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

$$\therefore \vec{A} \text{x} \vec{B} = AB\sin\theta\,\hat{n},$$

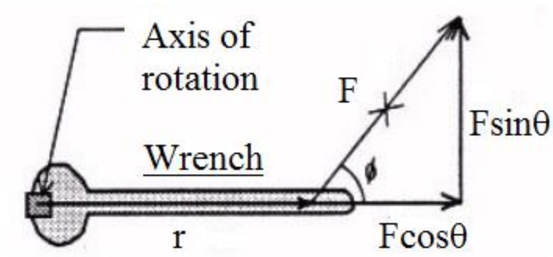
Where A - Magnitude of $\vec{A}$ B - Magnitude of $\vec{B}$
θ – Angle between $\vec{A}$ and $\vec{B}$
$\hat{n}$ - unit vector in the direction of $\vec{A}$x $\vec{B}$

The direction of the resulutant vector can be found from the right hand screw rule.

Examples :

Torque :

1. Torque is the tendency of a force to cause an object to rotate.
2. When you push on the end of a wrench, you are applying torque to make it rotate.
3. Only the part of the force perpendicular to the wrench will create torque. ⇒ τ = rFsinθ



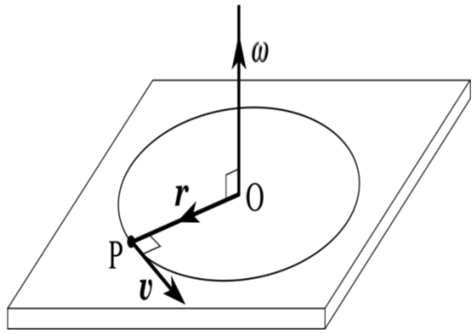
4. Torque is calculated by finding the cross product of the position vector $\vec{r}$ and the force vector $\vec{F}$.

$$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \tau = rF\sin\theta.$$

Linear velocity:

1. Consider a particle travelling in a circular path of radius r.
2. The linear velocity of particle is V and angular velocity is ω.
3. The linear velocity vector $\vec{V}$ is equal to the vector product of the radius vector $\vec{r}$and is the angular velocity vector$\vec{\omega}$.

$$\vec{V} = \vec{\omega}\, x\, \vec{r} \Rightarrow V = r\omega\sin\theta.$$



2.18Mention the properties of Cross product.

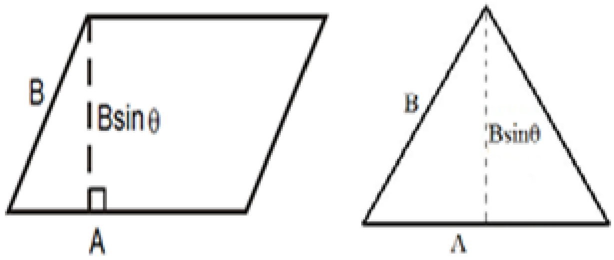
If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are the 3 vectors then

1. It gives a vector quantity.
2. It doesn't obeys commutative law.
   $\Rightarrow \vec{A}$x $\vec{B} \neq \vec{B}$x $\vec{A}$ but $\vec{A}$x $\vec{B}$ = -($\vec{B}$x $\vec{A}$)
3. It obeys distributive law.
   $\Rightarrow \vec{A}\text{x}(\vec{B} + \vec{C}) = \vec{A}\text{x}\vec{B} + \vec{A}\text{x}\vec{C}$
4. If two vectors are parallel their vector product is is a null vector.$\Rightarrow |\vec{A}\text{x}\vec{B}|$ =0.
5. If two vectors are perpendicular their vector product having maximum magnitude.
   $$\Rightarrow |\vec{A}\text{x}\vec{B}| = AB.$$
6. If two vectors are anti-parallel their vector product is a null vector.$\Rightarrow |\vec{A}\text{x}\vec{B}|$ = 0.
7. If $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vector then
   $|\vec{\imath}\text{x}\vec{\imath}| = |\vec{\jmath}\text{ x}\vec{\jmath}| = |\vec{k}\text{x}\vec{k}| = 0$ and
   $$\vec{\imath} \times \vec{\jmath} = \vec{k}, \vec{\jmath} \times \vec{k} = \vec{\imath} \text{ and } \vec{k} \times \vec{\imath} = \vec{\jmath}$$
   (or)
   $$\vec{\jmath} \times \vec{\imath} = -\vec{k}, \vec{k} \times \vec{\jmath} = -\vec{\imath} \text{ and } \vec{\imath} \times \vec{k} = -\vec{\jmath}$$
8. If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and
   $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{K}$ then
   $$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$
9. If θ is the angle between $\vec{A} \text{ and } \vec{B}$,

$$then \quad \sin\theta = \frac{|\vec{A}\text{x}\vec{B}|}{|\vec{A}||\vec{B}|} = \frac{|\vec{A}\text{x}\vec{B}|}{AB}$$

Area of parallelogram:

Let the two vectors $\vec{A} \text{ and } \vec{B}$ represents two adjacent sides of a parallelogram as shown in figure.
Area of parallelogram = base x heigh.
Here base = A and Height is Bsinθ.
⇒ Area = ABsinθ
∴Area of parallelogram = $|\vec{A}\text{x}\vec{B}|$



Area of Parallelogram　　Area of Triangle

Area of triangle:

Let the two vectors $\vec{A} \text{ and } \vec{B}$ represents two adjacent sides of a triangle as shown in figure.
Area of triangle = ½ base x heigh.
Here base = A and Height = Bsinθ.
⇒ Area = ½ ABsinθ
∴Area of triangle= $\frac{1}{2}|\vec{A}\text{x}\vec{B}|$

KINEMATICS

KINEMATICS: The branch of mechanics that studies the motion of a body or a system of bodies without consideration of cause of motion is called Kinematics.

DYNAMICS: The branch of mechanics that studies the motion of a body or a system of bodies and consideration of cause of motion is called Dynamics.

3.1Write the equations of motion in a straight line

1. $V = u + at$
2. $S = ut + ½ at^2$
3. $v^2 – u^2 = 2as$
4. $S_n = u + a\ (n - \frac{1}{2})$

Equations of motion for freely falling body:

Here v = v, u = 0, a = +g, s = h, t = t

1. $\mathbf{v = gt}$
2. $\mathbf{h = ½ gt^2}$
3. $\mathbf{v^2 = 2gh}$
4. $\mathbf{h_n th = g(n – ½\ )}$(or) $v = \sqrt{2gh}$

Equations of motion vertically upwards projected body:

Here v = v, u = u, a = -g, s = h, t = t

1. $\mathbf{v = u – gt}$ 2. $\mathbf{h = ut - ½ gt^2}$

3. $\mathbf{v^2 – u^2 = - 2gh}$ 4. $\mathbf{h_n th = u - g(n – ½\ )}$

Equations of motion vertically downward projected body:

Here v = v, u = u, a = +g, s = h, t = t

1. $\mathbf{v = u + gt}$ 2. $\mathbf{h = ut + ½ gt^2}$

3. $\mathbf{v^2 – u^2 = 2gh}$ 4. $\mathbf{h_n th = u + g(n – ½\ )}$

3.2 Explain the acceleration due to gravity

Acceleration due to gravity:

The acceleration produced in a freely falling body due to the gravitational pull of the earth is called acceleration due to gravity (g).

Dimensional formula - M^0LT^{-2} .

Unit - m/s^2 .

Properties:

1. The value **g** is same for all bodies at a given place.
2. The average value of **g** on the earth is $9.8\ m/s^2$.
3. It is always directed towards the earth.
4. Its value is maximum at poles and minimum at equator.
5. It is negative for vertically upward moving body.
6. It is positive for vertically downward moving body.

3.3 Derive expressions for vertical motion

a) Maximum Height, b) time of ascent, c) time of descent, and d) time of flight

Vertical Projection:

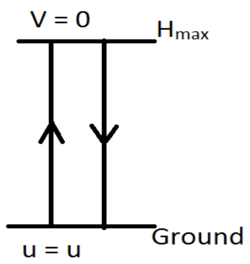
When a body projected making an angle 90^0 to the horizontal, then the projection is called as vertical projection.

	H_{max}	t_a	t_d	T
S	H_{max}	-	$u^2/2g$	0
U	u	U	0	u
V	0	0	-	-
a	-g	-g	+g	-g
T	-	t_a	t_d	T

1. $v=u+at$
2. $S = ut +1/2\ at^2$
3. $v^2 – u^2 = 2as$

a) Expression for maximum height:

Definition: The maximum vertical distance travelled by a vertically projected body before its velocity becomes zero is called maximum height (Hmax).



We have, $v^2 – u^2 = 2as$

But here, $v = 0, u = u,$

$a = -g, s = H_{max}$

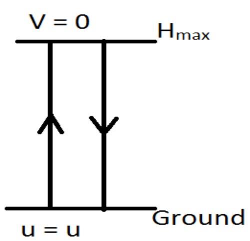
$\Rightarrow 0 - u^2 = -2g\ H_{max},$

$\Rightarrow u^2 = 2g\ H_{max}$

$$\Rightarrow \mathbf{H_{max}} = \frac{u^2}{2g}$$

b) **Expression for time of ascent (t_a)**:

Definition: The time taken by a vertically projected body to reach the maximum height from ground is called time of ascent.



We have, $v = u + at$

But here, $v = 0, u = u,$

$$a = -g, t = t_a$$

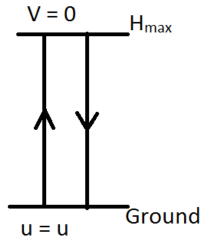
$$\Rightarrow 0 = u - gt_a$$

$$\Rightarrow u = gt_a$$

$$\Rightarrow \mathbf{t_a = \frac{u}{g}}$$

c) **Expression for time of descent (t_d)**:

Definition: The time taken by a vertically projected body to reach the ground from maximum height is called time of descent.



We have, $S = ut + ½ at^2$

But here, $S = H_{max} = \frac{u^2}{2g}$, $u = 0,$

$$a = +g, t = t_d$$

$$\Rightarrow \frac{u^2}{2g} = (0 \times t_d) + ½ gt_d^2$$

$$\Rightarrow \frac{u^2}{2g} = \frac{gt_d^2}{2}$$

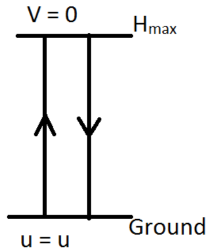
$$\Rightarrow u^2 = g^2 t_d^2$$

$$\Rightarrow u = gt_d$$

$$\Rightarrow \mathbf{t_d = \frac{u}{g}}$$

d) **Expression for time of flight (T)**:

Definition: The time taken by a vertically projected body remains in air is called time of flight.



We have, $s = ut + ½ at^2$

But here, $s = 0,$

$$u = u,$$

$$a = -g,$$

$$t = T$$

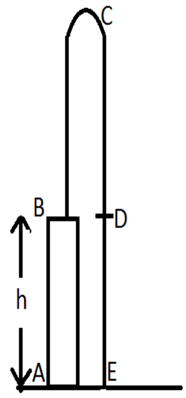
$$\Rightarrow 0 = uT - ½ gT^2$$

$$\Rightarrow uT = ½ gT^2$$

$$\Rightarrow T = u \times \frac{2}{g}$$

$$\Rightarrow \mathbf{T = \frac{2u}{g}}$$

3.4 Derive height of a tower when a body projected vertically upward from the top of a tower:



1. Consider a tower AB of height h above the ground. (∴ AB = h)
2. A body is projected from the top of the tower B with a velocity u.
3. The body travels in the path BCD and reaches the ground at E as shown in figure.
4. The net displacement of the body is

   $S = BC + CD + DE$

   $= BC - BC + DE$ (∵ $BC = - CD$)

   $= -h$ (∵ $DE = - AB$)
5. For a body projected vertically up from the top of tower the displacement is

   $S = ut + ½ at^2$

   Here we have $S = -h,$

   $u = u,$

   $a = - g,$

   $t = t.$

   $\Rightarrow -h = ut - ½ gt^2$

$$\Rightarrow \mathbf{h = - ut + ½ gt^2}$$

This is the expression for height of the tower when a body projected vertically up from its top.

If a body dropped from the balloon moving vertically upwards with uniform velocity is similar to the body projected vertically upwards from its top.

$$\mathbf{h = - ut + ½ gt^2}$$

For tower:

h – height of the tower

u- velocity of the body with which it is projected.

t – Time taken by the body to reach the ground.

$$\mathbf{h = - ut + ½ gt^2}$$

For balloon:

h – height of the balloon from where it is dropped

u- uniform velocity of the balloon moving upward.

t – Time taken by the body to reach the ground.

3.5 Explain projectile motion with examples

Definition: A projectile is an object projected into air with some velocity at an angle other than 90^0 with the horizontal.

(or)

A projectile is a body making two dimensional motion under the influence of gravity.

Projectile: The body which is in projectile motion is called projectile.

Trajectory: The path followed by the projectile is called its trajectory.

There are two types of projectile motions –

1) Horizontal Projection $\theta=0^0$

2) Oblique projection $0^0<\theta<90^0$

Examples of Projectiles:

1. A Javelin thrown by an athlete.
2. A body dropped from a horizontally moving aeroplane.
3. A bullet fired from a gun.
4. A shot-put thrown by an athlete.
5. A cricket ball thrown by a fielder.

3.6 Explain Horizontal projection

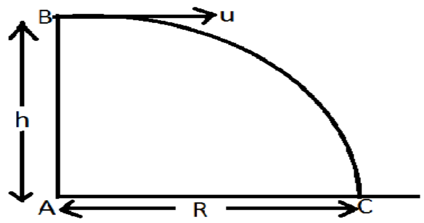
Definition: If a body projected parallel ($\theta = 0^0$)to the horizontal plane and moves in a parabolic path, then the projection is called Horizontal projection.

Ex: 1.A body dropped from a moving aeroplane.

1. A bullet fired from a gun.
2. A shotput thrown by an athlete.
3. A stone thrown by the catapult.

Expression for Time of flight:

Definition: The time taken by a horizontally projected body to reach the ground is called its time of flight.



We have, $s = ut + \frac{1}{2} at^2$

But here, s = h,

u = 0,

a = +g,

t = T

$\Rightarrow h = (0 \times T) + \frac{1}{2} gT^2$

$\Rightarrow h = \frac{1}{2} gT^2$

$\Rightarrow T^2 = h \times \frac{2}{g}$

$$\Rightarrow \mathbf{T} = \sqrt{\frac{2h}{g}}$$

Expression for Horizontal Range(R):

Definition: The maximum horizontal distance travelled by a horizontal projectile during its flight is called its horizontal range.

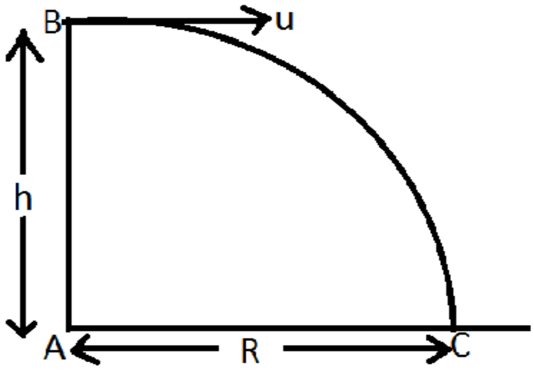
For horizontal projection, the velocity in the horizontal direction is equal to the velocity of projection.

∴Horizontal Range

R = horizontal velocity x Time of flight

$= u \times T = u \times \sqrt{\frac{2h}{g}}$

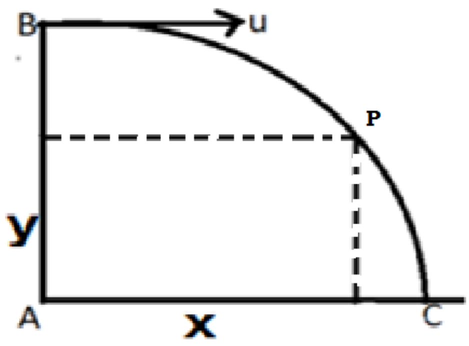
$$\therefore \mathbf{R} = u\sqrt{\frac{2h}{g}}$$



3.7 Derive an expression for the path of a projectile in horizontal projection

1. If a body projected in the horizontally, the motion of the body will be (i) uniform velocity in the horizontal direction and (ii) uniform acceleration due to gravity in the vertically downward direction.
2. Consider a body is thrown with an initial velocity **u** along the horizontal direction.
3. After a time **t** the body at a point **P** as shown in figure.
4. We will study the motion along x and y axis separately.
5. We will take the starting point to be at the origin.

	horizotal	Vertical
Initial velocity (u)	u	0
Acceleration(a)	0	+g
Displacement(s)	x	y



In horizontal direction:

5.Component of initial velocity along x-axis is **u_x=u**

6. Acceleration along x-axis is **a_x=0**
(Because no force is acting along the horizontal direction)

7.The displacement along x-axis at any instant **t**

$$s = ut + \frac{1}{2}at^2$$
$$x = ut + \frac{1}{2} \times 0 \times t^2$$

= ut + 0

x = u t

t = x/u---------(1)

In vertical direction:

8.Component of initial velocity along y-axis is **u_y=0.**

9. Acceleration along y-axis **a_y=g.**
(Because gravitational force is acting along the vertical direction)

10.The displacement along y-axis at any instant **t**

$$s = ut + \frac{1}{2}at^2$$

y= 0 x t + $\frac{1}{2}$ (g) t²

y= 0 + $\frac{1}{2}$ gt²

y = $\frac{1}{2}$gt²---------(2)

Equation of trajectory:

11.Fromequations (1) and (2) we can write,

$$y = \frac{1}{2} \times g \times \left(\frac{x}{u}\right)^2$$
$$\Rightarrow y = \left(\frac{g}{2u^2}\right)x^2$$
$$\therefore y = kx^2 \text{ where } \mathbf{k} = \left(\frac{g}{2u^2}\right)$$

12. This is the equation of a parabola. Thus, the path of a horizontal projectile is a parabola.

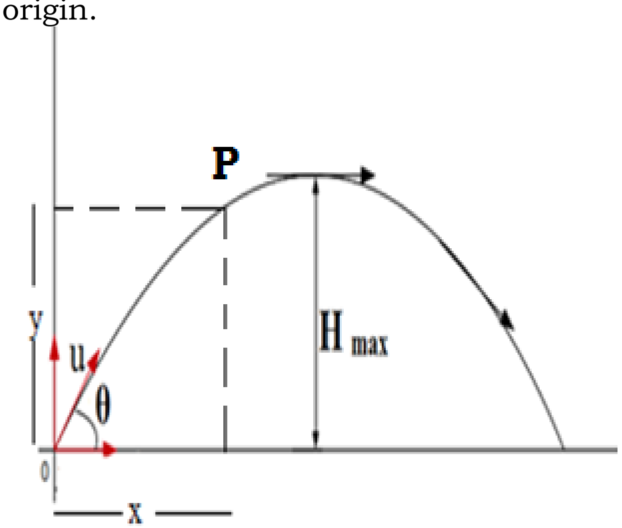
3.8 Explain oblique projection

Definition: If a body projected with an angle between 0^0 and 90^0 to the horizontal plane and moves in a parabolic path, then the projection is called oblique projection.

Ex:
1. A ball hit by a bat.
2. A javelin thrown by an athlete.

3.9 Derive an expression for the path of projectile in oblique projection

1. If a body projected in the obliquelly , the motion of the body will be (i) uniform velocity in the horizontal direction and (ii) uniform acceleration due to gravity in the vertically downward direction.
2. Consider a body is thrown with an initial velocity **u** obliquely.
3. After a time **t** the body at a point **P** as shown in figure.
4. We will study the motion along x and y axis separately.
5. We will take the starting point to be at the origin.



	horizotal	Vertical
Initial velocity (u)	u_x = ucosθ	u_y = usinθ
Acceleration(a)	0	-g
Displacement	x	y

In horizontal direction:

5. Component of initial velocity along x-axis.
u_x=u cosθ

6. Acceleration along x-axis is **a_x=0**
(Because no force is acting along the horizontal direction)

7.The displacement along x-axis at any instant **t**

$$s = ut + \frac{1}{2}at^2$$

$$x = u_x t + \frac{1}{2} \times a_x \times t^2$$

$$= u_x t + 0$$

$$x = u\cos\theta \times t$$

$$t = \frac{x}{u\cos\theta} \text{ ---------(1)}$$

In vertical direction:

8. Component of initial velocity along y-axis.

$u_y = u \sin\theta$

9. Acceleration along y-axis $a_y = -g$.
(Because gravitational force is acting along the vertical direction)

10.The displacement along y-axis at any instant **t**

$$s = ut + \frac{1}{2}at^2$$

$$y = u_y t + \frac{1}{2}(-g)\,t^2$$

$$y = (u\sin\theta \times t) - \frac{1}{2}gt^2 \text{-------(2)}$$

Equation of trajectory :

11. Subsituting for **t** we get

$$y = \left[u\sin\theta \times \left(\frac{x}{u\cos\theta}\right)\right] - \left[\frac{1}{2} \times g \times \left(\frac{x}{u\cos\theta}\right)^2\right]$$

$$\Rightarrow y = \left[\left(\frac{\sin\theta}{\cos\theta}\right) \times x\right] - \left[\frac{g}{2} \times \frac{x^2}{u^2\cos^2\theta}\right]$$

$$\Rightarrow y = [\tan\theta \times x] - \left[\frac{g}{2u^2\cos^2\theta} \times x^2\right]$$

$$\therefore y = Ax - Bx^2$$

where $A = \tan\theta$ and $B = \frac{g}{2u^2\cos^2\theta}$

12. This is the equation of a parabola. Thus, the path of an oblique projectile is a parabola.

3.10Deriveformulaefor projectile in oblique projection

a) Maximum Height, b) time of ascent, c) time of descent, and d) time of flight e) Horizontal Range, f) Maximum range

	H_{max}	t_a	t_d	T
S	H_{max}	-	$\frac{u^2\sin^2\theta}{2g}$	0
U	Usinθ	usinθ	0	usinθ
V	0	0	-	-
a	-g	-g	+g	-g
t	-	t_a	t_d	T

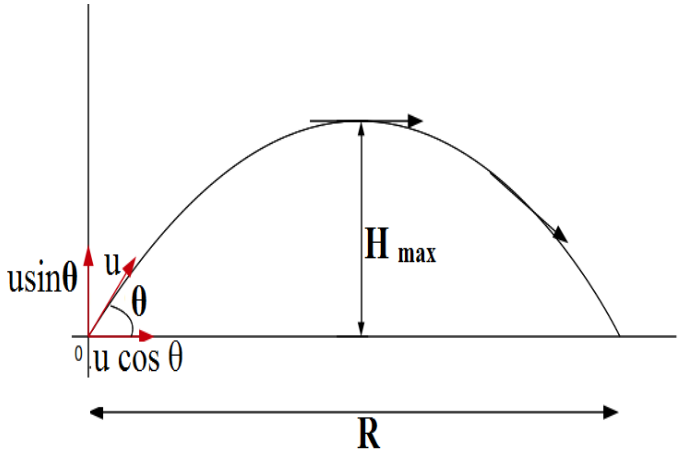
1.v=u+at

2. $S = ut + 1/2\, at^2$

3.$v^2 - u^2 = 2as$

a)Expression for maximum height:

Definition: The maximum vertical distance travelled by a obliquely projected body is called maximum height (Hmax).



We have, $v^2 - u^2 = 2as$

But here,v = 0,

u = usinθ,

a = -g,

$s = H_{max}$

$\Rightarrow 0 - u^2\sin^2\theta = -2g\,H_{max}$,

$\Rightarrow u^2\sin^2\theta = 2g\,H_{max}$

$$\Rightarrow H_{max} = \frac{u^2\sin^2\theta}{2g}$$

b)Expression for time of ascent (t_a):

Definition: The time taken by a obliquelly projected body to reach the maximum height from ground is called time of ascent.

We have, v = u + at

But here,v = 0,

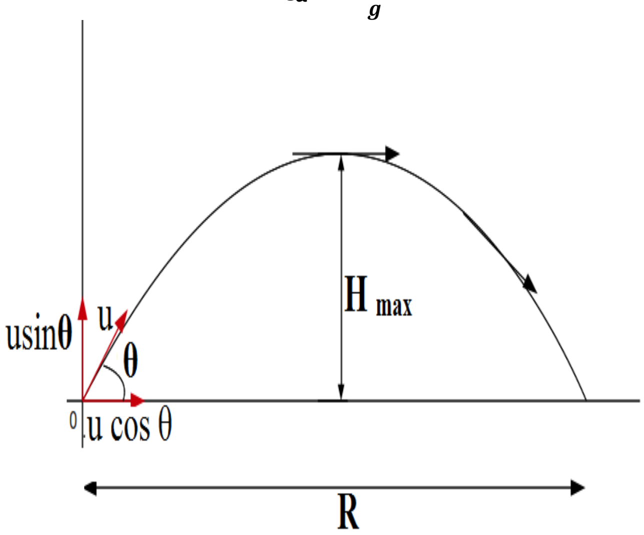
u = usinθ,

$$a = -g, \quad t = t_a$$

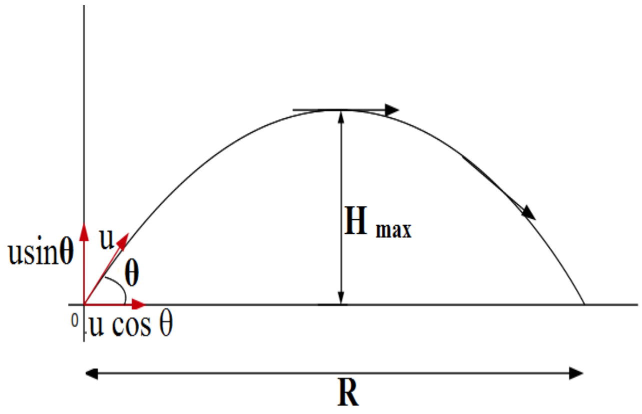
$$\Rightarrow 0 = u\sin\theta - gt_a$$

$$\Rightarrow u\sin\theta = gt_a$$

$$\Rightarrow \mathbf{t_a} = \frac{u\sin\theta}{g}$$



c) Expression for time of descent (t_a):
Definition: The time taken by a obliquelly projected body to reach the ground from maximum height is called time of descent.



We have, $S = ut + \frac{1}{2}at^2$

But here, $S = H_{max} = \frac{u^2\sin^2\theta}{2g}$,

$u = 0,$

$a = +g,$

$t = t_d$

$$\Rightarrow \frac{u^2\sin^2\theta}{2g} = (0 \times t_d) + \frac{1}{2}gt_d^2$$

$$\Rightarrow \frac{u^2\sin^2\theta}{2g} = \frac{1}{2}gt_d^2$$

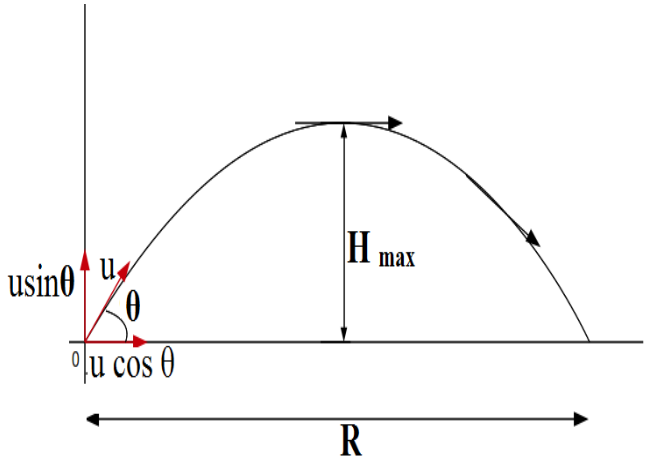
$$\Rightarrow u^2\sin^2\theta = g^2t_d^2$$

$$\Rightarrow u\sin\theta = gt_d$$

$$\Rightarrow \mathbf{t_d} = \frac{u\sin\theta}{g}$$

d) Expression for time of flight (T):
Definition: The time taken by a obliquelly projected body remains in air is called time of flight.

We have, $s = ut + \frac{1}{2}at^2$



But here, $s = 0$,

$u = u\sin\theta,$

$a = -g,$

$t = T$

$$\Rightarrow 0 = u\sin\theta \times T - \frac{1}{2}gT^2$$

$$\Rightarrow u\sin\theta \times T = \frac{1}{2}gT^2$$

$$\Rightarrow T = u\sin\theta \times \frac{2}{g}$$

$$\Rightarrow \mathbf{T} = \frac{2u\sin\theta}{g}$$

e) Expression for horizontal range (R):
Definition: The maximum horizontal distance travelled by a oblique projectile during its flight is called its horizontal range.

$S = ut + \frac{1}{2}at^2$

But here, $S = R$,

$u = u_x = u\cos\theta,$

$a = 0,$

$t = T$

$R = u\cos\theta \times T$

∴Horizontal Range

R = horizontal velocity x Time of flight

For oblique projection,
the velocity in the horizontal direction $u_x = u\cos\theta$

time of flight $T = \frac{2u\sin\theta}{g}$

$$\Rightarrow R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{u^2(2\sin\theta\cos\theta)}{g}$$

But $\sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \mathbf{R} = \frac{u^2\sin 2\theta}{g}$$

f) Expression for maximum Range(R):
The horizontal range of the oblique projectile is

$$R = \frac{u^2 \sin 2\theta}{g}$$

For the angle of projection θ = 45° the horizontal range of the oblique projectile is maximum.

Because, sin2θ = sin (2 x 45°) = sin90° = 1

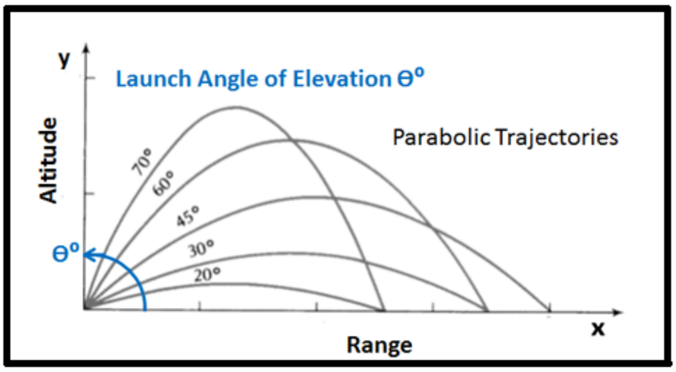
Maximum range $R_{max} = \frac{u^2 \times 1}{g}$

$$\Rightarrow \mathbf{R_{max}} = \frac{u^2}{g}$$

Two angles of projection for same range :

The horizontal range of the oblique projectile is

$$R = \frac{u^2 \sin 2\theta}{g}$$



But sin2θ = sin (180° – 2θ) = sin 2(90°-θ)

Therefore the angle of projectionsθ and (90-θ) have same range. Because

$$\sin 2\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

∴ **Ex :** Angles (15°,75°), (30°,60°) , (40°,50°) ------ have same range.

RELATION BETWEEN TIME OF FLIGHT(T) AND MAXIMUM HEIGHT (H) OF A PROJECTILE.

We know that

Time of flight of a projectile is

$T = \frac{2u\sin\theta}{g} \Rightarrow u = \frac{gT}{2\sin\theta}$ -------(1)

Maximum height of the projectile is

$$H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow H = \left(\frac{gT}{2\sin\theta}\right)^2 \times \frac{\sin^2\theta}{2g}$$

$$= \frac{g^2T^2}{4\sin^2\theta} \times \frac{\sin^2\theta}{2g} = \frac{1}{8}gT^2$$

$$\therefore \boldsymbol{H = \frac{1}{8}gT^2}$$

RELATION BETWEEN ANGLE OF PROJECTION(θ),HORIZONTAL RANGE(R) AND MAXIMUM HEIGHT (H) OF A PROJECTILE.

We know that,

Mximum height $H = \frac{u^2 \sin^2\theta}{2g}$ ------(1)

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$ -------(2)

$$\frac{H}{R} = \frac{u^2 \sin^2\theta}{2g} \times \frac{g}{u^2 \sin 2\theta} = \frac{\sin^2\theta}{2} \times \frac{1}{2\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{4\cos\theta} = \frac{\tan\theta}{4}$$

$$\therefore \boldsymbol{\frac{H}{R} = \frac{\tan\theta}{4} (or) \tan\theta = \frac{4H}{R}}$$

RELATION BETWEEN TIME OF FLIGHT(T), ANGLE OF PROJECTION(Θ) AND HORIZONTAL RANGE(R) OF A PROJECTILE.

We know that

Time of flight $T = \frac{2u\sin\theta}{g}$ ---------(1)

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$ -------(2)

$$\frac{T^2}{R} = \left(\frac{2u\sin\theta}{g}\right)^2 \times \frac{g}{u^2 \sin 2\theta}$$

$$= \frac{4u^2 \sin^2\theta}{g^2} \times \frac{g}{u^2 \times 2\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta}{g\cos\theta} = 2\tan\theta$$

$$\therefore R\tan\theta = \frac{1}{2}gT^2$$

-------------------THE END ------------------