Design of Steel Structures (Web Course)

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Detailed Syllabus

. Introduction (2 lectures)
Properties of Structural Steel, I. S. Rolled Sections, I. S. Specifications

II. Design Approach (2 lectures)
Factor of Safety, Permissible and Working Stresses, Elastic Method, Plastic Method, Introduction to Limit States of Design

III. Connections (5 lectures)
Type of Connections, Riveted, Bolted and Welded Connections, Strength, Efficiency and Design of Joints, Modes of Failure of a Riveted Joint, Advantages and Disadvantages of Welded Joints, Design of Fillet and Butt Welds, Design of Eccentric Connections

IV. Tension Members (8 lectures)
Net Sectional Area, Permissible Stress, Design of Axially Loaded Tension Member, Design of Member Subjected to Axial Tension and Bending

V. Compression Members (9 lectures)
Modes of Failure of a Column, Buckling Failure: Euler's Theory, Effective Length, Slenderness Ratio, Design Formula: I.S. Code Formula, Design of Compression Members, Design of Built-Up Compression Members: Laced and Battened Columns

VI. Beams (8 lectures)
Design Procedure, Built-Up Sections, Plate Thickness, Web Crippling, Web Buckling, Connections and Curtailment of Flange Plates

VII. Beam Column (3 lectures)
Eccentricity of Load, Interaction Formulae, Design Procedure, Eccentrically Loaded Base Plates

VIII. Column Base (3 lectures)
Slab Base, Gusseted Base, Grillage Foundation

INTRODUCTION

1.1 Introduction

When the need for a new structure arises, an individual or agency has to arrange the funds required for its construction. The individual or agency henceforth referred to as the owner then approaches an architect. The architect plans the layout so as to satisfy the functional requirements and also ensures that the structure is aesthetically pleasing and economically feasible. In this process, the architect often decides the material and type of construction as well. The plan is then given to a structural engineer who is expected to do locate the structural elements so as to cause least interference to the function and aesthetics of the structure. He then makes the strength calculations to ensure safety and serviceability of the structure. This process is known as structural design. Finally, the structural elements are fabricated and erected by the contractor. If all the people work as a team then a safe, useful, aesthetic and economical structure is conceived. However in practice, many structures fulfill the requirements only partially because of inadequate coordination between the people involved and their lack of knowledge of the capabilities and limitations of their own and that of others. Since a structural engineer is central to this team, it is necessary for him to have adequate knowledge of the architects and contractors work. It is his responsibility to advise both the architect and the contractor about the possibilities of achieving good structures with economy. Ever since steel began to be used in the construction of structures, it has made possible some of the grandest structures both in the past and also in the present day (Fig. 1.1). In the following paragraph, some of the aspects of steel structures, which every structural engineer should know, are briefly discussed.

Forth bridge in UK	Eiffel tower in France	Empire State Building in US
Golden gate bridge in US	Howrah bridge in India	Petronas tower in Malaysia

Fig 1.1. Some notable structures built with steel

Steel is by far the most useful material for building structures with strength of approximately ten times that of concrete, steel is the ideal material for modern construction. Due to its large strength to weight ratio, steel structures tend to be more economical than concrete structures for tall buildings and large span buildings and bridges. Steel structures can be constructed very fast and this enables the structure to be used early thereby leading to overall economy. Steel structures are ductile and robust and can withstand severe loadings such as earthquakes. Steel structures can be easily repaired and retrofitted to carry higher loads. Steel is also a very eco-friendly material and steel structures can be easily dismantled and sold as scrap. Thus the life-cycle cost of steel structures, which includes the cost of construction, maintenance, repair and dismantling, can be less than that for concrete structures. Since steel is produced in the factory under better quality control, steel structures have higher reliability and safety.

To get the most benefit out of steel, steel structures should be designed and protected to resist corrosion and fire. They should be designed and detailed for easy fabrication and erection. Good quality control is essential to ensure proper fitting of the various structural elements. The effects of temperature should be considered in design. To prevent development of cracks under fatigue and earthquake loads the connections and in particular the welds should be designed and detailed properly. Special steels and protective measures for corrosion and fire are available and the designer should be familiar with the options available.

1.2 Metallurgy of steel

When carbon in small quantities is added to iron, 'Steel' is obtained. Since the influence of carbon on mechanical properties of iron is much larger than other alloying elements. The atomic diameter of carbon is less than the interstices between iron atoms and the carbon goes into solid solution of iron. As carbon dissolves in the interstices, it distorts the original crystal lattice of iron.

This mechanical distortion of crystal lattice interferes with the external applied strain to the crystal lattice, by mechanically blocking the dislocation of the crystal lattices. In other words, they provide mechanical strength. Obviously adding more and more carbon to iron (upto solubility of iron) results in more and more distortion of the crystal lattices and hence provides increased mechanical strength. However, solubility of more carbon influences negatively with another important property of iron called the 'ductility' (ability of iron to undergo large plastic deformation). The a-iron or ferrite is very soft and it flows plastically. Hence we see that when more carbon is added, enhanced mechanical strength is obtained, but ductility is reduced. Increase in carbon content is not the only way, and certainly not the desirable way to get increased strength of steels. More amount of carbon causes problems during the welding process. We will see later, how both mechanical strength and ductility of steel could be improved even with low carbon content. The iron-carbon equilibrium diagram is a plot of transformation of iron with respect to carbon content and temperature. This diagram is also called iron-iron carbon phase diagram (Fig. 1.2). The important metallurgical terms, used in the diagram, are presented below.

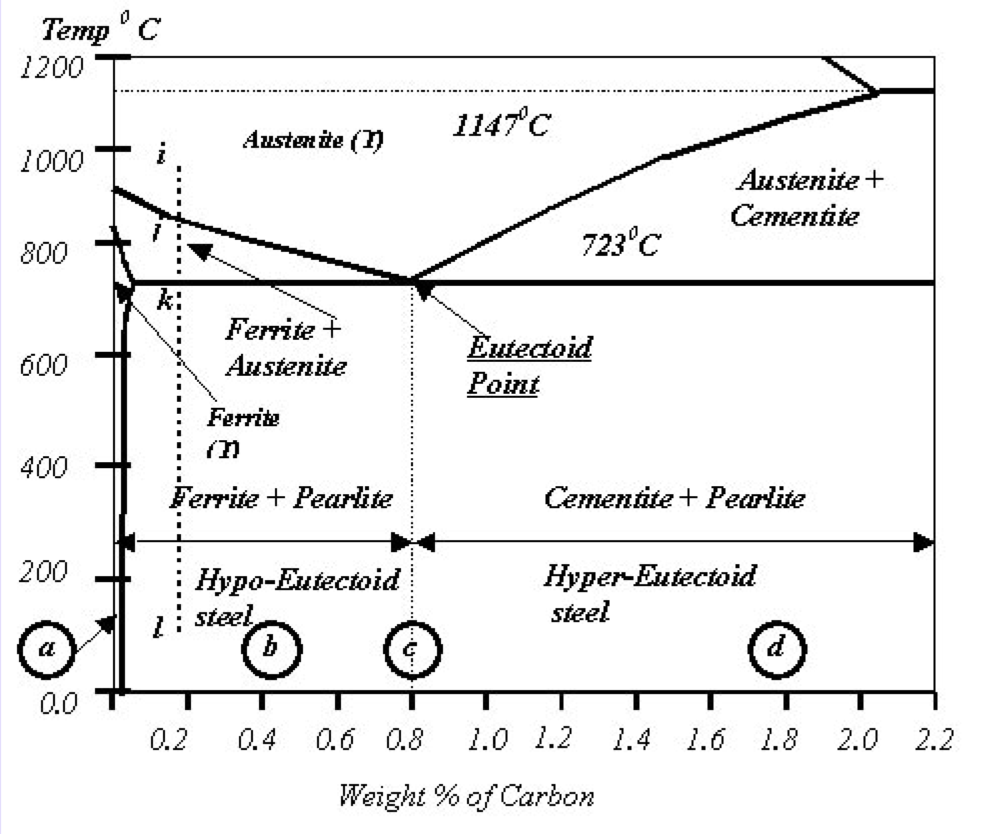

Fig 1.2. The iron – iron carbon phase diagram

Ferrite (α): Virtually pure iron with body centered cubic crystal structure (bcc). It is stable at all temperatures upto 9100C. The carbon solubility in ferrite depends upon the temperature; the maximum being 0.02% at 723^{o}C.

Cementite: Iron carbide (Fe3C), a compound iron and carbon containing 6.67% carbon by weight.

Pearlite: A fine mixture of ferrite and cementite arranged in lamellar form. It is stable at all temperatures below 723^{o}C.

Austenite (γ): Austenite is a face centred cubic structure (fcc). It is stable at temperatures above 723^{o}C depending upon carbon content. It can dissolve upto 2% carbon.

The maximum solubility of carbon in the form of Fe3C in iron is 6.67%. Addition of carbon to iron beyond this percentage would result in formation of free carbon or graphite in iron. At 6.67% of carbon, iron transforms completely into cementite or Fe3C (Iron Carbide). Generally carbon content in structural steels is in the range of 0.12-

0.25%. Upto 2% carbon, we get a structure of ferrite + pearlite or pearlite + cementite depending upon whether carbon content is less than 0.8% or beyond 0.8%. Beyond 2% carbon in iron, brittle cast iron is formed.

1.2.1 The structural steels or ferrite – pearlite steels

The iron-iron carbide portion of the phase diagram that is of interest to structural engineers is shown in Fig.1.2. The phase diagram is divided into two parts called “hypoeutectoid steels” (steels with carbon content to the left of eutectoid point [0.8% carbon]) and “hyper eutectoid steels” which have carbon content to the right of the eutectoid point. It is seen from the figure that iron containing very low percentage of carbon (0.002%) called very low carbon steels will have 100% ferrite microstructure (grains or crystals of ferrite with irregular boundaries) as shown in Fig 1.2. Ferrite is soft and ductile with very low mechanical strength. This microstructure at ambient temperature has a mixture of what is known as ‘pearlite and ferrite’ as can be seen in Fig. 1.2. Hence we see that ordinary structural steels have a pearlite + ferrite microstructure. However, it is important to note that steel of 0.20% carbon ends up in pearlite + ferrite microstructure, only when it is cooled very slowly from higher temperature during manufacture. When the rate of cooling is faster, the normal pearlite + ferrite microstructure may not form, instead some other microstructure called bainite or martensite may result.

We will consider how the microstructures of structural steel are formed by the slow cooling at 0.2% carbon. At about 900^{o}C, this steel has austenite microstructure. This is shown as point ‘i’ in Fig. 1.2. When steel is slowly cooled, the transformation would start on reaching the point ‘j’. At this point, the alloy enters a two-phase field of ferrite and austenite. On reaching the point, ferrite starts nucleating around the grain boundaries of austenite as shown in Fig. 1.3(a). By slowly cooling to point 'k', the ferrite grains grow in size and diffusion of carbon takes place from ferrite regions into the austenite regions as shown in Fig. 1.3(b), since ferrite cannot retain carbon above 0.002% at room temperature.

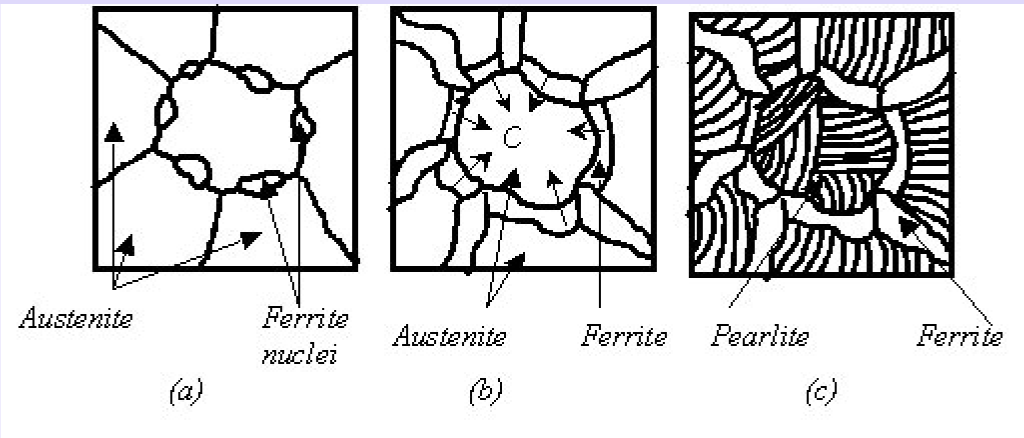

Fig 1.3. Different stages of formation of pearlite

At this point it is seen that a network of ferrite crystals surrounds each austenite grain. On slow cooling to point 'l' the remaining austenite gets transformed into 'pearlite' as shown in Fig 1.3(c). Pearlite is a lamellar mixture of ferrite and cementite. The amount of 'pearlite' for a given carbon content is usually calculated using the lever rule assuming 0% carbon in ferrite as given below:

$$\text{Volume fraction of Pearlite} = \frac{\text{\% of Carbon}}{\text{0.8\% of Carbon}}$$

For example for microstructure of a 0.2% carbon steel would consist of a quarter of pearlite and three- quarters of ferrite. As explained earlier, ferrite is soft and ductile and pearlite is hard and it imparts mechanical strength to steel. The higher the carbon content, the higher would be the pearlite content and hence higher mechanical strength. Conversely, when the pearlite content increases, the ferrite content decreases and hence the ductility is reduced.

1.2.2 Strengthening structural steels

Cooling rate of steel from austenite region to room temperature produces different microstructures, which impart different mechanical properties. In the case of structural steels, the (pearlite + ferrite) microstructure is obtained after austenitising, by cooling it very slowly in a furnace. This process of slow cooling in a furnace is called 'annealing'. As, mentioned in the earlier section, the formation of pearlite, which is responsible for mechanical strength, involves diffusion of carbon from ferrite to austenite. In the annealing process sufficient time is given for the carbon diffusion and other transformation processes to get completed. Hence by full annealing we get larger size pearlite crystals as shown in the cooling diagram in Fig. It is very important to note that the grain size of crystal is an important parameter in strengthening of steel. The yield strength of steel is related to grain size by the equation

$$f_y = f_0 + \frac{k}{\sqrt{d}} \qquad \textbf{\textit{(1.1)}}$$

Where f_y is the yield strength, f_0 is the yield strength of very large isolated crystals (for mild steel this is taken as 5 N/mm^2) and 'k' is a constant, which for mild steel is 38 $N/mm^{3/2}$. From Eq.1.1 we see that decreasing the grain size could enhance the yield strength. We will see in the following section as to how this reduction of grain size could be controlled. The grain size has an influence both in the case of mechanical strength and the temperature range of the ductile-brittle transition (temperature at which steel would become brittle from a ductile behaviour). When steel is fully annealed, there is enough time for the diffusion or shuffling of carbon atoms and larger crystallization is possible. However, if we increase the cooling rate, then transformation that generally needs a specified time, would not keep up with the falling temperature. When we normalise (cool in air) steel, we obtain a small increase in the ferrite content and a finer lamellar pearlite. Since pearlite is responsible for mechanical strength, decrease in its grain size we get improved mechanical strength. Hence we see that another method of increasing the mechanical strength of steel is by normalising.

Structural steel sections are produced by hot rolling process, which involves the temperature range of austenite. During rolling at this high temperature, the heavy mechanical deformation results in finer size grains. In addition to that, rolling at the temperature of austenite, they are allowed to cool in air (normalising) and hence both the procedures aid the formation of smaller size crystals and hence increased mechanical strength.

1.2.3 Rapid cooling of steels

In the earlier section we saw that steel is made to under-cool by normalizing (by giving lesser cooling time than required by the equilibrium state of the constitutional diagram), it results in finer microstructure. However, if we cool steel very rapidly, say quenching in cold water, there is insufficient time for the shuffling or diffusion of carbon atoms and hence the formation of ferrite + pearlite is prevented. However, such a fast cooling results in 'martensite'. Slightly less rapid cooling could result in a product called 'bainite' which is dependent on the composition of steel. Bainite is formed above a temperature of about 300°C and between a cooling rate of 8.4°C/sec to 0.0062°C/sec. Martensite is formed by rapid cooling rate less than 8.4°C/sec. Very slow cooling, say full annealing does not form both Martensite and Bainite.

Martensite is very hard and less ductile. Martensitic structure is not desirable in structural steel sections used in construction, because its welding becomes very difficult. However, high strength bolts and some other important accessories have predominantly martensitic structure. The hardness of martensite is a function of carbon content. When martensite is heated to a temperature of 600°C it softens and the toughness is improved. This process of reheating martensite is called tempering. This process of quenching and tempering results in very many varieties of steel depending upon the requirement for hardness, wear resistance, strength and toughness.

1.2.4 Inclusions and alloying elements in steel

Steel contains impurities such as phosphorous and sulphur and they eventually form phosphides and sulphides which are harmful to the toughness of the steel. Hence it is desirable to keep these elements less than 0.05%. Phosphorous could be easily removed compared to sulphur. If manganese (Mn) is added to steel, it forms a less harmful manganese sulphide (MnS) rather than the harmful iron sulphide. Sometimes calcium, cerium, and other rare earth elements are added to the refined molten steel. They combine with sulphur to form less harmful elements. Steel treated this way has good toughness and such steels are used in special applications where toughness is the criteria. The addition of manganese also increases the under cooling before the start of the formation of ferrite+ pearlite. This gives fine-grained ferrite and more evenly divided pearlite. Since the atomic diameter of manganese is larger that the atomic diameter of iron, manganese exists as 'substitutional solid solution' in ferrite crystals, by displacing the smaller iron atoms. This improves the strength of ferrite because the distortion of crystal lattice due to the presence of manganese blocks the mechanical movement of the crystal lattices. However, manganese content cannot be increased unduely, as it might become harmful. Increased manganese content increases the formation of martensite and hence hardness and raises its ductile to brittle transition temperature (temperature at which steel which is normally ductile becomes brittle). Because of these reasons, manganese is restricted to 1.5% by weight. Based on the manganese content, steels are classified as carbon-manganese steels (Mn>1%) and carbon steels (Mn<1%). In recent years, micro alloyed steels or high strength low alloy (HSLA) steels have been developed. They are basically carbon manganese steels in which small amounts of aluminium, vanadium, mobium or other elements are used to help control the grain size.

These steels are controlled rolled and/or controlled cooled to obtain fine grain size. They exhibit a best combination of strength and toughness and also are generally weldable without precautions such as preheating or post heating. Sometimes 0.5% molybdenum is added to refine the lamellar spacing in pearlite, and to make the pearlite evenly distributed. Today steel with still higher performance are being developed all over the world to meet the following specifications such as: (a) high strength with yield strength of 480 MPa and 690 MPa, (b) excellent weldability without any need for preheating, (c) extremely high toughness with charpy V notch values of 270 N-m @ 23°C compared with current bridge design requirement of 20 N.-m @ 23°C, and (d) corrosion resistance comparable to that of weathering steel. (The terminology used above has been discussed later in this chapter). The micro alloyed steels are more expensive than ordinary structural steels, however, their strength and performance outweighs the extra cost.

Some typical structural steels with their composition range and properties and their relevant codes of practice, presently produced in India, are given in Tables 1.1. These steels are adequate in many structural applications but from the perspective of ductile response, the structural engineer in cautioned against using unfamiliar steel grades, without checking the producer supplied properties. Weldability of steel is closely related to the amount of carbon in steel. Weldability is also affected by the presence of other elements. The combined effect of carbon and other alloying elements on the weldability is given by “carbon equivalent value (Ceq)”, which is given by

$$C_{eq} = \%C + \%Mn/6 + (\%Cr + \%Mo + \%V)/5 + (\%Ni + \%Cu)/15$$

The steel is considered to be weldable without preheating, if Ceq < 0.42%. However, if carbon is less than 0.12% then Ceq can be tolerated upto 0.45%.

Table 1.1 Chemical composition of some typical structural steels

Type of steel	Designation	IS code:	C	S	Mn	P	Si	Cr		Carbon equivalent
Standard structural steel	Fe410A	2062	0.23	0.50	1.5	0.50	-	-	SK	0.42
	Fe410B	2062	0.22	0.45	1.5	0.45	0.4	-	Sk	0.41
	Fe410C	2062	0.20	0.40	1.5	0.40	0.4	-	K	0.39
Micro alloyed high strength steel	Fe440	8500	0.20	0.50	1.3	0.50	0.45	-	-	0.40
	Fe540	8500	0.20	0.45	1.6	0.45	0.45	-	-	0.44
	Fe590	8500	0.22	0.45	1.8	0.45	0.45	-	-	0.48
K- killed steel SK- Semi Killed steel (Explained in section 1.4.2)										

1.2.5 Stainless steels

In an iron-chromium alloy, when chromium content is increased to about 11%, the resulting material is generally classified as a stainless steel. This is because at this minimum level of chromium, a thin protective passive film forms spontaneously on steel, which acts as a barrier to protect the steel from corrosion. On further increase in chromium content, the passive film is strengthened and achieves the ability to repair itself, if it gets damaged in the corrosive environment. 'Ni' addition in stainless steel improves corrosion resistance in reducing environments such as sulphuric acid. It also changes the crystal structure from bcc to fcc thereby improving its ductility, toughness and weldability. 'Mo' increases pitting and crevice corrosion in chloride environments.

Stainless steel is attractive to the architects despite its high cost, as it provides a combined effect of aesthetics, strength and durability.

Stainless steels are available in variety of finishes and it enhances the aesthetics of the structure. On Life Cycle cost Analysis (LCA), stainless steel works out to be economical in many situations. Increased usage of stainless steel in the construction sector is expected, as awareness on LCA improves among architects and consulting engineers.

1.3 Mechanical properties of steel

1.3.1 Stress – strain behaviour: tensile test

The stress-strain curve for steel is generally obtained from tensile test on standard specimens as shown in Fig.1.4. The details of the specimen and the method of testing is elaborated in IS: 1608 (1995). The important parameters are the gauge length 'L_c' and the initial cross section area S_o. The loads are applied through the threaded or shouldered ends. The initial gauge length is taken as 5.65 $(S_o)^{1/2}$ in the case of rectangular specimen and it is five times the diameter in the case of circular specimen. A typical stress-strain curve of the tensile test coupon is shown in Fig.1.5 in which a sharp change in yield point followed by plastic strain is observed. After a certain amount of the plastic deformation of the material, due to reorientation of the crystal structure an increase in load is observed with increase in strain. This range is called the strain hardening range. After a little increase in load, the specimen eventually fractures. After the failure it is seen that the fractured surface of the two pieces form a cup and cone arrangement. This cup and cone fracture is considered to be an indication of ductile fracture. It is seen from Fig.1.5 that the elastic strain is up to ey followed by a yield plateau between strains ey and esh and a strain hardening range start at esh and the specimen fail at eult where ey, esh and eult are the strains at onset of yielding, strain hardening and failure respectively.

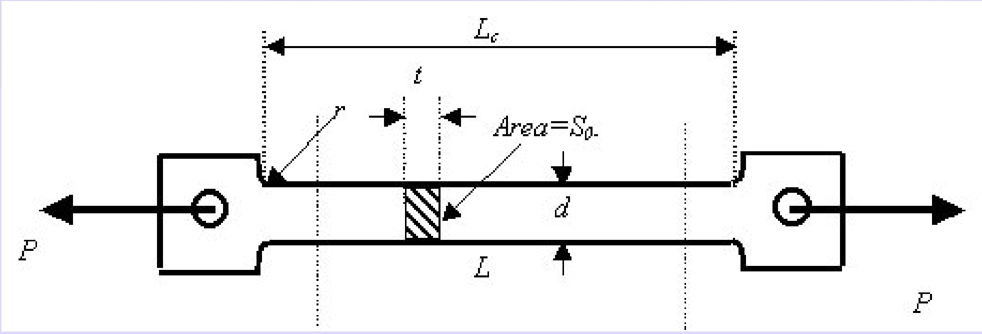

Fig 1.4. Standard tensile test specimen

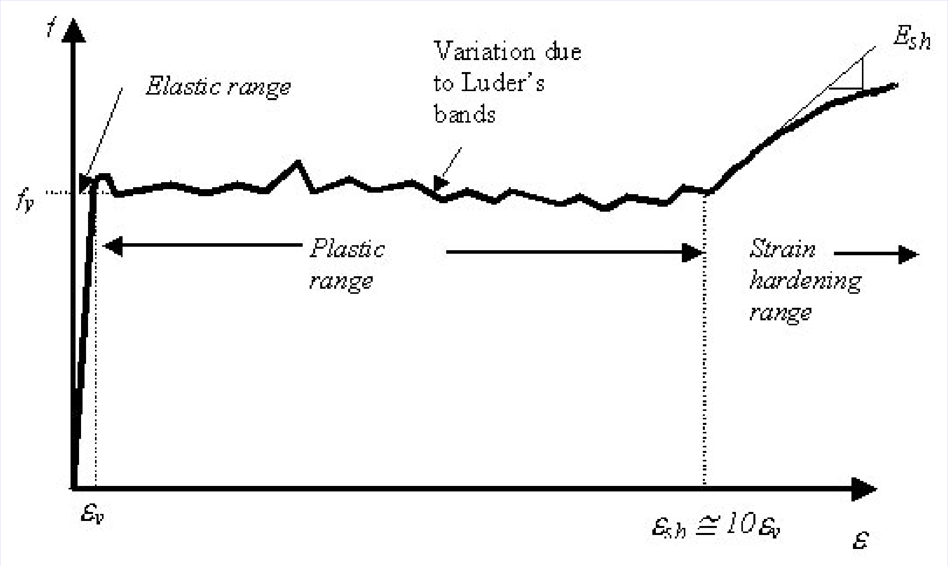

Fig 1.5. Stress strain curve for mild steel

Depending on the steel used, ε_{sh} generally varies between 5 and 15 ε_y, with an average value of 10 ε_y typically used in many applications. For all structural steels, the modulus of elasticity can be taken as 205,000 MPa and the tangent modus at the onset of strain hardening is roughly 1/30th of that value or approximately 6700 MPa.

High strength steels, due to their specific microstructure, do not show a sharp yield point but rather they yield continuously as shown in Fig. 1.6. For such steels the yield stress is always taken as the stress at which a line at 0.2% strain, parallel to the elastic portion, intercepts the stress strain curve. This is shown in Fig. 1.6.

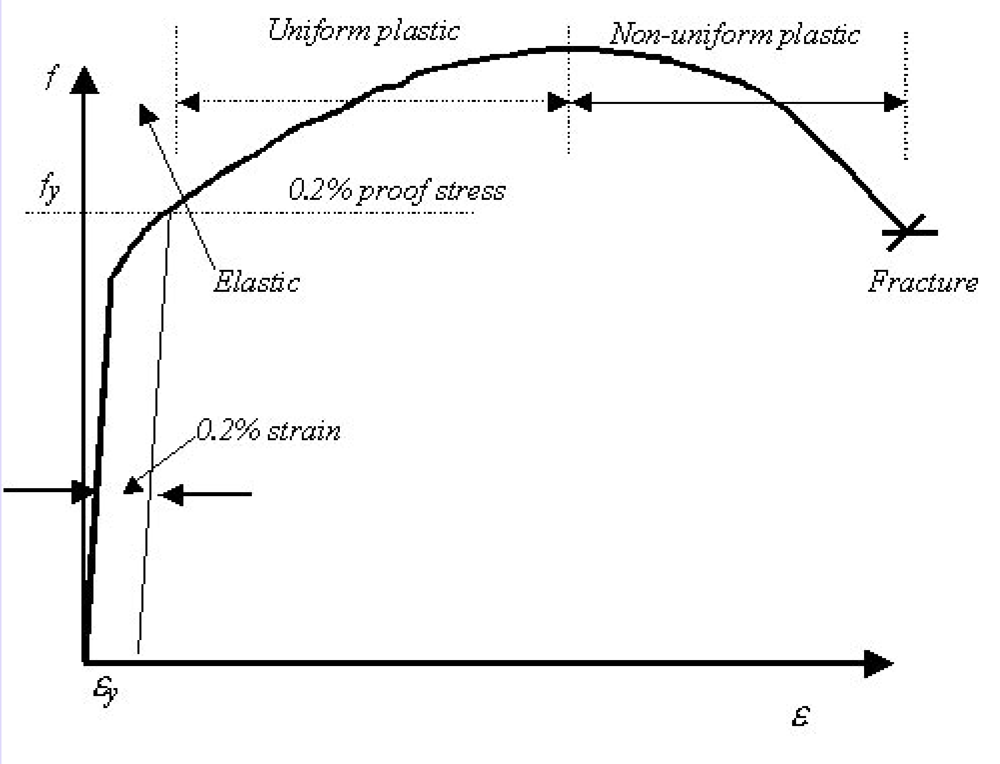

Fig 1.6. Stress strain curve for high strength steel

The nominal stress or the engineering stress is given by the load divided by the original area. Similarly, the engineering strain is taken as the ratio of the change in length to original length.

1.3.2 Hardness

Hardness is regarded as the resistance of a material to indentations and scratching. This is generally determined by forcing an indentor on to the surface. The resultant deformation in steel is both elastic and plastic. There are several methods using which the hardness of a metal could be found out. They basically differ in the form of the indentor, which is used on to the surface. They are presented in Table 1.2.

In all the above cases, hardness number is related to the ratio of the applied load to the surface area of the indentation formed. The testing procedure involves forcing the indentor on to the surface at a particular road. On removal, the size of indentation is measured using a microscope. Based on the size of the indentation, hardness is worked out. For example, Brinell hardness (BHN) is given by the ratio of the applied load and spherical area of the indentation i.e.

Table 1.2 Hardness testing methods and their indentors

Hardness Testing Method		Indentor
(a)	Brinell hardness	Steel ball
(b)	Vickers hardness	Square based diamond pyramids of 135^o included angle
(c)	Rockwell hardness	Diamond core with 120^o included angle
Note: Rockwell hardness testing is not normally used for structural steels.		

$$\mathrm{BHN} = \frac{P}{\pi(d/2)\left[D - \sqrt{D^2 - d^2}\right]} \qquad \textbf{\textit{(1.2)}}$$

Where P is the load, D is the ball diameter, d is the indent diameter. The Vickers test gives a similar hardness value (VHN) as given by

$$\mathrm{VHN} = \frac{1.854P}{L^2} \qquad \textbf{\textit{(1.3)}}$$

Where L is the diagonal length of the indent.

Both the BHN and VHN for steel range from 150 to 190.

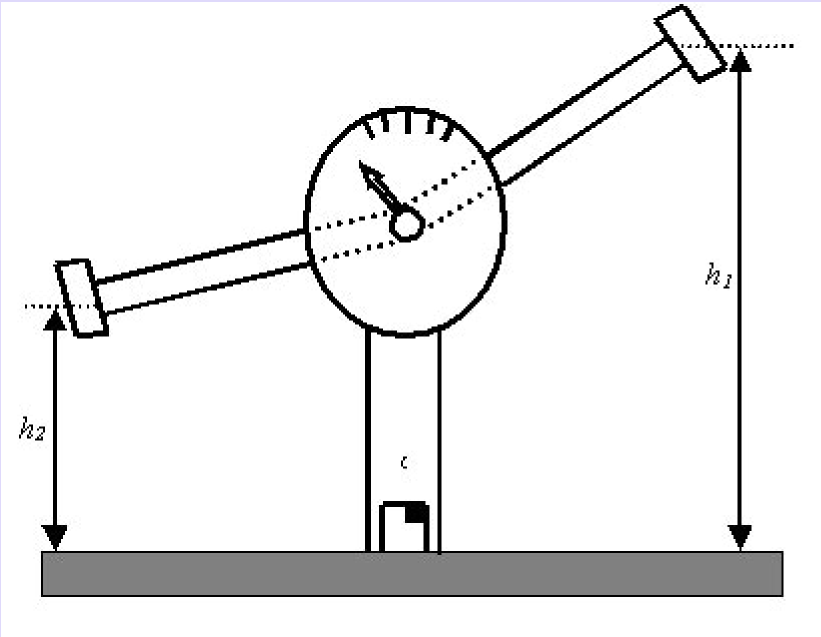

Fig 1.7. Experimental set up for notch toughness test

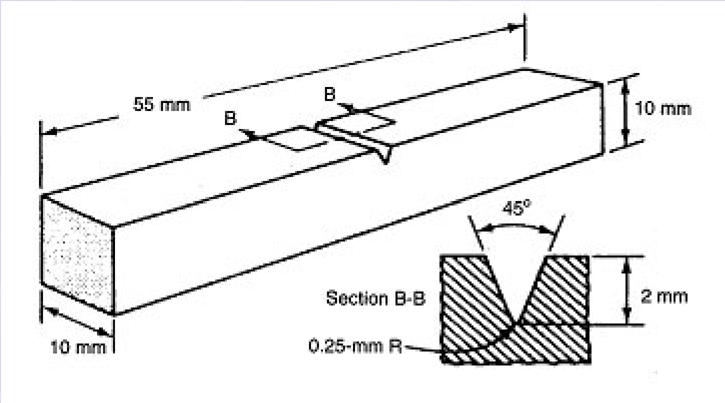

Fig 1.8.Test specimen for notch toughness test

1.3.3 Notch-toughness

There is always a possibility of microscopic cracks in a material or the material may develop such cracks as a result of several cycles of loading. Such cracks may grow rapidly without detection and lead to sudden collapse of the structure. To ensure that this does not happen, materials in which the cracks grow slowly are preferred. Such steels are known as notch-tough steels and the amount of energy they absorb is measured by impacting a notched specimen with a heavy pendulum as in Izod or

Charpy tests. A typical test set up for this is shown in Fig. 1.7 and the specimen used is shown in Fig. 1.8.

The important mechanical properties of steel produced in India are summarized in Table 1.3. In Table 1.3, the UTS represent the minimum guaranteed Ultimate Tensile Strength at which the corresponding steel would fail.

Table 1.3 Mechanical properties of some typical structural steels

Type of steel	Designation	UTS (MPa)	Yield strength(MPa) Thickness (mm)			Elongation Gauge $5.65\sqrt{S_0}$	Charpy V - notch values Joules (min)
			<20	20-40	>40		
Standard structural steel	Fe410A	410	250	240	230	23	27
	Fe410B	410	250	240	230	23	27
	Fe410C	410	250	240	230	23	27
<16 16-40 41-63							
Micro alloyed high strength steel	Fe440	440	300	290	280	22	-
	Fe540	540	410	390	380	20	-
	Fe590	590	450	430	420	20	-

1.4 The manufacturing of steel structures

For design of structures, the structural engineer uses long and flat products. The long products include: angles; channels; joists/beams; bars and rods; while the flat products comprise: plates; hot rolled coils (HRC) or cold rolled coils (CRC)/sheets in as annealed or galvanised condition. The starting material for the finished products is as given below:

· Blooms in case of larger diameter/cross-section long products

· Billets in case of smaller diameter/cross-section long products

· Slabs for hot rolled coils/sheets

· Hot rolled coils in case of cold rolled coils/sheets

· Hot/Cold rolled coils/sheets for cold formed sections

1.4.1 Electric arc or induction furnace route for steel making in mini or midi steel plants

The production process depends upon whether the input material to the steel plant is steel scrap or the basic raw materials i.e. iron ore. In case of former, the liquid steel is produced in Electric Arc Furnace (EAF) or Induction Furnace (IF) and cast into ingots or continuously cast into blooms/billets/slabs for further rolling into desired product. The steel mills employing this process route are generally called as mini or midi steel plants. Since liquid steel after melting contains impurities like sulphur and phosphorus beyond desirable limits and no refining is generally possible in induction furnace. The structural steel produced through this process is inferior in quality. Through refining in EAF, any desired quality (i.e. low levels of sulphur and phosphorus and of inclusion content) can be produced depending upon the intended application. Quality can be further improved by secondary refining in the ladle furnace, vacuum degassing unit or vacuum arc-degassing (VAD) unit.

1.4.2 Iron making and basic oxygen steel making in integrated steel plants

When the starting input material is iron ore, then the steel plant is generally called the integrated steel plant. In this case, firstly hot metal or liquid pig iron is produced in a vertical shaft furnace called the blast furnace (BF). Iron ore, coke (produced by carbonisation of coking coal) and limestone [Fig.] in calculated proportion are charged at the top of the blast furnace. Coke serves two purposes in the BF (Fig). Firstly it provides heat energy on combustion and secondly carbon for reduction of iron ore into iron. Limestone on decomposition at higher temperature provides lime, which combines with silica present in the iron ore to form slag. It also combines with sulphur in the coke and reduces its content in the liquid pig iron or hot metal collected at the bottom of the BF.

The hot metal contains very high level of carbon content around 4%; silicon in the range of 0.5-1.2%; manganese around 0.5%; phosphorus in the range 0.03-0.12%; and somewhat higher level of sulphur around 0.05%. Iron with this kind of composition is highly brittle and cannot be used for any practical purposes. Hot metal is charged in to steel making vessel called LD converter or the Basic Oxygen Furnace (BOF). Open-hearth process is also used in some plants, though it is gradually being phased out [Fig.]. Oxygen is blown into the liquid metal in a controlled manner, which reduces the carbon content and oxidises the impurities like silicon, manganese, and phosphorus. Lime is charged to slag off the oxidised impurities. Ferro Manganese (FeMn), Ferro Silicon (FeSi) and/or Aluminium (Al) are added in calculated amount to deoxidise the liquid steel, since oxygen present in steel will appear as oxide inclusions in the solid state, which are very harmful. Ferro alloy addition also helps to achieve the desired composition. Generally the structural steel contains: carbon in the range 0.10-0.25%; manganese in the range 0.4-1.2%; sulphur 0.025-0.050%; phosphorus 0.025-0.050% depending upon specification and end use. Some micro alloying elements can also be added to increase the strength level without affecting its weldability and impact toughness.

If the oxygen content is brought down to less than 30 parts per million (PPM), the steel is called fully killed, whereas if the oxygen content is around 150 PPM, then the steel is called semi-killed. During continuous casting, only killed steel is used. However, both semi-killed and killed steels are cast in the form of ingots. The present trend is to go in for casting of steel through continuous casting, as it improves the quality, yield as well as the productivity.

1.4.3 Casting and primary/finish rolling

Liquid steel is cast into ingots [Fig.], which after soaking at 1280-1300^{o}C in the soaking pits [Fig] are rolled in the blooming and billet mill into blooms/billets [Fig.] or in slabbing mill into slabs. The basic shapes such as ingots, cast slabs, bloom and billets are shown in Fig.. The blooms are further heated in the reheating furnaces at 1250-1280^{o}C and rolled into billets or to large structural [Fig]. The slabs after heating to similar temperature are rolled into plates in the plate mill. Even though the chemical composition of steel dictates the mechanical properties, its final mechanical properties are strongly influenced by rolling practice, finishing temperature, and cooling rate and subsequent heat treatment.

The slabs or blooms or the billets can directly be continuously cast from the liquid state and thereafter are subjected to further rolling after heating in the reheating furnaces.

In the hot rolling operation the material passes through two rolls where the gap between rolls is lower than the thickness of the input material. The material would be repeatedly passed back and forth through the same rolls several times by reducing the gap between them during each pass. Plain rolls (Fig) are used for flat products such as plate, strip and sheet, while grooved rolls (Fig.1.9) are used in the production of structural sections, rails, rounded and special shapes. The rolling process, in addition to shaping the steel into the required size, improves the mechanical properties by refining the grain size of the material.

Final rolling of structural, bars/rods and HRC/CRC or sheet product is done in respective mills. In case of cold rolled sheets/coils, the material is annealed and skin passed to provide it the necessary ductility and surface finish

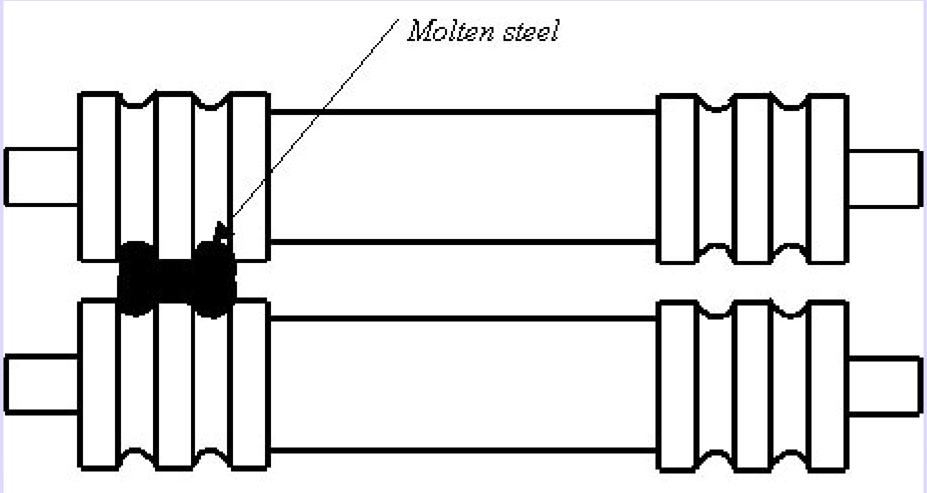

Fig 1.9. Primary rolls for structural shapes

1.4.4 Steel products and steel tables

The long products are normally used in the as-hot-rolled condition. Plates are used in hot rolled condition as well as in the normalised condition to improve their mechanical properties particularly the ductility and the impact toughness.

The structural sections produced in India include open sections such as beams, channels, tees and angles (see Fig.1.10). Closed (hollow) sections such as rectangular and circular tubes are available only in smaller sizes. Solid sections like bars, flats and strips are available. Steel plates are also available in various sizes and thicknesses. These sections are designated in a standard manner with the letters IS indicating that they satisfy the prescriptions of the Indian Standards Specifications (SP 6(1)) followed by the letter indicating the classification and type of section and a number indicating the size of the section. Usually the depth of the section is chosen to indicate its size. The beam sections are classified as ISLB (light), ISJB (junior), ISMB (medium), ISHB (heavy) and ISWB (wide-flanged) sections. Similarly, Channel sections are designated as ISLC, ISMC etc. and angles are designated as ISA followed by the size of each leg and the thickness. Both equal and unequal angles are available. Sometimes two different sections have the same designation but their weight per unit length is slightly different. In such cases, the weight per unit length is also specified as ISMB 600 @ 48.5 kg/m.

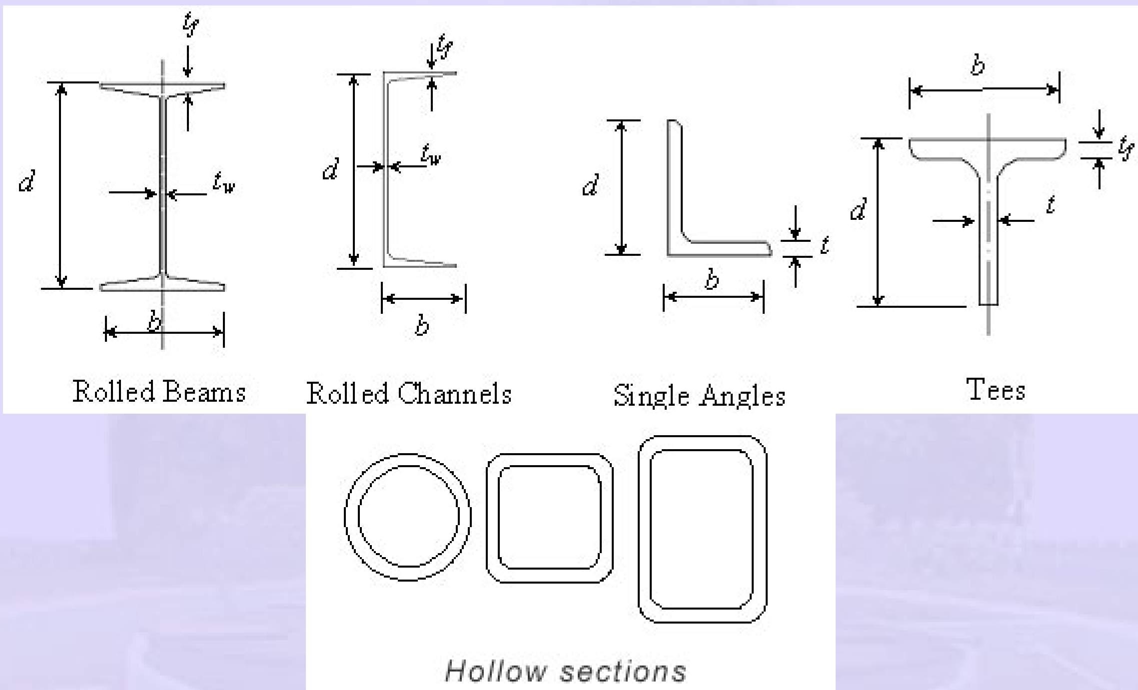

Fig 1.10 Standard shapes of rolled steel sections

The properties of sections, including the geometric details such as average thickness, area, moment of inertia about various axes and preferred location and diameter of holes for bolts etc are tabulated in the steel tables such as SP6(1). Such tables are of great use to designers for selecting a suitable section for a member.

1.4.5 Cold rolling and cold forming

Cold rolling, as the term implies involves reducing the thickness of unheated material into thin sheets by applying rolling pressure at ambient temperature. The common colds rolled products are coils and sheets. Cold rolling results in smoother surface and improved mechanical properties. Cold rolled sheets could be made as thin as 0.3 mm. Cold forming is a process by which the sheets (hot rolled / cold rolled) are folded in to desired section profile by a series of forming rolls in a continuous train of roller sets. Such thin shapes are impossible to be produced by hot rolling. The main advantage of cold-formed sheets in structural application is that any desired shape can be produced. In other words it can be tailor-made into a particular section for a desired member performance. These cold formed sheet steels are basically low carbon steels (<0.1 % carbon) and after rolling these steel are reheated to about 650 - 723oC and at this stage ferrite is recrystalised and also result in finer grain size. Because of the presence of ferrite, the ductility is enhanced. The design of cold-formed steel sections is covered by IS 801.

1.6 Steel structures subjected to fire

In this section a brief review of aspects of structural steel work subjected to fire is given. The strength of all engineering materials reduces as their temperature increases. Steel is no exception. However, a major advantage of steel is that it is incombustible and it can fully recover its strength following a fire, most of the times. During the fire steel absorbs a significant amount of thermal energy. After this exposure to fire, steel returns to a stable condition after cooling to ambient temperature. During this cycle of heating and cooling, individual steel members may become slightly bent or damaged, generally without affecting the stability of the whole structure. From the point of view of economy, a significant number of steel members may be salvaged following a post-fire review of a fire affected steel structure. Using the principle " If the member is straight after exposure to fire – the steel is O.K", many steel members could be left undisturbed for the rest of their service life. Steel members which have slight distortions may be made dimensionally reusable by simple straightening methods and the member may be put to continued use with full expectancy of performance with its specified mechanical properties. The members which have become unusable due to excessive deformation may simply be scrapped. In effect, it is easy to retrofit steel structures after fire. On the other hand concrete exposed to fire beyond say 600oC, may undergo an irreversible degradation in mechanical strength and spolling However it is useful to know the behaviour of steel at higher temperatures and methods available to protect it from damage done to fire. Provisions related to fire protections are given in section 16 of the IS 800 code.

1.6.1 Fire loads and fire resistance

Table 1.5 Fire load on steel structures

Examples of fire load in various structures	
Type of steel structure	Kg wood / m^2
School	15
Hospital	20
Hotel	25

Office	35
Departmental store	35
Textile mill show room	>200

The term ‘fire load’ in a compartment of a structure is the maximum heat that can be theoretically generated by the combustible items and contents of the structure. The fire load could be measured as the weight of the combustible material multiplied by the calorific value per unit weight. Fire load is conveniently expressed in terms of the floor space as MJ/m^2 or $Mcal/m^2$. More often it would be expressed in terms of equivalent quantity of wood and expressed as Kg wood / m^2 (1 Kg wood = 18MJ). The commonly encountered fire loads are presented in Table 1.5. The values are just an indication of the amount of fire load and the values may change from one environment to the other and also from country to country.

The fire ratings of steel structures are expressed in units of time ½, 1, 2, 3 and 4 hours etc. The specified time neither represents the time duration of the real fire nor the time required for the occupants to escape. The time parameters are basically a convenient way of comparative grading of buildings with respect to fire safety. Basically they represent the endurance of structural steel elements under standard laboratory conditions. Fig. 1.18 represents the performance of protected and unprotected steel in a laboratory condition of fire. The rate of heating of the unprotected steel is obviously quite high as compared to the fire-protected steel. We shall see in the following sections that these two types of fire behaviour of steel structure give rise to two different philosophies of fire design. The time equivalence of fire resistance for steel structures or the fire rating could be calculated as

$$T_{eq}\,(\text{Minutes}) = CWQ_f \qquad \textbf{\textit{(1.4)}}$$

Where Qf is the fire load MJ/m^2 which is dependent on the amount of combustible material, ‘W’ is the ventilation factor relating to the area and height and width of doors and windows and ‘C’ is a coefficient related to the thermal properties of the walls, floors and ceiling. As an illustration, the "W" value for a building with large openings could be chosen as 1.5 and for highly insulating materials "C" value could be chosen as 0.09.

We need to know about the mechanical properties of steel at elevated temperatures in the case of fire resistant design of structural steel work. The variations of the non-dimensional modulus of elasticity, yield strength and coefficient of thermal expansion with respect to temperature are shown in Fig1.19. The corresponding equations are given below (Cl.16.5). The variation of modulus of elasticity ratio $\overline{E}$ with respect to the corresponding value at 20°C, with respect to temperature is given by

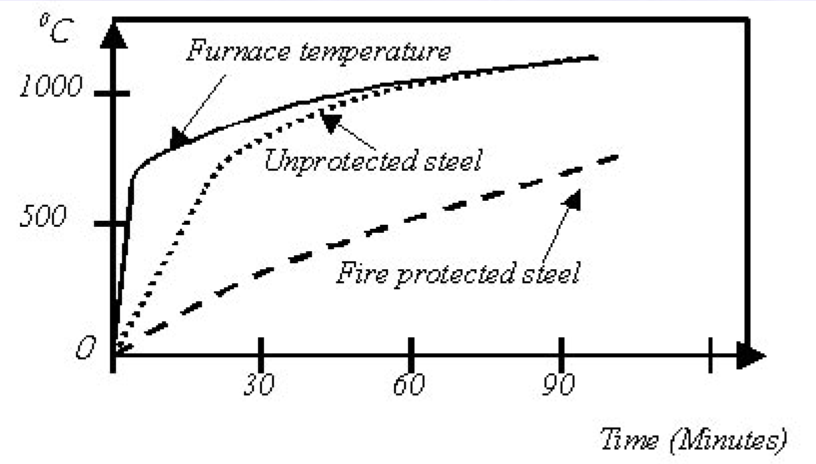

Fig 1.18 Rate of heating of structural steel work

$$\overline{E} = \frac{E(T)}{E(20^0 C)} = 1.0 + \frac{T}{2000 \ln\left[\frac{T}{1100}\right]} \quad \text{for } 0^0 C < T < 600^0 C \qquad \textbf{(1.5)}$$

$$= \frac{690\left(1.0 - \frac{T}{1000}\right)}{T - 53.5} \quad \text{for } 600^0 C < T < 1000^0 C$$

The yield stress of steel remains unchanged up to a temperature of about 215°C and then loses its strength gradually. The yield stress ratio $\overline{f}$ (with respect to yield stress at 20°C) vs. temperature relation is given by

$$\overline{f} = \frac{f_{y(T)}}{f_y(20)} = 1.0 \qquad 0^0 C < T < 215^0 C$$

$$= \frac{905 - T}{690} \qquad 215^0 C < T < 905^0 C \qquad \textbf{(1.6)}$$

Similarly the coefficient of thermal expansion also varies with temperature by a simple relation

$$\alpha(T) = \left(12.0 + \frac{T}{100}\right) x10^{-6} \quad \left({}^{0}C\right)^{-1} \qquad \textbf{(1.7)}$$

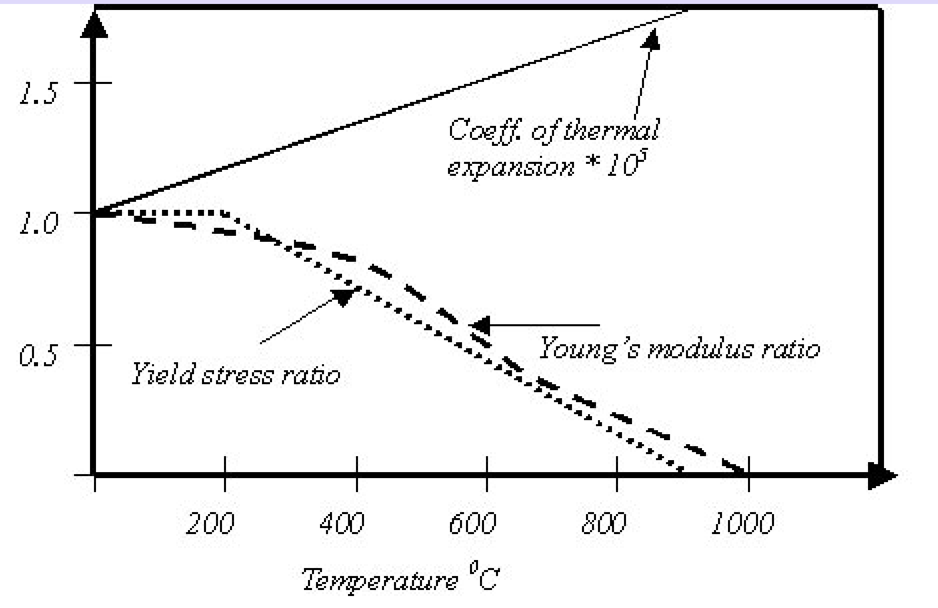

Fig 1.19 Mechanical properties of steel at elevated temperatures

These equations are very useful when one is interested in the analysis of steel structures subjected to fire.

In the codes of practice for steel structures subjected to fire, strength curves are generally provided for structural steel work at elevated temperatures. In these curves the strain at which the strength is assessed in an important parameter. For example the BS: 5950 part 8 has used 1.5% strain as the strain limit as against 2% for Eurocode 3 Part10. A lower strain of 0.5% may be used for columns or components with brittle fire protection materials.

Fire resistant steel

Fire safety in steel structures could also be brought about by the use of certain types of steel, which are called ‘Fire Resistant Steels (FRS)’. These steels are basically thermo-mechanically treated (TMT) steels which perform much better structurally under fire than the ordinary structural steels. These steels have the ferrite – pearlite microstructure of ordinary structural steels but the presence of Molybdenum and Chromium stabilises the microstructure even at 600^{o}C. The composition of fire resistant steel is presented in Table.1.2

Table 1.5 Chemical composition of fire resistant steel

C		Mn	Si	S	P	Mo + Cr
FRS	≤0.20%	≤1.50%	≤0.50%	≤0.040%	≤0.040%	≤1.00%
Mild Steel	≤0.23%	≤1.50%	≤0.40%	≤0.050%	≤0.050%	-

The fire resistant steels exhibit a minimum of two thirds of its yield strength at room temperature when subjected to a heating of about 600^{o}C. In view of this, there is an innate protection in the steel for fire hazards. Fire resistant steels are weldable without pre-heating and are commercially available in the market as joists, channels and angles.

1.6.2 Fire engineering of steel structures

The study of steel structures under fire and its design provision are known as ‘fire engineering’. The basic idea is that the structure should not collapse prematurely without giving adequate time for the occupants to escape to safety. As briefly outlined earlier, there are two ways of providing fire resistance to steel structures. In the first method of fire engineering, the structure is designed using ordinary temperature of the material and then the important and needed members may be insulated against fire. For the purpose of fire protection the concept of 'section factor' is used. In the case of fire behaviour of structures, an important factor which affects the rate of heating of a given section, is the section factor which is defined as the ratio of the perimeter of section exposed to fire (Hp) to that of the cross-sectional area of the member (A). As seen from Fig. 1.20, a section, which has a low (Hp/A) value, would normally be heated at a slower rate than the one with high (Hp/A) value, and therefore achieve a higher fire resistance. Members with low Hp/A value would require less insulation. For example sections at the heavy end (deeper sections) of the structural range have low Hp/A value and hence they have slow heating rates. The section factor can be used to describe either protected or unprotected steel. The section factor is used as a measure of whether a section can be used without fire protection and also to ascertain the amount of protection that may be required. Typical values of Hp of some fire-protected sections are presented in Fig. 1.21.

In the second method of fire engineering, the high temperature property of steel is taken into account in design using the Equations 1.5, 1.6and 1.7. If these are taken into account in the design for strength, at the rated elevated temperature, then no insulation will be required for the member. The structural steel work then may be an unprotected one. There are two methods of assessing whether or not a bare steel member requires fire protection. The first is the load ratio method which compares the 'design temperature' i.e. maximum temperature experienced by the member in the required fire resistance time, and the 'limiting temperatures', which is the temperature at which the member fails.

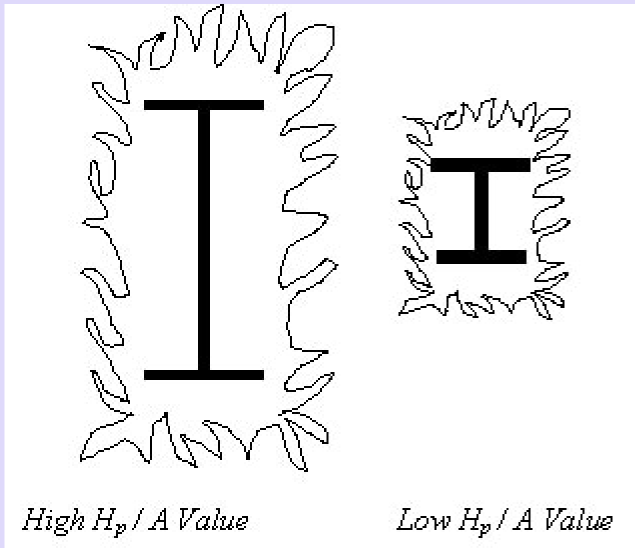

Fig 1.20 The section factor concept

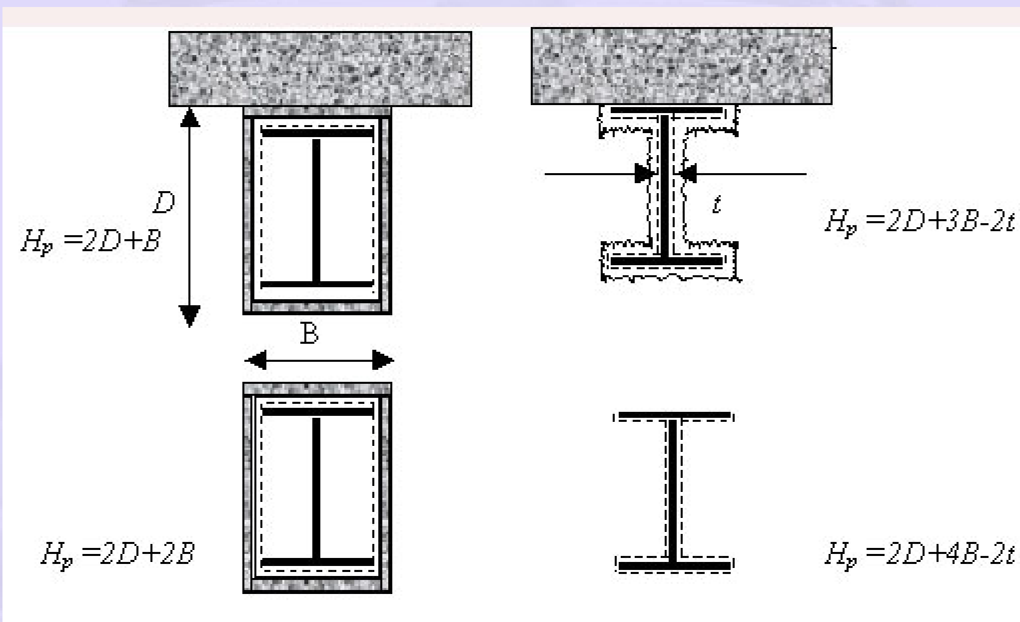

Fig 1.21 Some typical values of HP of fire protected steel sections

The limiting temperatures for various structural members are available in the relevant codes of practice. The load ratio may be defined as:

$$\text{Load ratio} = \frac{\text{Load applied at the fire limit state}}{\text{Load causing the member to fail under normal conditions}}$$

If the load ratio is less than 1, then no fire protection is required. In the second method, which is applicable to beams, the moment capacity at the required fire resistance time is compared with the applied moment. When the moment capacity under fire exceeds the applied moment, no fire protection is necessary.

Methods of fire protection

Fire protection methods are basically dependent on the fire load, fire rating and the type of structural members. The commonly used fire protection methods are briefly enumerated below.

Spray protection: The thickness of spray protection depends on the fire rating required and size of the job. This is a relatively low cost system and could be applied rapidly. However due to its undulating finish, it is usually preferred in surfaces, which are hidden from the view.

Board protection: This is effective but an expensive method. Board protection is generally used on columns or exposed beams. In general no preparation of steel is necessary prior to applying the protection.
Intumescent coating: These coatings expand and form an insulating layer around the member when the fire breaks out. This type of fire protection is useful in visible steelwork with moderate fire protection requirements. This method does not increase the overall dimensions of the member. Certain thick and expensive intumescent coatings will give about 2-hour fire protection. But these type of coatings require blast cleaned surface and a priming coat.

Concrete encasement: This used to be the traditional fire proofing method but is not employed in structures built presently. The composite action of the steel and concrete can provide higher load resistance in addition to high fire resistance. However this method results in increases dead weight loading compared to a protected steel frame. Moreover, carbonation of concrete aids in encouraging corrosion of steel and the presence of concrete effectively hides the steel in distress until it is too late.

1.7 Fatigue of steel structures

A component or structure, which is designed to carry a single monotonically increasing application of static load, may fracture and fail if the same load or even smaller load is, applied cyclically a large number of times. For example a thin rod bent back and forth beyond yielding fails after a few cycles of such repeated bending. This is termed as the 'fatigue failure'. Examples of structures, prone to fatigue failure, are bridges, cranes, offshore structures and slender towers, etc., which are subjected to cyclic loading. The fatigue failure is due to progressive propagation of flaws in steel under cyclic loading. This is partially enhanced by the stress concentration at the tip of such flaw or crack. As we can see from Fig. 1.22, the presence of a hole in a plate or simply the presence of a notch in the plate has created stress concentrations at the points 'm' and 'n'. The stress at these points could be three or more times the average applied stress. These stress concentrations may occur in the material due to some discontinuities in the material itself. These stress concentrations are not serious when a ductile material like steel is subjected to a static load, as the stresses redistribute themselves to other adjacent elements within the structure.

At the time of static failure, the average stress across the entire cross section would be the yield stress as shown in Fig.1.23. However when the load is repeatedly applied or the load fluctuates between tension and compression, the points m, n experience a higher range of stress reversal than the applied average stress. These fluctuations involving higher stress ranges, cause minute cracks at these points, which open up progressively and spread with each application of the cyclic load and ultimately lead to rupture.

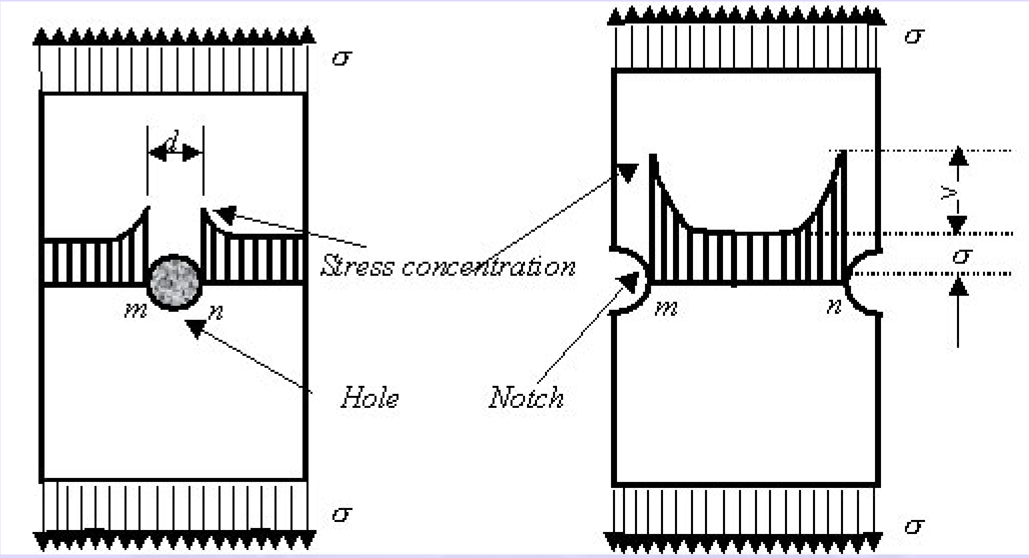

Fig 1.22 Stress concentrations in the presence of notches and holes

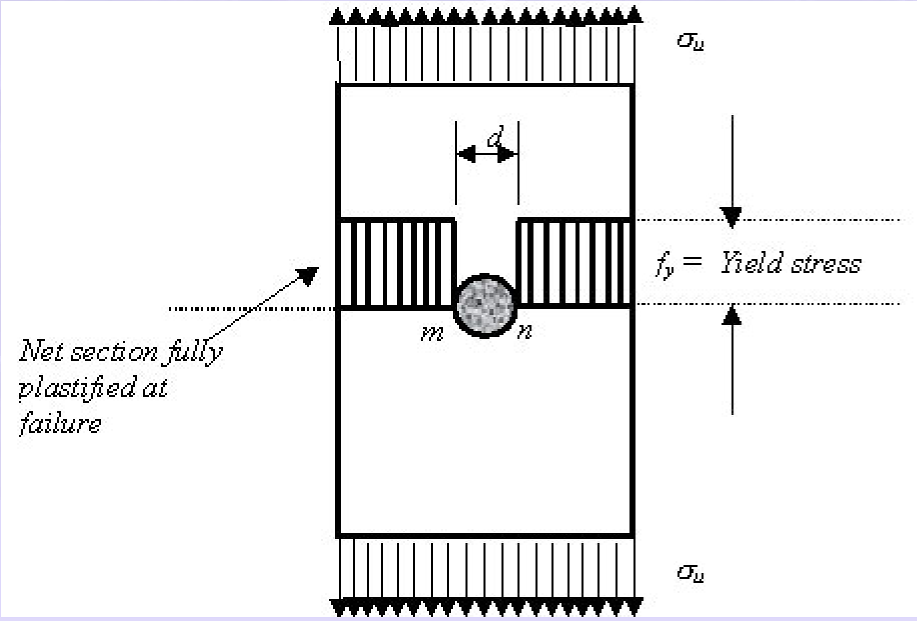

Fig 1.23 Stress pattern at the point of static failure

The fatigue failure occurs after four different stages, namely:

1. Crack initiation at points of stress concentration
2. Crack growth
3. Crack propagation
4. Final rupture

The development of fatigue crack growth and the various stages mentioned above are symbolically represented in Fig. 1.24. Fatigue failure can be defined as the number of cycles and hence time taken to reach a pre-defined or a threshold failure criterion. Fatigue failures are classified into two categories namely the high cycle and low cycle fatigue failures, depending upon the number of cycles necessary to create rupture. Low cycle fatigue could be classified as the failures occurring in few cycles to a few tens of thousands of cycles, normally under high stress/ strain ranges. High cycle fatigue requires about several millions of cycles to initiate a failure. The type of cyclic stresses applied on structural systems and the terminologies used in fatigue resistant design are illustrated in Fig. 1.25.

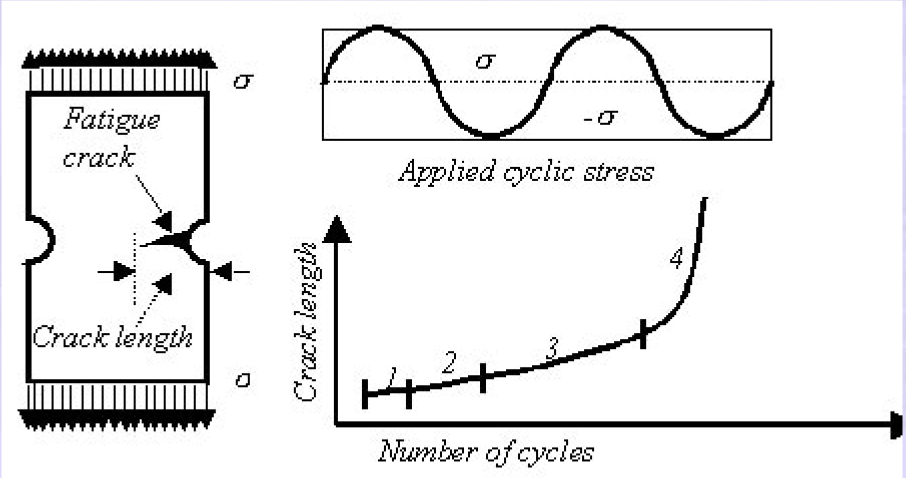

Fig 1.24 Crack growth and fatigue failure under cyclic load

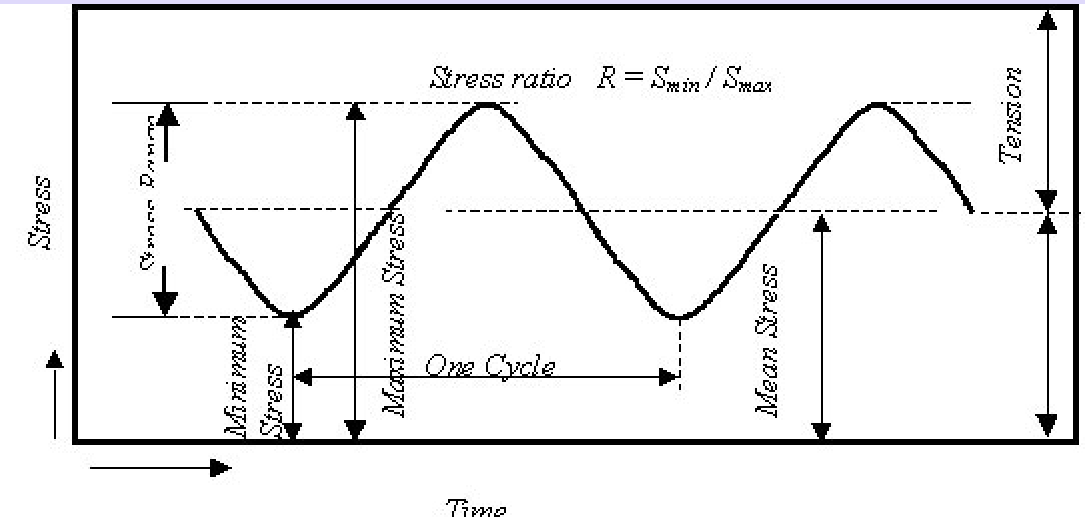

Fig 1.25 Terminology used in fatigue resistant design of structural steel work

S-N curves and fatigue resistant design

The common form of presentation of fatigue data is by using the S-N curve, where the total cyclic stress (S) is plotted against the number of cycles to failure (N) in logarithmic scale. A typical S-N curve is shown in Fig. 1.26.

It is seen from Fig. 1.26 that the fatigue life reduces with respect to increase in stress range and at a limiting value of stress, the curve flattens off. The point at which the S-N curve flattens off is called the 'endurance limit'. To carry out fatigue life predictions, a linear fatigue damage model is used in conjunction with the relevant S-N curve. One such fatigue damage model is that postulated by Wohler as shown in Fig.1.26. The relation between stress and the number of cycles for failure could be written as

$$\log N = \log C - m \log S \qquad \textbf{\textit{(1.8)}}$$

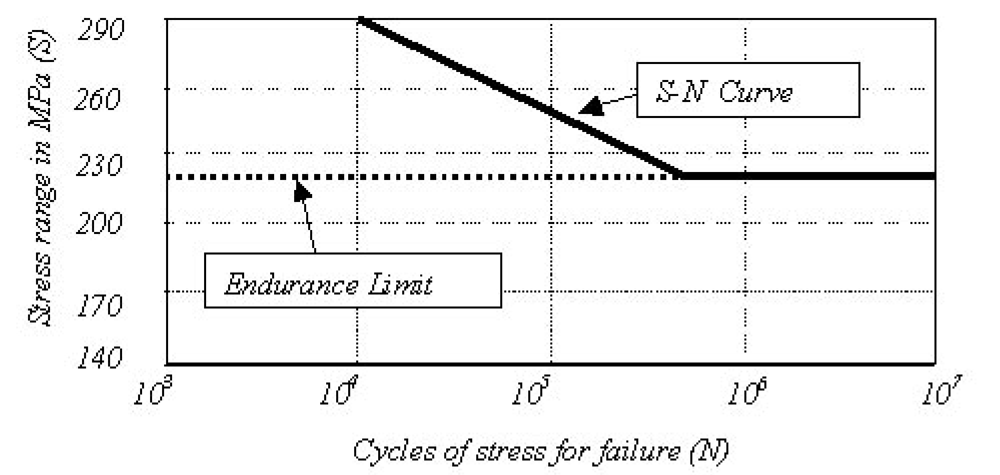

Fig 1.26 S-N diagram for fatigue life assessment

where ‘N’ is the number of cycles to failure, ‘C’ is the constant dependant on detailing category, ‘S’ is the applied constant amplitude stress range and ‘m‘ is the slope of the S-N curve. For the purpose of design it is more convenient to have the maximum and minimum stresses for a given life as the main parameters. For this reason the modified Goodman diagram, as shown in Fig, is mostly used. The maximum stresses are plotted in the vertical ordinate and minimum stresses as abscissa. The line OA represents alternating cycle (R = -1), line OB represents pulsating cycle (R = 0) and OC the static load (R = 1). Different curves for different values of fatigue life ‘N’ can be drawn through point ‘C’ representing the fatigue strength for various numbers of cycles. The vertical distance between any point on the ‘N’ curve and the 450 line OC through the origin represents the stress range. As discussed earlier, the stress range is the important parameter in the fatigue resistant design. Higher the stress range a component is subjected to, lower would be its fatigue life and lower the stress range, higher would be the fatigue life.

It becomes very important to avoid any local structural discontinuities and notches by good design and this is the most effective means of increasing fatigue life. Where a structure is subjected to fatigue, it is important that welded joints are considered carefully. Indeed, weld defects and poor weld details are the major contributors of fatigue failures. The fatigue performance of a joint can be enhanced by the use of techniques such as proper weld geometry, improvements in welding methods and better weld quality control using non-destructive testing (NDT) methods. The following general points are important for the design of a welded structure with respect of fatigue strength: (a) use butt welds instead of fillet welds (b) use double sided welds instead of single sided fillet welds (c) pay attention to the detailing which may cause stress concentration and (d) in very important details subjected to high cyclic stresses use any non-destructive testing (NDT) method to ensure defect free details. From the point of view of fatigue design, the codes of practice classify various structural joints and details depending upon their vulnerability to fatigue cracks.

Each categories denoted by a number which corresponds to the stress for 5 x 106, IS: 800 classifies the detailing in the structural steel work in to several categories depending upon their vulnerability to stress concentrations. It provides S-N curves for all the categories. Using these curves the allowable stress (S) for a given life time (N) may be obtained. (Cl.13.5). The accuracy of any fatigue life calculation is highly dependent on a good understanding of the expected loading sequence during the whole life of a structure. Once a global load pattern has been developed, then a more detailed inspection of particular area of a structure where the effects of loading may be more important called the ‘hot spot stresses’ which are basically the areas of stress concentrations.

2. LIMIT STATE DESIGN

2.1 Introduction to Limit State Design

A Civil Engineering Designer has to ensure that the structures and facilities he designs are (i) fit for their purpose (ii) safe and (iii) economical and durable. Thus safety is one of the paramount responsibilities of the designer. However, it is difficult to assess at the design stage how safe a proposed design will actually be. There is, in fact, a great deal of uncertainty about the many factors, which influence both safety and economy. The uncertainties affecting the safety of a structure are due to

· Uncertainty about loading

· Uncertainty about material strength and

· Uncertainty about structural dimensions and behaviour.

These uncertainties together make it impossible for a designer to guarantee that a structure will be absolutely safe. All that the designer can ensure is that the risk of failure is extremely small, despite the uncertainties.

An illustration of the statistical meaning of safety is given in Fig.2.1. Let us consider a structural component (say, a beam) designed to carry a given nominal load. Bending moments (B.M.) produced by loads are first computed. These are to be compared with the resistance or strength (R.M.) of the beam. But the resistance (R.M.) itself is not a fixed quantity, due to variations in material strengths that might occur between nominally same elements. The statistical distribution of these member strengths (or resistances) will be as sketched in (a).

Similarly, the variation in the maximum loads and therefore load effects (such as bending moment) which different structural elements (all nominally the same) might encounter in their service life would have a distribution shown in (b). ***The uncertainty here is both due to variability of the loads applied to the structure, and also due to the variability of the load distribution through the structure.*** Thus, if a particularly weak structural component is subjected to a heavy load which exceeds the strength of the structural component, clearly failure could occur.

Unfortunately it is not practicable to define the probability distributions of loads and strengths, as it will involve hundreds of tests on samples of components. Normal design calculations are made using a single value for each load and for each material property and taking an appropriate safety factor in the design calculations. The single value used is termed as ***"Characteristic Strength or Resistance"*** and ***"Characteristic Load".***

Characteristic resistance of a material (such as Concrete or Steel) is defined as that value of resistance below which not more than a prescribed percentage of test results may be expected to fall. (For example the characteristic yield stress of steel is usually defined as that value of yield stress below which not more than 5% of the test values may be expected to fall). In other words, this strength is expected to be exceeded by 95% of the cases.

Similarly, ***the characteristic load is that value of the load, which has an accepted probability of not being exceeded during the life span of the structure.*** Characteristic load is therefore that load which will not be exceeded 95% of the time.

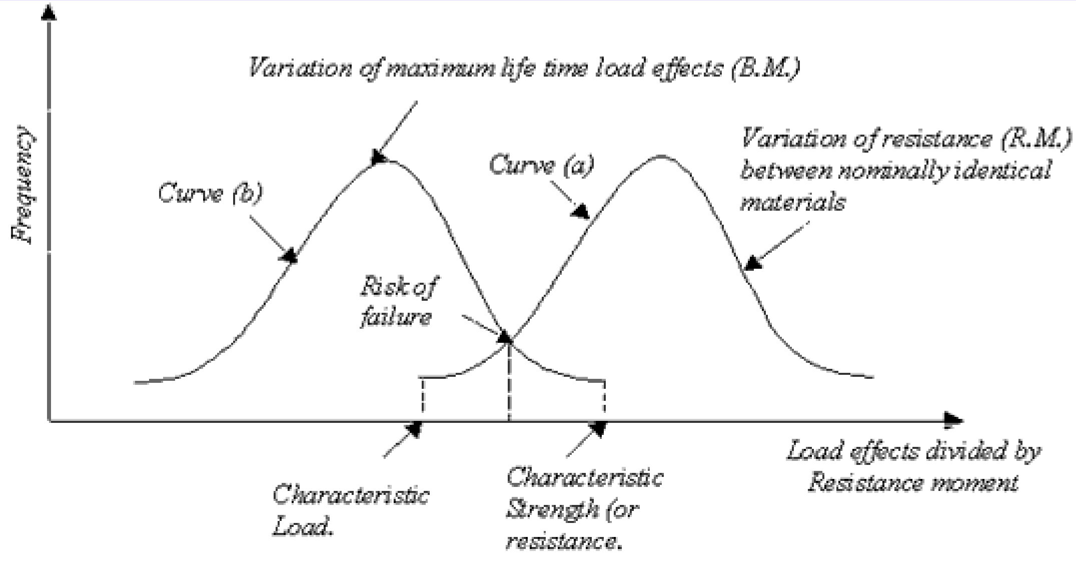

Fig 2.1 Statistical meaning of Safety

Most structural designs are based on experience. If a similar design has been built successfully elsewhere, there is no reason why a designer may not consider it prudent to follow aspects of design that have proved successful, and adopt standardised design rules. As the consequences of bad design can be catastrophic, the society expects designers to explain their design decisions. It is therefore advantageous to use methods of design that have proved safe in the past. Standardised design methods can help in comparing alternative designs while minimising the risk of the cheapest design being less safe than the others. The regulations and guidelines to be followed in design are given in the ***Codes of Practices*** which help in ensuring the safety of structures.

The development of linear elastic theories in the 19^{th} century enabled indeterminate structures to be analysed and the distribution of bending and shear stresses to be computed correctly. In the *Working Stress Method* (WSM) of design, the first attainment of yield stress of steel was generally taken to be the onset of failure as it represents the point from which the actual behaviour will deviate from the analysis results. Also, it was ensured that non-linearity and buckling effects were not present. It was ensured that the stresses caused by the working loads are less than an allowable stress obtained by dividing the yield stress by a factor of safety. The factor of safety represented a margin for uncertainties in strength and load. The value of factor of safety in most cases is taken to be around 1.67.

$$\text{AllowableStress} = \frac{\text{Yield Stress}}{\text{Factor Of Safety}}$$

In general, each member in a structure is checked for a number of different combinations of loads. Some loads vary with time and this should be taken care of. It is unnecessarily severe to consider the effects of all loads acting simultaneously with their full design value, while maintaining the same factor of safety or safety factor. Using the same factor of safety or safety factor when loads act in combination would result in uneconomic designs. A typical example of a set of load combinations is given below, which accounts for the fact that the dead load, live load and wind load are all unlikely to act on the structure simultaneously at their maximum values:

(Stress due to dead load + live load) ≤ allowable stress

(Stress due to dead load + wind load) ≤ allowable stress

(Stress due to dead load + live load + wind) ≤ 1.33 times allowable stress.

In practice there are severe limitations to this approach. The major limitation stems from the fact that yielding at any single point does not lead to failure. This means that the actual factor of safety is generally different from the assumed factor of safety and varies from structure to structure. There are also the consequences of material non-linearity, non-linear behaviour of elements in the post-buckled state and the ability of the steel components to tolerate high local stresses by yielding and redistributing the loads. The elastic theory does not consider the larger safety factor for statically indeterminate structures which exhibit redistribution of loads from one member to another before collapse. These are addresses in a more rational way in Limit State Design.

2.2 Analysis procedures and design philosophy

An improved design philosophy to make allowances for the shortcomings in the *Working Stress Method* was developed in the late 1970's and has been extensively incorporated in design standards and codes. The probability of operating conditions not reaching failure conditions forms the basis of *Limit State Method* (LSM). The Limit State is the condition in which a structure would be considered to have failed to fulfill the purpose for which it was built. In general two limit states are considered at the design stage and these are listed in Table 2.1.

Limit State of Collapse is a catastrophic state, which requires a larger reliability in order to reduce the probability of its occurrence to a very low level. Limit State of Serviceability refers to the limit on acceptable service performance of the structure. Not all the limit states can be covered by structural calculations. For example, corrosion is covered by specifying forms of protection (like painting) and brittle fracture is covered by material specifications, which ensure that steel is sufficiently ductile.

The major innovation in the Limit State Method is the introduction of the partial safety factor format which essentially splits the factor of safety into two factors – one for the material and one for the load. In accordance with these concepts, the safety format used in Limit State Codes is based on probable maximum load and probable minimum strengths, so that a consistent level of safety is achieved.

Table 2.1: Types of limit states

Limit State of Strength	**Limit State of Serviceability**
Yielding, Crushing and Rupture	Deflection
Stability against buckling, overturning and sway	Vibration
Fracture due to fatigue	Fatigue checks (including reparable damage due to fatigue)
Brittle Fracture	Corrosion

Thus, the design requirements are expressed as follows:

$$F_d \leq S_d \qquad \textbf{\textit{(2.1)}}$$

Where F_d = value of internal forces and moments caused by the factored design loads F_d

$F_d = \gamma_f$ * Characteristic Loads.

γ_f= partial safety factor for load (load factor)

S_d = factored design resistance as a function of the material design strength F_d

$F_d = \gamma_m$ * Characteristic strength

γ_m = partial safety factor for material strength,

Both the partial safety factors for load and material are determined on a *'probabilistic basis'* of the corresponding quantity. It should be noted that γ_f makes allowance for possible deviation of loads and also the reduced possibility of all loads acting together. On the other hand γ_m allows for uncertainties of element behaviour and possible strength reduction due to manufacturing tolerances and imperfections in the material. The partial safety factor for steel material failure by yielding or buckling γ_{m0} is given as 1.10 while for ultimate resistance it is given as γ_{m1}=1.25. For bolts and shop welds, the factor is 1.25 and for field welds it is 1.50.

Strength is not the only possible failure mode. Excessive deflection, excessive vibration, fracture etc. also contribute to Limit States. Fatigue is also an important design criterion for bridges, crane girders etc. Thus the following limit states may be identified for design purposes:

Collapse Limit States are related to the maximum design load capacity under extreme conditions. The partial load factors are chosen to reflect the probability of extreme conditions, when loads act alone or in combination. Stability shall be ensured for the structure as a whole and for each of its elements. It includes overall frame stability against overturning and sway, uplift or sliding under factored loads.

Serviceability Limit States are related to the criteria governing normal use. Unfactored loads are used to check the adequacy of the structure. These include Limit State of Deflection, Limit State of Vibration, Limit State of Durability and Limit State of Fire Resistance. Load factor, γ_f, of value equal to unity shall be used for all loads leading to serviceability limit states.

Fatigue Limit State is important where distress to the structure by repeated loading is a possibility. Stress changes due to fluctuations in wind loading normally need not be considered. Fatigue design shall be as per Section 13 of this code. When designing for fatigue, the load factor for action, γ_f, equal to unity shall be used for the load causing stress fluctuation and stress range.

The design considerations for Durability, Fire Resistance and Fatigue have already been discussed in the previous chapter.

The above limit states are provided in terms of partial factors, reflects the severity of the risks. An illustration of partial safety factors suggested in the revised IS: 800 for ultimate load conditions is given in Table 2.2.

The basic load values are specified in IS 875- Except for earthquake load. The dead load which includes the self weight of the member and the weight of any permanent fixture such as a wall can be obtained by knowing the unit weight of the materials. Live loads for residential buildings are given as 3 km/m^2 and the office buildings as 4 km/m^2. Wind load may be worked out based on the basic wind speed at the place and permeability of the build as described in IS 875-part3. The calculation of loads is given in IS 1893-2002.

Table 2.2: Partial safety factors (Cl.5.3.3)

Combination	**Limit State of Strength**					**Limit state of Serviceability**			
	DL	**LL**		**WL/EL**	**AL**	**DL**	**LL**		**WL/EL**
		Leading	Accom-panying				Leading	Accom-panying	
DL+LL+CL	1.5	1.5	1.05	-	-	1.0	1.0	1.0	-
DL+LL+CL +WL/EL	1.2 1.2	1.2 1.2	1.05 0.53	0.6 1.2	-	1.0	0.8	0.8	0.8
DL+WL/EL	1.5 (0.9)*	-	-	1.5	-	1.0	-	-	1.0
DL+ER	1.2 (0.9)	1.2	-	-	-	-	-	-	-
DL+LL+AL	1.0	0.35	0.35	-	1.0	-	-	-	-

* This value is to be considered when the dead load contributes to stability against overturning is critical or the dead load causes reduction in stress due to other loads.

* When action of different live loads is simultaneously considered, the leading live load is one which causes the higher load effects in the member/section and all other live loads are classified as accompanying.

Abbreviations: DL= Dead Load, LL= Imposed Load (Live Loads), WL= Wind Load, SL= Snow Load, CL= Crane Load (Vertical/horizontal), AL=Accidental Load, ER= Erection Load, EL= Earthquake Load.

Note: The effects of actions (loads) in terms of stresses or stress resultants may be obtained from an appropriate method of analysis as in Section 4.

2.3 Other design requirements

Sections normally used in steel structures are I-sections, Channels or angles etc. which are called open sections, or rectangular or circular tubes which are called closed sections. These sections can be regarded as a combination of individual plate elements connected together to form the required shape. The strength of compression members made of such sections depends on their slenderness ratio. Higher strengths can be obtained by reducing the slenderness ratio *i.e.* by increasing the moment of inertia of the cross-section. Similarly, the strengths of beams can be increased, by increasing the moment of inertia of the cross-section. For a given cross-sectional area, higher moment of inertia can be obtained by making the sections thin-walled. However, the buckling of the plate elements of the cross section under compression/shear may take place before the overall column buckling or overall beam failure by lateral buckling or yielding. This phenomenon is called *local buckling.* Thus, local buckling imposes a limit to the extent to which sections can be made thin-walled.

Local buckling has the effect of reducing the load carrying capacity of columns and beams due to the reduction in stiffness and strength of the locally buckled plate elements. It is useful to classify sections based on their tendency to buckle locally before overall failure of the member takes place. The codes also specify the limiting width-thickness ratios $\beta = b/t$ for component plates, which enables the classification to be made. The cross-sections are classified into plastic, compact, semi-compact and slender depending upon their width-thickness ratios $\beta = b/t$ for component plates. This will be discussed in more detail in the chapter on beams.

Fabrication and erection are important aspects to be considered in the design of any steel structure. Fabrication includes the process of straightening, bending, cutting, machining and drilling. The difficult involved in performing these operation will have a major influence on the cost of the structure. Fabrication may be done either entirely in the stags, or entirely in the field or partly in both places. Similarly case of erection also influences the design.

It should be noted that the code gives only guidelines for design which when followed will reduce the probability of a structure collapsing. However, it is the designer's responsibility to ensure that the structure does not collapse due to loads or actions which are special to the particular structure, improper construction and erection techniques, mistakes in calculations etc.

2.4 Summary

Limit state design philosophy takes into account the statistical nature of loads and material strengths, there by providing consistent levels of safety. Its also considers the other requirements such as serviceability and durability. The IS 800-2005(Draft) code advocates limit state design. However, it is the responsibility of the designer to ensure that the structure does not collapse or become unserviceable due to any reason.

2.5 REFERENCES

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3. IS 875-1987 part 1 to 5; Code of Practice for Design Loads (other than earthquake) for building structures, Bureau of Indian Standards, New Delhi.

4. IS 1893-2002; Criteria for Earthquake Resistant Design of Structures; Bureau of Indian Standards, New Delhi.

5. Teaching resources for structural steel design (Volume 1 to 3), INSDAG publication, Calcutta2000.

6. Dowling P.J., Knowles P and Owens GW., ‘ Structural Steel Design’ , The Steel construction Institute, 1988.

3 Connections

3.1 Introduction

Connections form an important part of any structure and are designed more conservatively than members. This is because, connections are more complex than members to analyse, and the discrepancy between analysis and actual behaviour is large. Further, in case of overloading, we prefer the failure confined to an individual member rather than in connections, which could affect many members. Connections account for more than half the cost of structural steelwork and so their design and detailing are of primary importance for the economy of the structure.

The type of connection designed has an influence on member design and so must be decided even prior to the design of the structural system and design of members. For example, in the design of bolted tension members, the net area is calculated assuming a suitable number and diameter of bolts based on experience. Therefore, it is necessary to verify the net area after designing the connection. Similarly in the analysis of frames, the member forces are determined by assuming the connections to be pinned, rigid, or semi-rigid, as the actual behaviour cannot be precisely defined.

Just as members are classified as bending members or axially loaded members depending on the dominant force/moment resisted, connections are also classified into idealised types while designing. But the actual behaviour of the connection may be different and this point should always be kept in mind so that the connection designed does not differ significantly from the intended type. Take for example, the connection of an axially loaded truss member at a joint. If the truss is assumed to be pin jointed, then the member should ideally be connected by means of a single pin or bolt. However, in practice, if the pin or bolt diameter works out to be larger than that possible, more than one bolt will be used. The truss can then be considered pin-jointed only if the bending due to self-weight or other superimposed loads beta joints is negligible. Note that the connection behaviour will also influence the calculation of the effective length for the buckling analysis of struts.

Connection elements consist of components such as cleats, gusset plates, brackets, connecting plates and connectors such as rivets, bolts, pins, and weld. The connections provided in steel structures can be classified as 1) riveted 2) bolted and 3) welded connections. Riveted connections were once very popular and are still used in some cases but will gradually be replaced by bolted connections. This is due to the low strength of rivets, higher installation costs and the inherent inefficiency of the connection. Welded connections have the advantage that no holes need to be drilled in the member and consequently have higher efficiencies. However, welding in the field may be difficult, costly, and time consuming. Welded connections are also susceptible to failure by cracking under repeated cyclic loads due to fatigue which may be due to working loads such as trains passing over a bridge (high-cycle fatigue) or earthquakes (low-cycle fatigue). A special type of bolted connection using High Strength Friction Grip (HSFG) bolts has been found to perform better under such conditions than the conventional black bolts used to resist predominantly static loading. Bolted connections are also easy to inspect and replace. The choice of using a particular type of connection is entirely that of the designer and he should take his decision based on a good understanding of the connection behaviour, economy and speed of construction. Ease of fabrication and erection should be considered in the design of connections. Attention should be paid to clearances necessary for field erection, tolerances, tightening of fasteners, welding procedures, subsequent inspection, surface treatment and maintenance.

3.2 Bolted connections

3.2.1 Connection classification

(a) Classification based on the type of resultant force transferred: The bolted connections are referred to as concentric connections (force transfer in tension and compression member), eccentric connections (in reaction transferring brackets) or moment resisting connections (in beam to column connections in frames).

Ideal concentric connections should have only one bolt passing through all the members meeting at a joint [Fig.3.1 (*a*)]. However, in practice, this is not usually possible and so it is only ensured that the centroidal axes of the members meet at one point [See Fig.3.1 (*b*)].

The Moment connections are more complex to analyse compared to the above two types and are shown in Fig.3.2 (*a*) and Fig.3.2 (*b*). The connection in Fig.3.2 (*a*) is also known as bracket connection and the resistance is only through shear in the bolts.

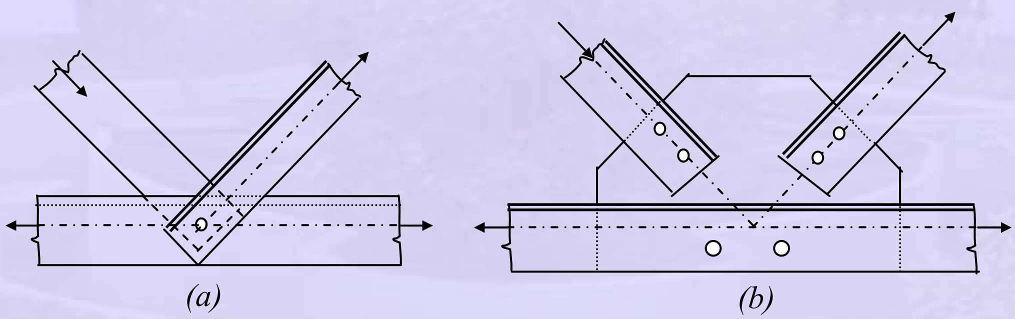


Fig. 3.1 Concentric connections

The connection shown in Fig.3.2 (*b*) is often found in moment resisting frames where the beam moment is transferred to the column. The connection is also used at the base of the column where a base plate is connected to the foundation by means of anchor bolts. In this connection, the bolts are subjected to a combination of shear and axial tension.

(b) Classification based on the type of force experienced by the bolts: The bolted connections can also be classified based on geometry and loading conditions into three types namely, shear connections, tension connections and combined shear and tension connections.

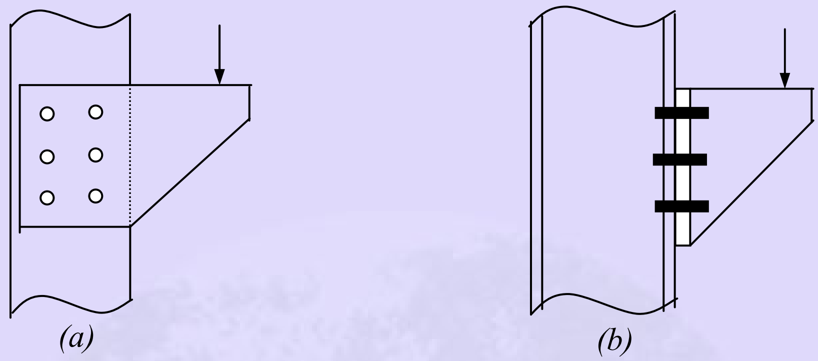
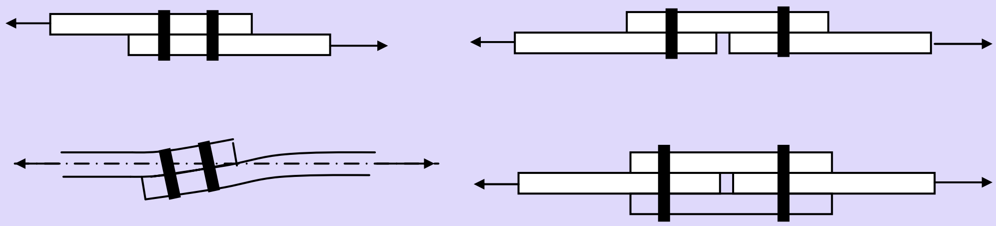


Fig 3.2 Moment connections

Typical shear connections occur as a *lap* or a *butt* joint used in the tension members [See Fig.3.3]. While the lap joint has a tendency to bend so that the forces tend to become collinear, the butt joint requires *cover plates*. Since the load acts in the plane of the plates, the load transmission at the joint will ultimately be through shearing forces in the bolts.

In the case of lap joint or a single cover plate butt joint, there is only one shearing plane, and so the bolts are said to be in *single shear*. In the case of double cover butt joint, there are two shearing planes and so the bolts will be in *double shear*. It should be noted that the single cover type butt joint is nothing but lap joints in series and also bends so that the centre of the cover plate becomes collinear with the forces. In the of single cover plate (lap) joint, the thickness of the cover plate is chosen to be equal to or greater than the connected plates. while in double cover plate (butt) joint, the combined thickness of the cover plates should be equal to or greater than the connected plates.



(a) Lap Connection *(b) Butt Connection*

Fig 3.3 Shear connections

A hanger connection is shown in Fig.3.4 (*a*). In this connection, load transmission is by pure tension in the bolts. In the connection shown in Fig.3.4 (*b*), the bolts are subjected to both tension and shear.

(c) Classification based on force transfer mechanism by bolts: The bolted connections are classified as bearing type (bolts bear against the holes to transfer the force) or friction type (force transfer between the plates due to the clamping force generated by the pre–tensioning of the bolts). The force transfer in either case is discussed in more detail later.

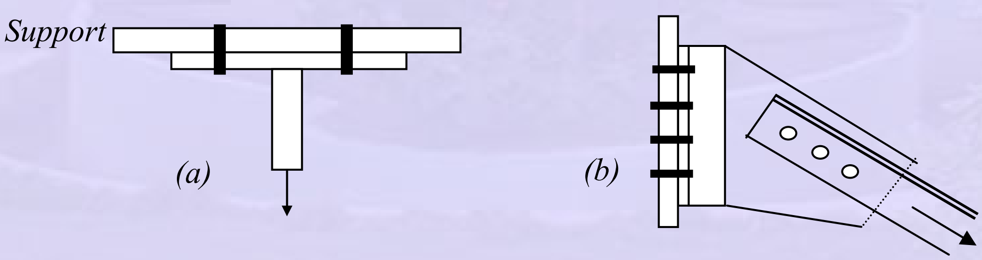


Fig 3.4(a) Tension connection (b) Tension plus shear connection

3.2.2 Bolts and bolting

Bolts used in steel structures are of three types: 1) Black Bolts 2) Turned and Fitted Bolts and 3) High Strength Friction Grip (HSFG) Bolts.

The International Standards Organisation designation for bolts, also followed in India, is given by Grade *x.y*. In this nomenclature, *x* indicates one-tenth of the minimum ultimate tensile strength of the bolt in kgf/mm^2 and the second number, *y*, indicates one-tenth of the ratio of the yield stress to ultimate stress, expressed as a percentage. Thus, for example, grade 4.6 bolt will have a minimum ultimate strength 40 kgf/mm^2 (392 Mpa) and minimum yield strength of 0.6 times 40, which is 24 kgf/mm^2 (235 Mpa).

Black bolts are unfinished and are made of mild steel and are usually of Grade 4.6. Black bolts have adequate strength and ductility when used properly; but while tightening the nut snug tight ("Snug tight" is defined as the tightness that exists when all plies in a joint are in firm contact) will twist off easily if tightened too much. Turned –and-fitted bolts have uniform shanks and are inserted in close tolerance drilled holes and made snug tight by box spanners. The diameter of the hole is about 1.5 to 2.0 mm larger than the bolt diameter for ease in fitting. High strength black bolts (grade 8.8) may also be used in connections in which the bolts are tightened snug fit.]

In these *bearing type of connections*, the plates are in firm contact but may slip under loading until the hole surface bears against the bolt .The load transmitted from plate to bolt is therefore by bearing and the bolt is in shear as explained in the next section. Under dynamic loads, the nuts are liable to become loose and so these bolts are not allowed for use under such loading. In situations where small slips can cause significant effects as in beam splices, black bolts are not preferred. However, due to the lower cost of the bolt and its installation, black bolts are quite popular in simple structures subjected to static loading.

Turned and fitted bolts are available from grade 4.6 to grade 8.8. For the higher grades there is no definite yield point and so 0.2% proof stress is used.

High Strength Friction Grip bolts (HSFG) provide extremely efficient connections and perform well under fluctuating/fatigue load conditions. These bolts should be tightened to their proof loads and require hardened washers to distribute the load under the bolt heads. The washers are usually tapered when used on rolled steel sections. The tension in the bolt ensures that no slip takes place under working conditions and so the load transmission from plate to the bolt is through friction and not by bearing as explained in the next section. However, under ultimate load, the friction may be overcome leading to a slip and so bearing will govern the design.

HSFG bolts are made from quenched and tempered alloy steels with grades from 8.8 to 10.9. The most common are the so-called, general grade of 8.8 and have medium carbon content, which makes them less ductile. The 10.9 grade have a much higher tensile strength, but lower ductility and the margin between the 0.2% yield strength and the ultimate strength is also lower.

The tightening of HSFG bolts can be done by either of the following methods (IS 4000-..):

1. *Turn-of-nut tightening method*: In this method the bolts are first made snug tight and then turned by specific amounts (usually either half or three-fourth turns) to induce tension equal to the proof load (Fig 3.5(a)).

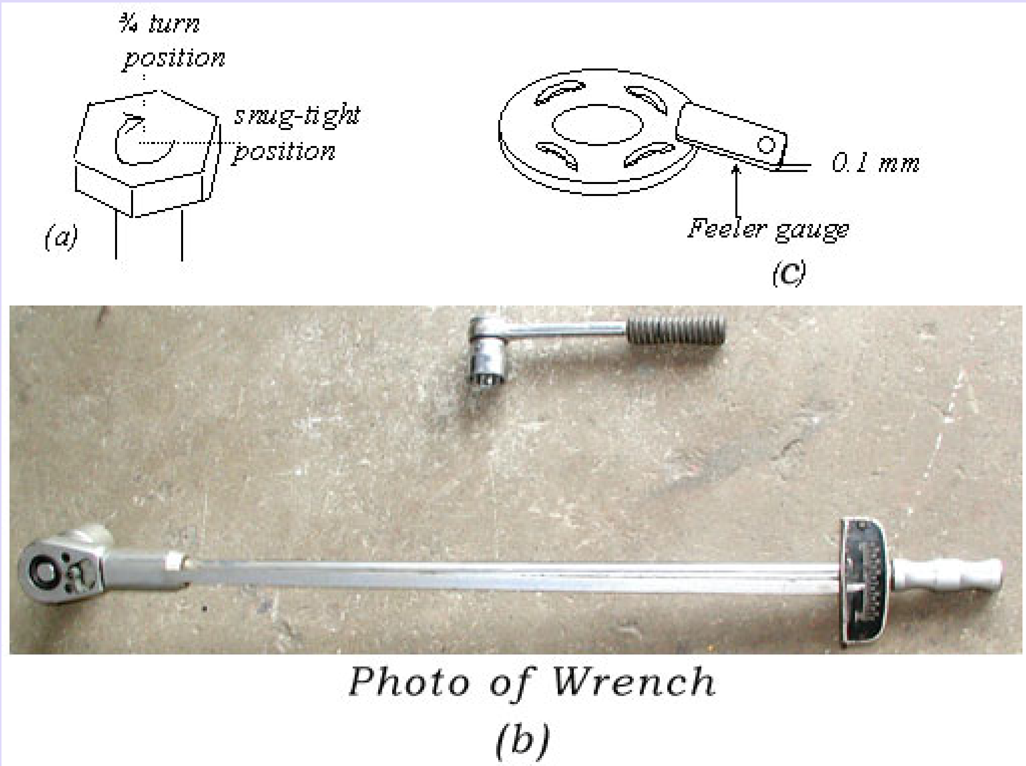
2. *Calibrated wrench tightening method*: In this method the bolts are tightened by a wrench (Fig 3.5(b)) calibrated to produce the required tension.

3. *Alternate design bolt installation*: In this method special bolts are used which indicate the bolt tension. Presently such bolts are not available in India.

4. *Direct tension indicator method*: In this method special washers with protrusions are used [Fig.3.5(*c*)]. As the bolt is tightened, these protrusions are compressed and the gap produced by them gets reduced in proportion to the load. This gap is

measured by means of a feeler gauge, consisting of small bits of steel plates of varying thickness, which can be inserted into the gap.

Fig 3.5 Tightening of HSFG bolts



Since HSFG bolts under working loads, do not rely on resistance from bearing, holes larger than usual can be provided to ease erection and take care of lack-of-fit. Typical hole types that can be used are standard, extra large and short or long slotted. These are shown in Fig.3.6. However the type of hole will govern the strength of the connection.

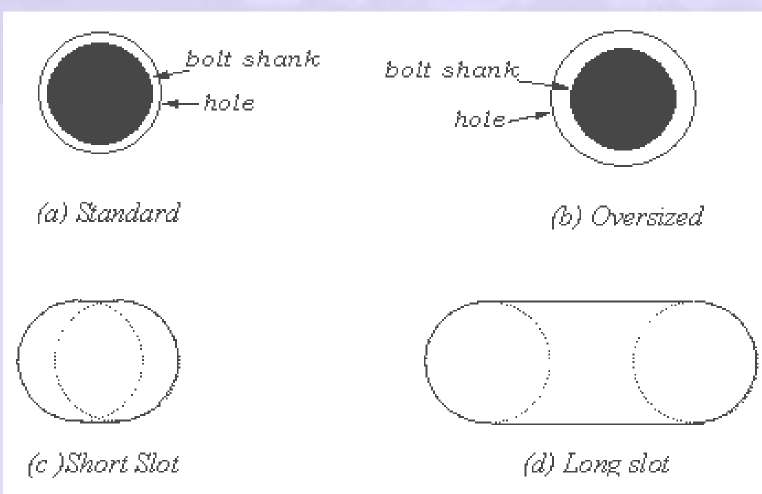


Fig. 3.6 Hole types for HSFG bolts

Holes must also satisfy pitch and edge/end distance criteria (Cl.10.2). A minimum pitch is usually specified for accommodating the spanner and to limit adverse interaction between the bearing stresses on neighbouring bolts. A maximum pitch criterion takes care of buckling of the plies under compressive loads.

3.2.3 Shear connections with bearing type bolts

In this section the force transfer mechanisms of bearing and friction type of bolted connections are described. This would help in identifying the modes of failure discussed in the next section.

3.2.3.1 Force transfer of bearing type bolts

Fig. 3.7 shows the free body diagram of the shear force transfer in bearing type of bolted connection. It is seen that tension in one plate is equilibrated by the bearing stress between the bolt and the hole in the plate. Since there is a clearance between the bolt and the hole in which it is fitted, the bearing stress is mobilised only after the plates slip relative to one another and start bearing on the bolt .The section *x-x* in the bolt is critical section for shear. Since it is a lap joint there is only one critical section in shear (single shear) in the bolt .In the case of butt splices there would be two critical sections in the bolt in shear (double shear), corresponding to the two cover plates.

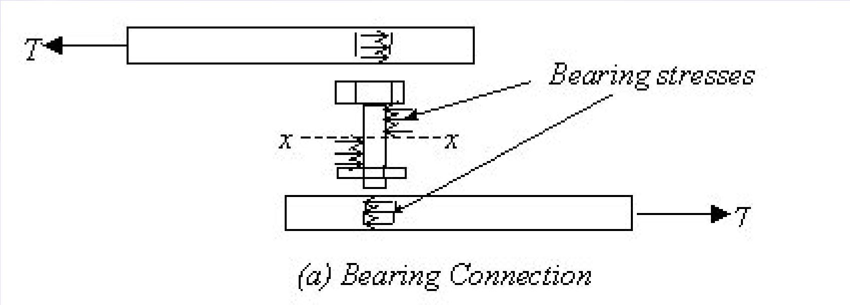


Fig. 3.7 Shear transfer by bearing type bolt

3.2.3.2 Design shear strength of bearing type bolts

The failure of connections with bearing bolts in shear involves either bolt failure or the failure of the connected plates. In this section, the failure modes are described along with the codal provisions for design and detailing shear connections.

In connections made with bearing type of bolts, the behaviour is linear until i) yielding takes place at the net section of the plate under combined tension and flexure or ii) shearing takes place at the bolt shear plane or iii) failure of bolt takes place in bearing, iv) failure of plate takes place in bearing and v) block shear failure occurs. Of these, i) and v) are discussed in the chapter on tension members. The remaining three are described below.

1. Shearing of bolts: The shearing of bolts can take place in the threaded portion of the bolt and so the area at the root of the threads, also called the tensile stress area A_t, is taken as the shear area A_s. Since threads can occur in the shear plane, the area A_e for resisting shear should normally be taken as the net tensile stress area, *An,* of the bolts. The shear area is specified in the code and is usually about 0.8 times the shank area. However, if it is ensured that the threads will not lie in the shear plane then the full area can be taken as the shear area.

A bolt subjected to a factored shear force (V_{sb}) shall satisfy

$V_{sb} \leq V_{nsb} / \gamma_{mb}$

Where

V_{nsb} = nominal shear capacity of a bolt, calculated as follows:

γ_{mb} = 1.25

$$V_{nsb} = \frac{f_u}{\sqrt{3}}\left(n_n A_{nb} + n_s A_{sb}\right) \qquad \textbf{(3.1)}$$

Where

f_u = ultimate tensile strength of a bolt

n_n = number of shear planes with threads intercepting the shear plane

n_s = number of shear planes without threads intercepting the shear plane

A_{sb} = nominal plain shank area of the bolt

A_{nb} = net tensile area at threads, may be taken as the area corresponding to root diameter at the thread

For bolts in single shear, either n_n or n_s is one and the other is zero. For bolts in double shear the sum of n_n and n_s is two.

2. Bearing failure: If the connected plates are made of high strength steel then failure of bolt can take place by bearing of the plates onthe bolts. If the plate material is weaker than the bolt material, then failure will occur by bearing of the bolt on the plate and the hole will elongate . The beating area is given by the nominal diameter of the bolt times the combined thickness of the plates bearing in any direction. – A bolt bearing on any plate subjected to a factored shear force (V_{sb}) shall satisfy

$$V_{sb} \leq V_{npb} / \gamma_{mb} \qquad \textbf{(3.2)}$$

Where, γ_{mb} = 1.25

V_{npb} = bearing strength of a bolt, calculated as

$$V_{npb} = 2.5 d t f_u \qquad \textbf{(3.3)}$$

Where

f_u = smaller of the ultimate tensile stress of the bolt and the ultimate tensile stress of the plate

d = nominal diameter of the bolt

t = summation of the thicknesses of the connected plates experiencing bearing stress in the same direction.

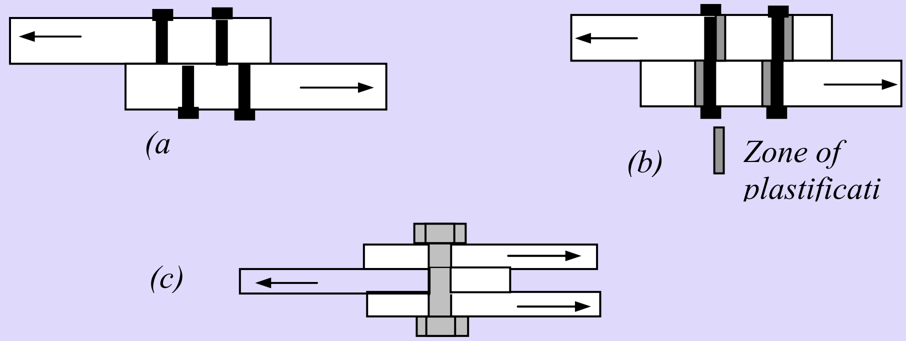


Fig. 3.8 Types of failures in a shear connection (a) Shearing of bolts (b) Bearing failure of plate (c) Bearing failure of bolt

The underlying assumption behind the design of bolted connections, namely that all bolts carry equal load is not true in some cases as mentioned below.

In long joints, the bolts farther away from the centre of the joint will carry more load than the bolts located close to the centre. Therefore, for joints having more than two bolts on either side of the building connection with the distance between the first and the last bolt exceeding 15*d* in the direction of load, the nominal shear capacity V_{ns}, shall be reduced by the factor, β_{lj}, given by (Cl.10.3.2.1)

$$\beta_{lj} = 1.075 - lj / (200\, d) \quad \text{but } 0.75 \le \beta_{lj} \le 1.0$$

Where, d= nominal diameter of the bolt

Similarly, if the grip length exceeds five times the nominal diameter, the strength is reduced as specified in IS 800. In multibolt connections, due to hole mismatch, all the bolts may not carry the same load. However, under ultimate load, due to high bearing ductility of the plates considerable redistribution of the load is possible and so the assumption that all bolts carry equal load may be considered valid.

3.2.4 Shear connection with HSFG bolts

3.2.4.1 Force transfer of HSFG bolts

The free body diagram of an HSFG connection is shown in Fig. 3.9. It can be seen that the pretension in the bolt causes clamping forces between the plates even before the external load is applied. When the external load is applied, the tendency of two plates to slip against one another is resisted by the friction between the plates. The frictional resistance is equal to the coefficient of friction multiplied by the normal clamping force between the plates. Until the externally applied force exceeds this frictional resistance the relative slip between the plates is prevented. The HSFG connections are designed such that under service load the force does not exceed the frictional resistance so that the relative slip is avoided during service. When the external force exceeds the frictional resistance the plates slip until the bolts come into contact with the plate and start bearing against the hole. Beyond this point the external force is resisted by the combined action of the frictional resistance and the bearing resistance.

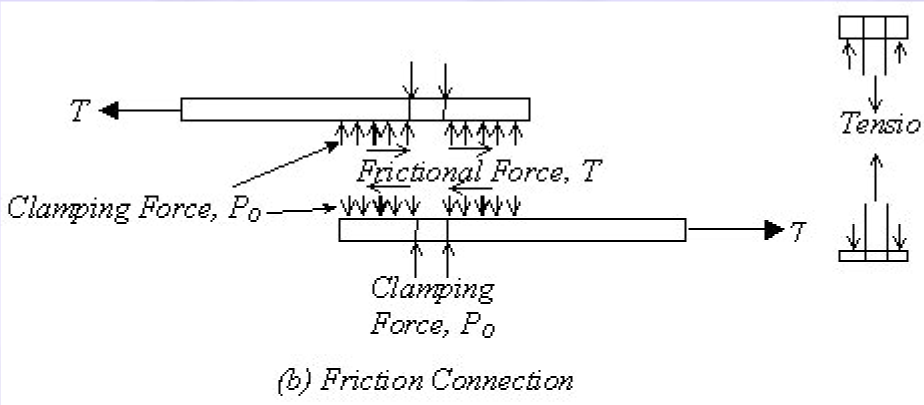


Fig. 3.9 Shear transfer by HSFG Bolt

3.2.4.2 Design shear strength of HSFG bolts

HSFG bolts will come into bearing only after slip takes place. Therefore if slip is critical (i.e. if slip cannot be allowed) then one has to calculate the slip resistance, which will govern the design. However, if slip is not critical, and limit state method is used then bearing failure can occur at the Limit State of collapse and needs to be checked. Even in the Limit State method, since HSFG bolts are designed to withstand working loads without slipping, the slip resistance needs to be checked anyway as a Serviceability Limit State.

1. Slip Resistance: Design for friction type bolting in which slip is required to be limited, a bolt subjected only to a factored design shear force, V_{sf}, in the interface of connections shall satisfy the following (Cl.10.4.3):

$$V_{sf} \le V_{nsf} / \gamma_{mf}$$

Where γ_{mf}= 1.25

V_{nsf} = nominal shear capacity of a bolt as governed by slip for friction type connection, calculated as follows:

$$V_{nsf} = \mu_f \, n_e \, K_h \, F_o \qquad \textbf{(3.4)}$$

Where, μ_f = coefficient of friction (slip factor) as specified in Table 3.1 ($\mu_f \le$ 0.55)(Table 3.1 of code).

n_e =number of effective interfaces offering frictional resistance to slip

K_h = 1.0 for fasteners in clearance holes

= 0.85 for fasteners in oversized and short slotted holes, and for fasteners in long slotted holes loaded perpendicular to the slot

= 0.7 for fasteners in long slotted holes loaded parallel to the slot.

γ_{mf}= 1.10 (if slip resistance is designed at service load)

γ_{mf}= 1.25 (if slip resistance is designed at ultimate load)

F_o= minimum bolt tension (proof load) at installation and may be taken as 0.8 A_{sb} F_o

A_{sb}= shank area of the bolt in tension

f_o= proof stress (= 0.70 f_{ub})

V_{ns} may be evaluated at a service load or ultimate load using appropriate partial safety factors, depending upon whether slip resistance is required at service load or ultimate load.

Table 3.1 Typical average values for coefficient of friction (μ_f)

Treatment of surface	Coeff. of friction (μf)
Clean mill scale	0.33
Sand blasted surface	0.48
Surfaces blasted with shot or grit and hot-dip galvanized	0.10

2. Bearing strength: The design for friction type bolting, in which bearing stress in the ultimate limit state is required to be limited, (V_{ub}=factored load bearing force) shall satisfy (Cl.10.4.4)

$$V_{bf} \leq V_{nbf} / \gamma_{mf}$$

Where γ_{mf}= 1.25

V_{nbf}= bearing capacity of a bolt, for friction type connection, calculated as follows:

$$V_{nbf} = 2.2\, d\, t\, f_{up} \leq 3\, d\, t\, f_{yp} \qquad \textbf{(3.5)}$$

Where

f_{up}= ultimate tensile stress of the plate

f_{yp} = tensile yield stress of the plate

d = nominal diameter of the bolt

t = summation of thicknesses of all the connected plates experiencing bearing stress in the same direction

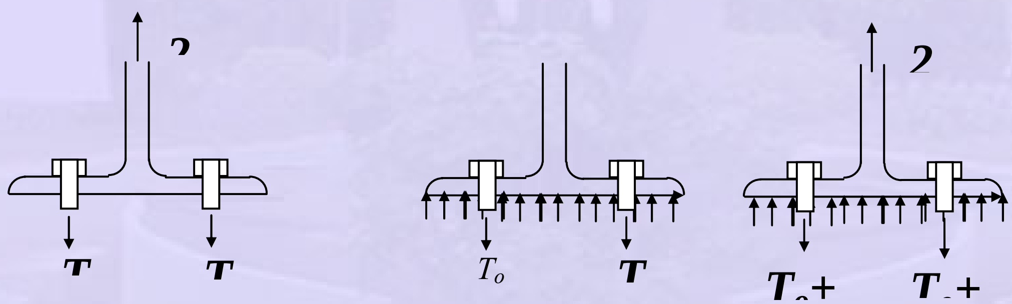
The block shear resistance of the edge distance due to bearing force shall also be checked.

3.2.5 Tension connections with bearing and HSFG bolts

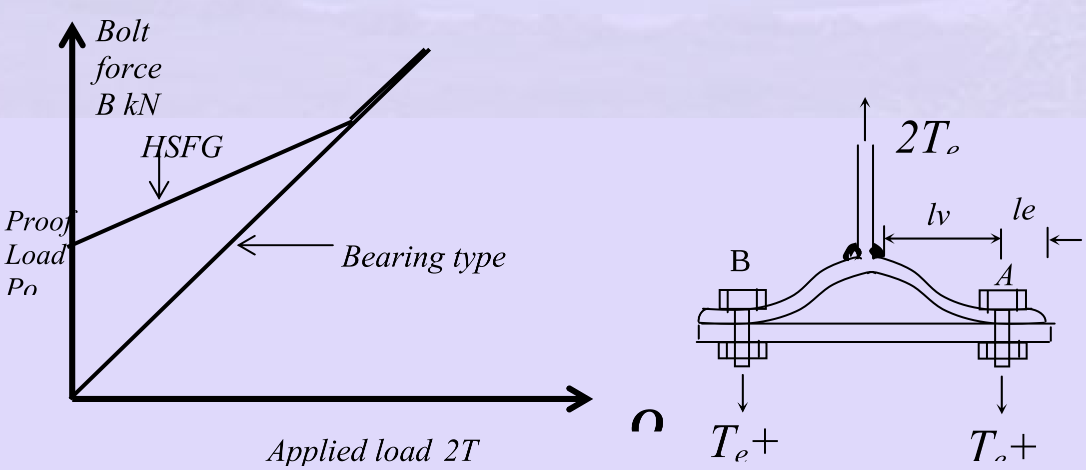
3.2.5.1 Force transfer by bearing and HSFG bolts

The free body diagram of the tension transfer in a bearing type of bolted connection is shown in Fig. 3.10(*a*). The variation of bolt tension due to externally applied tension is shown in Fig.3.10(*c*). It is seen that before any external tension is applied, the force in the bolt is almost zero, since the bolts are only snug tight. As the external tension is increased it is equilibrated by the increase in bolt tension. Failure is reached due to large elongation when the root of the bolt starts yielding. Depending on the relative flexibility of the plate and the bolt, sometimes the opening of the joint may be accompanied by prying action [Fig. 3.10(*d*)].

The free body diagram of an HSFG bolted connection is shown in Fig. 3.10(*b*). It is seen that even before any external load is applied, the force in the bolt is equal to proof load. Correspondingly there is a clamping force between the plates in contact. When the external load is applied, part of the load (nearly 10%) of the load is equilibrated by the increase in the bolt force. The balance of the force is equilibrated by the reduction in contact between the plates. This process continues and the contact between the plates is maintained until the contact force due to pre-tensioning is reduced to zero by the externally applied load. Normally, the design is done such that the externally applied tension does not exceed this level. After the external force exceeds this level, the behaviour of the bolt under tension is essentially the same as that in a bearing type of joint.



(a) Bearing type connection *(b) HSFG Connection*



(c) External Tension versus bolt force *(d) Prying Effect*

Fig. 3.10 Bolts under tension and prying effect

Where prying force, *Q*, is significant, prying force shall be calculated as given below and added to the tension in the bolt (Cl.10.4.7).

$$Q = \frac{l_v}{2\,l_e}\left[T_e - \frac{\beta\gamma f_o b_e t^4}{27 l_e l_v^2}\right] \qquad \textbf{(3.7)}$$

Where, l_v = distance from the bolt centreline to the toe of the fillet weld or to half the root radius for a rolled section; l_e = distance between prying force and bolt centreline and is the minimum of, either the end distance, or the value given by

$$l_e = 1.1\,t\sqrt{\frac{\beta f_o}{f_y}} \qquad \textbf{(3.8)}$$

Where,

β = 2 for non pre-tensioned bolt and 1 for pre-tensioned bolt

γ = 1.5

b_e = effective width of flange per pair of bolts

f_o = proof stress in consistent units

t = thickness of the end plate

Even if the bolts are strong enough to carry the additional prying forces, the plate can fail by developing a mechanism with yield lines at the centreline of the bolt and at the distance *b* from it. Therefore, the minimum thickness of the end plate (*t*), to avoid yielding of the plate, can be obtained by equating the moment in the plate at the bolt centreline (point A) and at the distance *b* from it (point B), to the plastic moment capacity of the plate M_p. Thus,

$$M_A = Qn; \quad M_B = Tb - Qn \qquad \textbf{(3.9)}$$

$$M_A = M_B = \frac{Tb}{2} = M_p \qquad \textbf{(3.10)}$$

taking M_p *as*

$$M_p = \frac{f_y}{1.15}\frac{wt^2}{4} \qquad \textbf{\textit{(3.11)}}$$

the minimum thickness for the end plate can be obtained as

$$t_{min} = \sqrt{\frac{1.15 \times 4 \times M_p}{f_y \times w}} \qquad \textbf{\textit{(3.12)}}$$

The corresponding prying force can then be obtained as $Q = M_p/n$. If the total force in the bolt *(T+Q)* exceeds the tensile capacity of the bolt, then the thickness of the end plate will have to be increased.

3.2.5.2 Design tensile strength of bearing and HSFG bolts

In a tension or hanger connection, the applied load produces tension in the bolts and the bolts are designed as tension members. If the attached plate is allowed to deform, additional tensile forces called prying forces are developed in the bolts.

Tension Capacity – A bolt subjected to a factored tension force (T_b) shall satisfy (Cl.10.3.4)

$$T_b \le T_{nb} / \gamma_{mb} \qquad \gamma_{mb} = 1.25$$

Where, T_{nb}= nominal tensile capacity of the bolt, calculated as follows:

$$T_{nb} = 0.90\, f_{ub}\, A_n \quad < f_{yb}\, A_{sb}\, (\gamma_{m1} / \gamma_{m0}) \qquad \gamma_{mo} = 1.10 \quad \textit{and} \quad \gamma_{mf} = 1.25$$

Where,

f_{ub} = ultimate tensile stress of the bolt

f_{yb} = yield stress of the bolt

A_n = net tensile stress area as specified in the appropriate Indian Standard. For bolts where the tensile stress area is not defined, A_n shall be taken as the area at the root of the threads (explained in next - chapter)

A_{sb} = shank area of the bolt

3.2.5.3 Combined shear and tension failure

Bolt Subjected to Combined Shear and Tension – A bolt required to resist both design shear force (V_{sd}) and design tensile force (T_{nd}) at the same time shall satisfy

$$\left(\frac{V}{V_{sd}}\right)^2 + \left(\frac{T_e}{T_{nd}}\right)^2 \leq 1.0 \qquad \textbf{\textit{(3.13)}}$$

Where, V = applied shear; V_{sd} = design shear capacity; T_e = externally applied tension and T_{nd} = design tension capacity. This gives a circular interaction curve as shown in Fig. 3.11.

Bolts in a connection for which slip in the serviceability limit state shall be limited, which are subjected to a tension force, T, and shear force, V, shall satisfy (Cl.10.4.6)

$$\left(\frac{V}{V_{sdf}}\right)^2 + \left(\frac{T_e}{T_{ndf}}\right)^2 \leq 1.0 \qquad \textbf{\textit{(3.14)}}$$

Where, V = applied shear at service load; V_{sdf} = design shear strength; T_e = externally applied tension at service load; T_{ndf} = design tension strength.

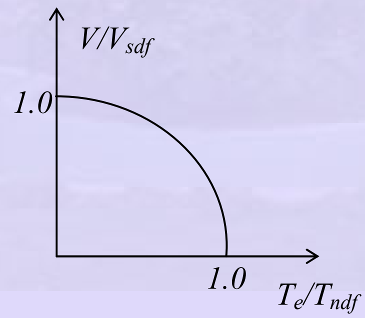


Fig. 3.11 Shear and Tension Interaction Curve

3.3 Welding and welded connections

Welding is the process of joining two pieces of metal by creating a strong metallurgical bond between them by heating or pressure or both. It is distinguished from other forms of mechanical connections, such as riveting or bolting, which are formed by friction or mechanical interlocking. It is one of the oldest and reliable methods of joining.

Welding offers many advantages over bolting and riveting. Welding enables direct transfer of stress between members eliminating gusset and splice plates necessary for bolted structures. Hence, the weight of the joint is minimum. In the case of tension members, the absence of holes improves the efficiency of the section. It involves less fabrication cost compared to other methods due to handling of fewer parts and elimination of operations like drilling, punching etc. and consequently less labour leading to economy. Welding offers air tight and water tight joining and hence is ideal for oil storage tanks, ships etc. Welded structures also have a neat appearance and enable the connection of complicated shapes. Welded structures are more rigid compared to structures with riveted and bolted connections. A truly continuous structure is formed by the process of fusing the members together. Generally welded joints are as strong or stronger than the base metal, thereby placing no restriction on the joints. Stress concentration effect is also considerably less in a welded connection.

Some of the disadvantages of welding are that it requires skilled manpower for welding as well as inspection. Also, non-destructive evaluation may have to be carried out to detect defects in welds. Welding in the field may be difficult due to the location or environment. Welded joints are highly prone to cracking under fatigue loading. Large residual stresses and distortion are developed in welded connections.

3.3.1 Fundamentals of welding

A welded joint is obtained when two clean surfaces are brought into contact with each other and either pressure or heat, or both are applied to obtain a bond. The tendency of atoms to bond is the fundamental basis of welding. The inter-diffusion between the materials that are joined is the underlying principle in all welding processes. The diffusion may take place in the liquid, solid or mixed state. In welding the metallic materials are joined by the formation of metallic bonds and a perfect connection is formed. In practice however, it is very difficult to achieve a perfect joint; for, real surfaces are never smooth. When welding, contact is established only at a few points in the surface, joins irregular surfaces where atomic bonding occurs. Therefore the strength attained will be only a fraction of the full strength. Also, the irregular surface may not be very clean, being contaminated with adsorbed moisture, oxide film, grease layer etc. In the welding of such surfaces, the contaminants have to be removed for the bonding of the surface atoms to take place. This can be accomplished by applying either heat or pressure. In practical welding, both heat and pressure are applied to get a good joint.

As pointed out earlier, any welding process needs some form of energy, often heat, to connect the two materials. The relative amount of heat and pressure required to join two materials may vary considerably between two extreme cases in which either heat or pressure alone is applied. When heat alone is applied to make the joint, pressure is used merely to keep the joining members together. Examples of such a process are Gas Tungsten Arc Welding (GTAW), Shielded Metal Arc Welding (SMAW), Submerged Arc Welding (SAW) etc. On the other hand pressure alone is used to make the bonding by plastic deformation, examples being cold welding, roll welding, ultrasonic welding etc. There are other welding methods where both pressure and heat are employed, such as resistance welding, friction welding etc. A flame, an arc or resistance to an electric current, produces the required heat. Electric arc is by far the most popular source of heat used in commercial welding practice.

3.3.2 Welding process

In general, gas and arc welding are employed; but, almost all structural welding is arc welding.

In gas welding a mixture of oxygen and some suitable gas is burned at the tip of a torch held in the welder's hand or by an automatic machine. Acetylene is the gas used in structural welding and the process is called oxyacetylene welding. The flame produced can be used both for cutting and welding of metals. Gas welding is a simple and inexpensive process. But, the process is slow compared to other means of welding. It is generally used for repair and maintenance work.

The most common welding processes, especially for structural steel, use electric energy as the heat source produced by the electric arc.IS:816 in this process, the base metal and the welding rod are heated to the fusion temperature by an electric arc. The arc is a continuous spark formed when a large current at a low voltage is discharged between the electrode and the base metal through a thermally ionised gaseous column, called plasma. The resistance of the air or gas between the electrode and the objects being welded changes the electric energy into heat. A temperature of 3300^0 C to 5500^0 C is produced in the arc. The welding rod is connected to one terminal of the current source and the object to be welded to the other. In arc welding, fusion takes place by the flow of material from the welding rod across the arc without pressure being applied. The Shielded Metal Arc Welding process is explained in the following paragraph.

In Shielded Metal Arc Welding or SMAW (Fig.3.12), heating is done by means of electric arc between a coated electrode and the material being joined. In case bare wire electrode (without coating) is employed, the molten metal gets exposed to atmosphere and combines chemically with oxygen and nitrogen forming defective welds. The electrode coating on the welding rod forms a gaseous shield that helps to exclude oxygen and stabilise the arc.

The coated electrode also deposits a slag in the molten metal, which because of its lesser density compared to the base metal, floats on the surface of the molten metal pool, shields it from atmosphere, and slows cooling. After cooling, the slag can be easily removed by hammering and wire brushing.

The coating on the electrode thus: shields the arc from atmosphere; coats the molten metal pool against oxidation; stabilises the arc; shapes the molten metal by surface tension and provides alloying element to weld metal.

Fig.3.12 Shielded metal arc welding (SMAW) process

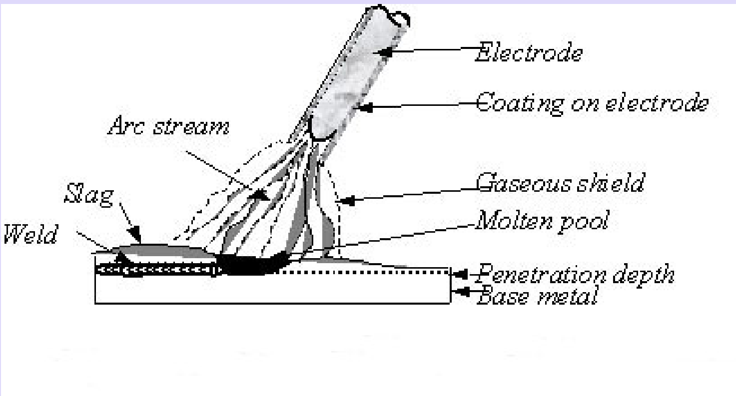


Fig.3.12 Shielded metal arc welding (SMAW) process

The type of welding electrode used would decide the weld properties such as strength, ductility and corrosion resistance. The type to be used for a particular job depends upon the type of metal being welded, the amount of material to be added and the position of the work. The two general classes of electrodes are lightly coated and heavily coated. The heavily coated electrodes are normally used in structural welding. The resulting welds are stronger, more corrosion resistant and more ductile compared to welds produced by lightly coated electrodes. Usually the SMAW process is either automatic or semi-automatic.

The term weldability is defined as the ability to obtain economic welds, which are good, crack-free and would meet all the requirements. Of great importance are the chemistry and the structure of the base metal and the weld metal. The effects of heating and cooling associated with fusion welding are experienced by the weld metal and the Heat Affected Zone (HAZ) of the base metal. The cracks in HAZ are mainly caused by high carbon content, hydrogen enbrittlement and rate of cooling. For most steels, weld cracks become a problem as the thickness of the plates increases.

3.3.3 Types of joints and welds

By means of welding, it is possible to make continuous, load bearing joints between the members of a structure. A variety of joints is used in structural steel work and they can be classified into four basic configurations namely, Lap joint, Tee joint, Butt joint and Corner joint.

For lap joints, the ends of two members are overlapped and for butt joints, the two members are placed end to end. The T- joints form a Tee and in Corner joints, the ends are joined like the letter L. Most common joints are made up of fillet weld or the butt (also calling groove) weld. Plug and slot welds are not generally used in structural steel work. Fig.3.14 Fillet welds are suitable for lap joints and Tee joints and groove welds for butt and corner joints. Butt welds can be of complete penetration or incomplete penetration depending upon whether the penetration is complete through the thickness or partial. Generally a description of welded joints requires an indication of the type of both the joint and the weld.

Though fillet welds are weaker than butt welds, about 80% of the connections are made with fillet welds. The reason for the wider use of fillet welds is that in the case of fillet welds, when members are lapped over each other, large tolerances are allowed in erection. For butt welds, the members to be connected have to fit perfectly when they are lined up for welding. Further butt welding requires the shaping of the surfaces to be joined as shown in Fig. 3.15. To ensure full penetration and a sound weld, a backup plate is temporarily provided as shown in Fig.3.15

Butt welds:

Full penetration butt welds are formed when the parts are connected together within the thickness of the parent metal. For thin parts, it is possible to achieve full penetration of the weld. For thicker parts, edge preparation may have to be done to achieve the welding. There are nine different types of butt joints: square, single V, double V, single U, double U, single J, double J, single bevel and double bevel. They are shown in Fig. 3.13 In order to qualify for a full penetration weld; there are certain conditions to be satisfied while making the welds.

Welds are also classified according to their position into flat, horizontal, vertical and overhead. Flat welds are the most economical to make while overhead welds are the most difficult and expensive.

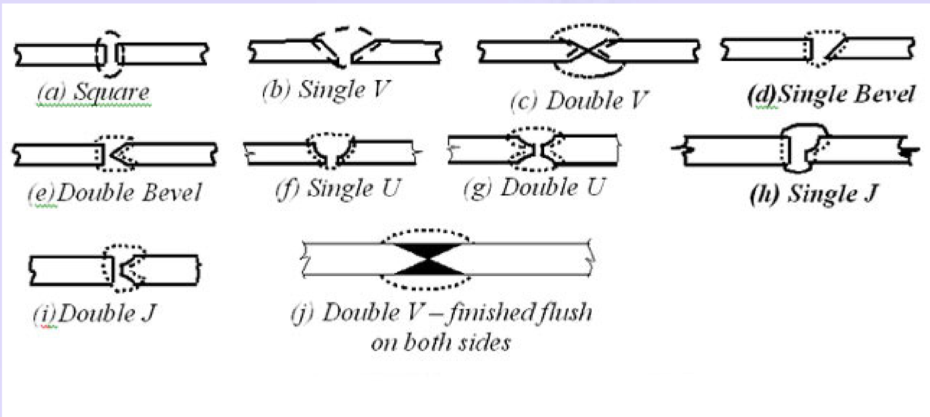


Fig. 3.13 Different types of butt welds

The main use of butt welds is to connect structural members, which are in the same plane. A few of the many different butt welds are shown in Fig. 3.16.There are many variations of butt welds and each is classified according to its particular shape. Each type of butt weld requires a specific edge preparation and is named accordingly. The proper selection of a particular type depends upon: Size of the plate to be joined; welding is by hand or automatic; type of welding equipment, whether both sides are accessible and the position of the weld.

Butt welds have high strength, high resistance to impact and cyclic stress. They are most direct joints and introduce least eccentricity in the joint. But their major disadvantages are: high residual stresses, necessity of edge preparation and proper aligning of the members in the field. Therefore, field butt joints are rarely used.

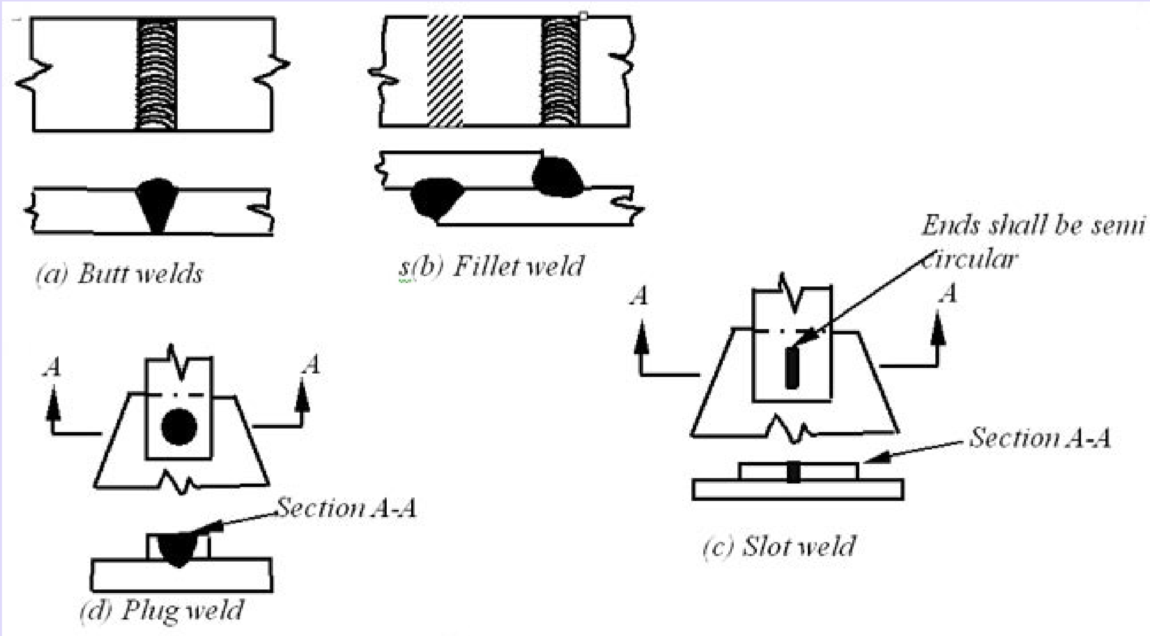


Fig.3.14 Common types of welds

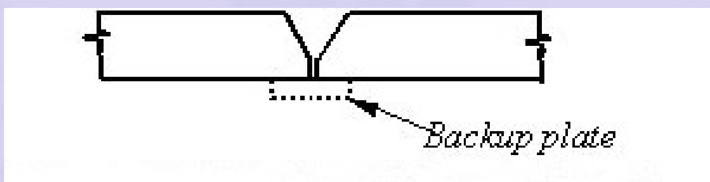


Fig.3.15 Shaping of surface and backup plate

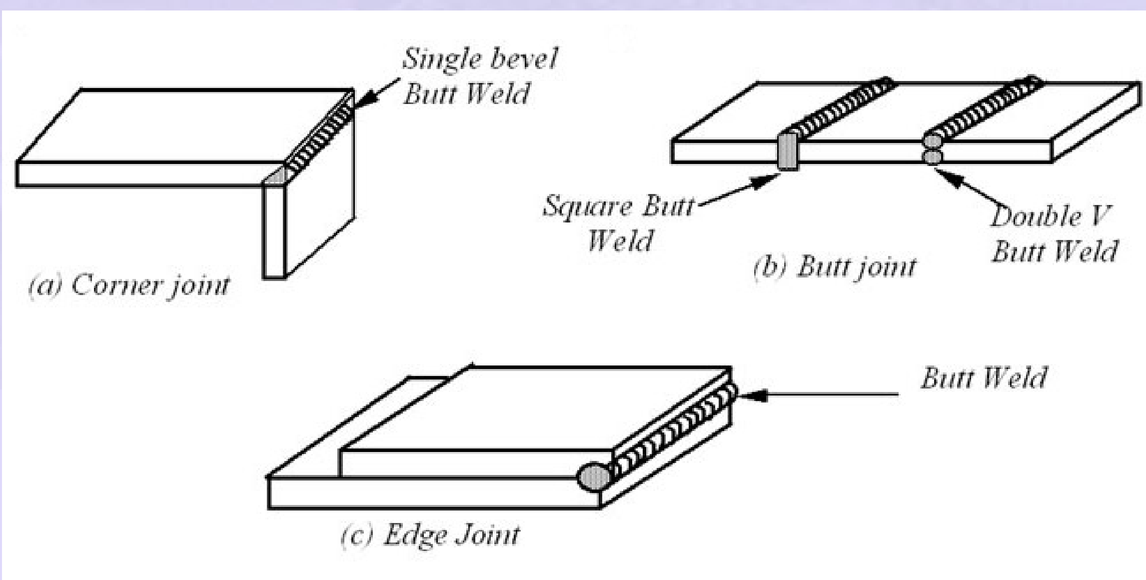


Fig.3.16 Typical connections with butt weld

To minimise weld distortions and residual stresses, the heat input is minimised and hence the welding volume is minimised. This reduction in the volume of weld also reduces cost. Hence for thicker plates, double Butt welds and U welds are generally used. For a butt weld, the root gap, *R*, is the separation of the pieces being joined and is provided for the electrode to access the base of a joint. The smaller the root gap the greater the angle of the bevel. The depth by which the arc melts into the plate is called the depth of penetration [Fig.3.17 (a)]. Roughly, the penetration is about 1 mm per 100A and in manual welding the current is usually 150 – 200 A. Therefore, the mating edges of the plates must be cut back if through-thickness continuity is to be established. This groove is filled with the molten metal from the electrode. The first run that is deposited in the bottom of a groove is termed as the root run [Fig.3.176 (*c*)]. For good penetration, the root faces must be melted. Simultaneously, the weld pool also must be controlled, preferably, by using a backing strip.

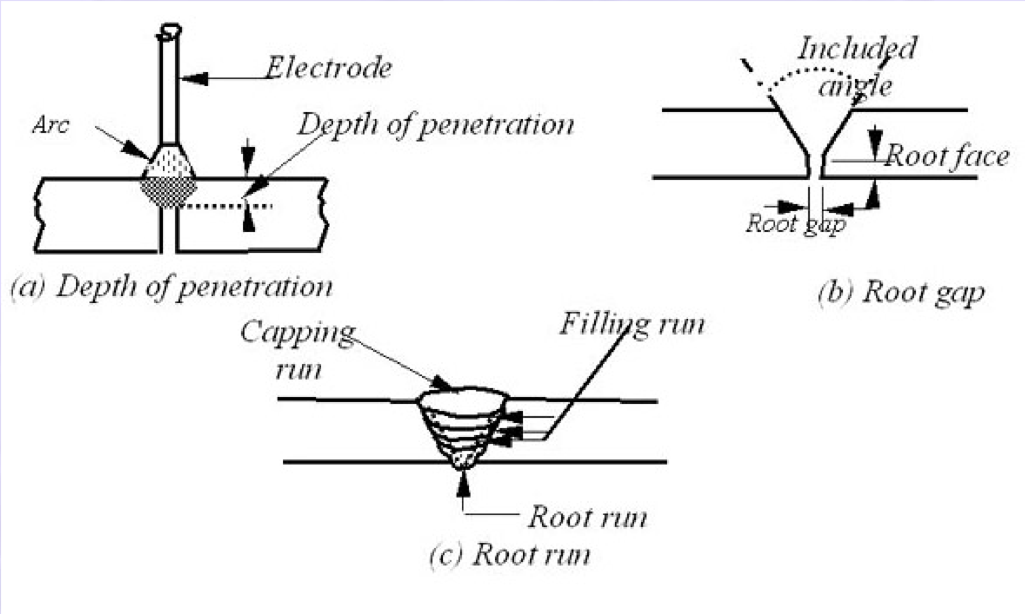


Fig.3.17 Butt weld details

Fillet welds:

Owing to their economy, ease of fabrication and adaptability, fillet welds are widely used. They require less precision in the fitting up because the plates being joined can be moved about more than the Butt welds. Another advantage of fillet welds is that special preparation of edges, as required by Butt welds, is not required. In a fillet weld the stress condition in the weld is quite different from that of the connected parts. A typical fillet weld is shown in Fig.3.18

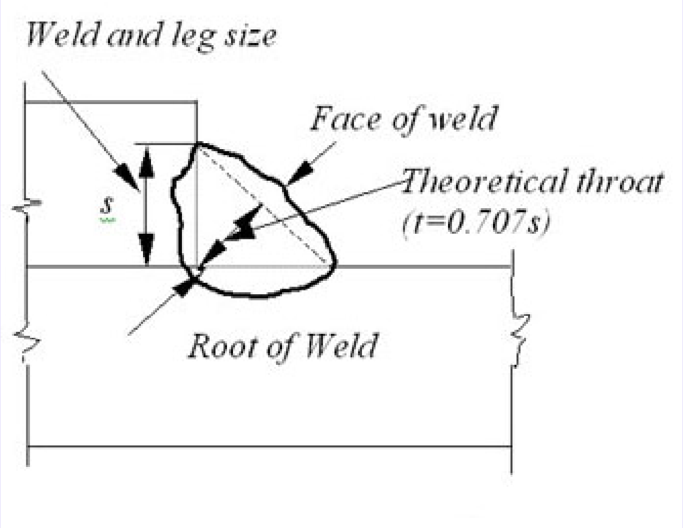


Fig. 3.18 Typical fillet weld

The root of the weld is the point where the faces of the metallic members meet. The theoretical throat of a weld is the shortest distance from the root to the hypotenuse of the triangle. The throat area equals the theoretical throat distance times the length of the weld.

The concave shape of free surface provides a smoother transition between the connected parts and hence causes less stress concentration than a convex surface. But it is more vulnerable to shrinkage and cracking than the convex surface and has a much reduced throat area to transfer stresses. On the other hand, convex shapes provide extra weld metal or reinforcement for the throat. For statically loaded structures, a slightly convex shape is preferable, while for fatigue – prone structures, concave surface is desirable.

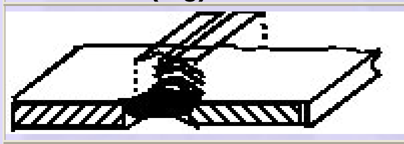
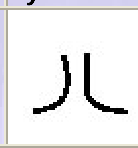
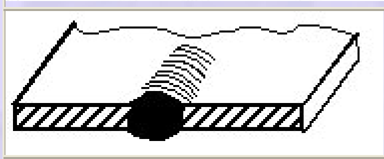
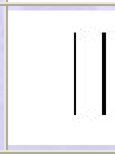
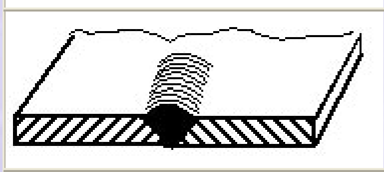
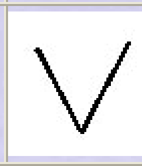
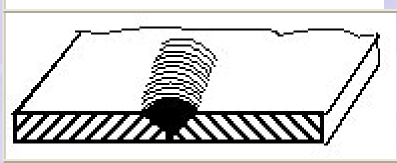
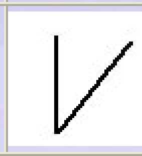
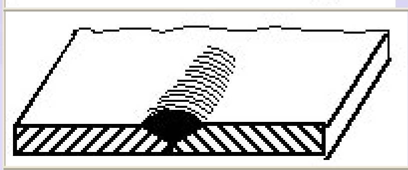
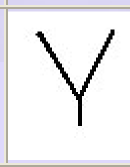
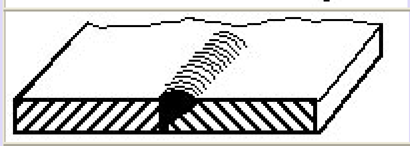
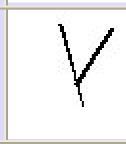
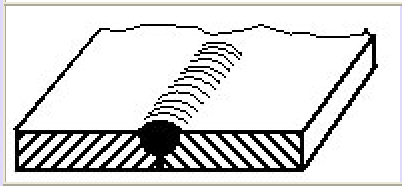
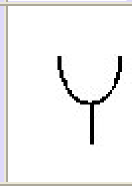
Large welds are invariably made up of a number of layers or passes. For reasons of economy, it is desirable to choose weld sizes that can be made in a single pass. Large welds scan be made in a single pass by an automatic machine, though manually, 8 *mm* fillet is the largest single-pass layer.

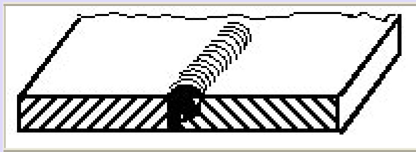
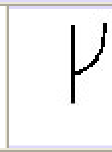
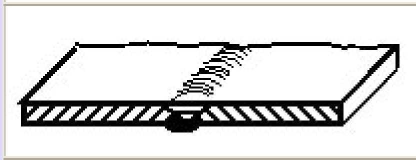
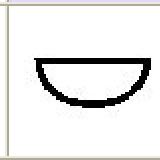
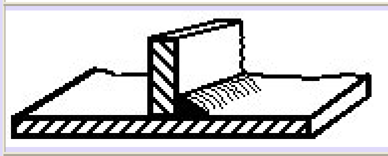
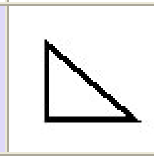
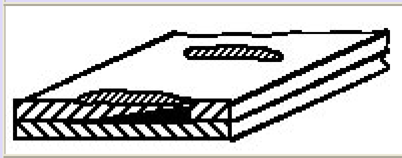
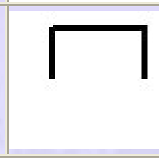
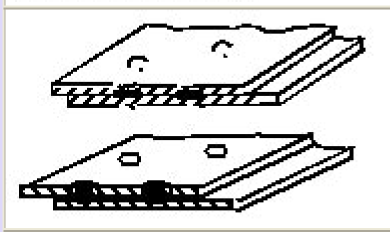
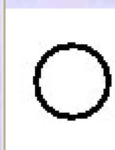
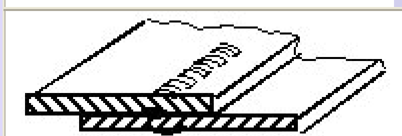
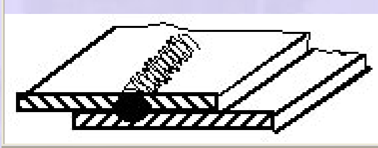
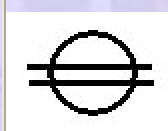
3.3.4 Weld symbols

The information concerning type, size, position, welding process etc. of the welds in welded joints is conveyed by standard symbols in drawings. The symbolic representation includes elementary symbols along with a) supplementary symbol, b) a means of showing dimensions, or c) some complementary indications. IS: 813 “Scheme of Symbols for Welding” gives all the details of weld representation in drawings.

Elementary symbols represent the various categories of the weld and look similar to the shape of the weld to be made. Combination of elementary symbols may also be used, when required. Elementary symbols are shown in Table 3.2.

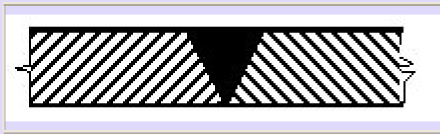
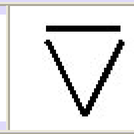
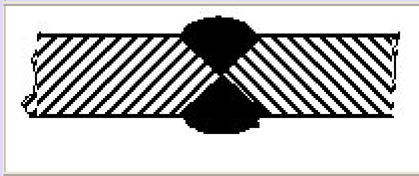
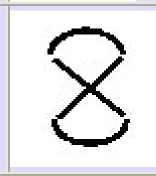
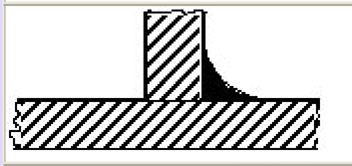
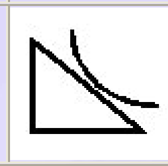
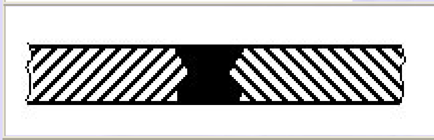
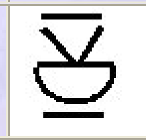
Table 3.2 Elementary symbols

Illustration (Fig)	Symbol	Description
		Butt weld between plates with raised edges*(the raised edges being melted down completely)
		Square butt weld
		Single-V butt weld
		Single-bevel butt weld
		Single – V butt weld with broad root face
		Single – bevel butt weld with broad root face
		Single – U butt weld (parallel or sloping sides)

		Single – J butt joint
		Backing run; back or backing weld
		Fillet weld
		Plug weld; plug or slot weld
		Spot weld
 		Seam weld

Supplementary symbols characterise the external surface of the weld and they complete the elementary symbols. Supplementary symbols are shown in Table 3.3. The weld locations are defined by specifying, a) position of the arrow line, b) position of the reference line, and c) the position of the symbol. More details of weld representation may be obtained from IS 813.

Table 3.3. Supplementary symbols

		Flat (flush) single – V butt weld
		Convex double – V butt weld
		Concave fillet weld
		Flat (flush) single – V butt with flat (flush) backing run

Position of symbols in drawings:

Apart from the symbols as covered earlier, the methods of representation (Fig.3.19) also include the following:

· An arrow line for each joint

. A dual reference line, consisting of two parallel lines, one continuous and the other dashed.

. A certain number of dimensions and conventional signs

The location of welds is classified on the drawings by specifying:

Position of the arrow line, position of the reference line and the position of the symbol

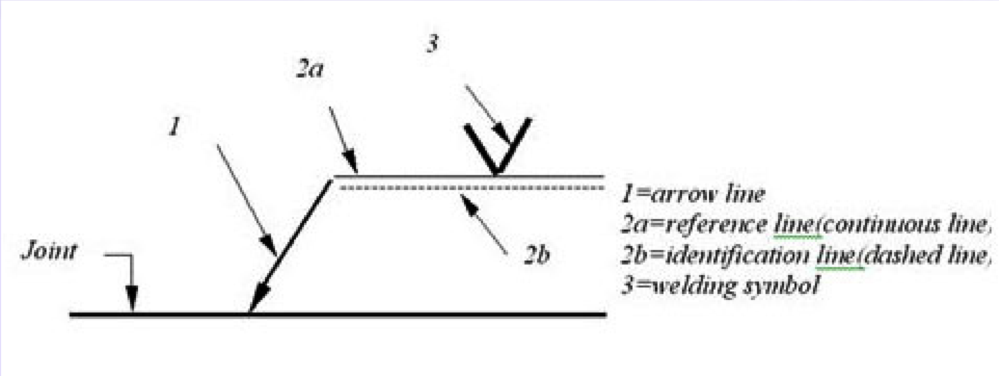


Fig. 3.19 Method of representation

The position of arrow line with respect to the weld has no special significance. The arrow line joins one end of the continuous reference line such that it forms an angle with it and shall be completed by an arrowhead or a dot. The reference line is a straight line drawn parallel to the bottom edge of the drawing.

The symbol is placed either above or beneath the reference line. The symbol is placed on the continuous side of the reference line if the weld is on the other side of the joint; the symbol is placed on the dashed line side

3.3.5 Design of welds

Design of butt welds:

For butt welds the most critical form of loading is tension applied in the transverse direction. It has been observed from tests conducted on tensile coupons containing a full penetration butt weld normal to the applied load that the welded joint had higher strength than the parent metal itself. The yield stress of the weld metal and the parent metal in the HAZ region was found to be much higher than the parent metal.

The butt weld is normally designed for direct tension or compression. However, a provision is made to protect it from shear. Design strength value is often taken the same as the parent metal strength. For design purposes, the effective area of the butt-welded connection is taken as the effective length of the weld times the throat size. Effective length of the butt weld is taken as the length of the continuous full size weld. The throat size is specified by the effective throat thickness. For a full penetration butt weld, the throat dimension is usually assumed as the thickness of the thinner part of the connection. Even though a butt weld may be reinforced on both sides to ensure full cross-sectional areas, its effect is neglected while estimating the throat dimensions. Such reinforcements often have a negative effect, producing stress concentration, especially under cyclic loads.

Unsealed butt welds of V, U, J and bevel types and incomplete penetration butt welds should not be used for highly stressed joints and joints subjected to dynamic and alternating loads. Intermittent butt welds are used to resist shear only and the effective length should not be less than four times the longitudinal space between the effective length of welds nor more than 16 times the thinner part. They are not to be used in locations subjected to dynamic or alternating stresses. Some modern codes do not allow intermittent welds in bridge structures.

For butt welding parts with unequal cross sections, say unequal width, or thickness, the dimensions of the wider or thicker part should be reduced at the butt joint to those of the smaller part. This is applicable in cases where the difference in thickness exceeds 25 % of the thickness of the thinner part or 3.0 mm, whichever is greater. The slope provided at the joint for the thicker part should not be steeper than one in five [Figs.3.20 (a) & (b)]. In instances, where this is not practicable, the weld metal is built up at the junction equal to a thickness which is at least 25 % greater than the thinner part or equal to the dimension of the thicker part [Fig.3.20 (*c*)]. Where reduction of the wider part is not possible, the ends of the weld shall be returned to ensure full throat thickness.

Stresses for butt welds are assumed same as for the parent metal with a thickness equal to the throat thickness (Cl.10.5.7.1). For field welds, the permissible stresses in shear and tension calculated using a partial factor γ_{mw} of 1.5. (Cl.10.5.7.2)

Design of fillet welds:

Fillet welds are broadly classified into side fillets and end fillets (Fig.3.21). When a connection with end fillet is loaded in tension, the weld develops high strength and the stress developed in the weld is equal to the value of the weld metal. But the ductility is minimal. On the other hand, when a specimen with side weld is loaded, the load axis is parallel to the weld axis. The weld is subjected to shear and the weld shear strength is limited to just about half the weld metal tensile strength. But ductility is considerably improved. For intermediate weld positions, the value of strength and ductility show intermediate values.

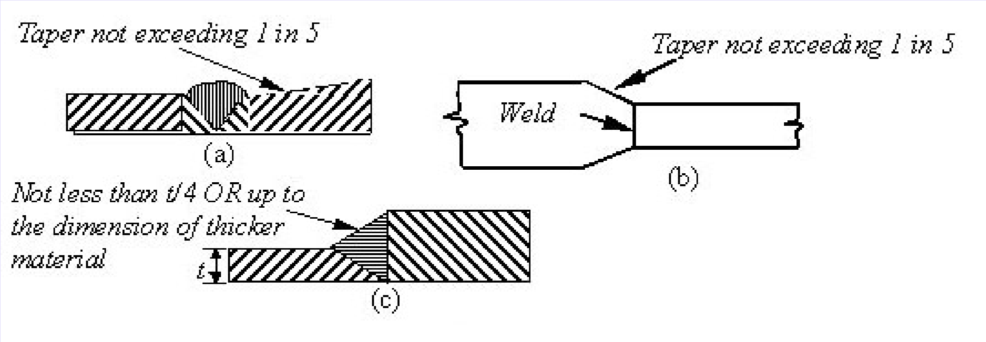


Fig.3.20 Butt welding of members with (a) & (b) unequal thickness (c) unequal width

In many cases, it is possible to use the simplified approach of average stresses in the weld throat (Fig. 3.22). In order to apply this method, it is important to establish equilibrium with the applied load. Studies conducted on fillet welds have shown that the fillet weld shape is very important for end fillet welds. For equal leg lengths, making the direction of applied tension nearly parallel to the throat leads to a large reduction in strength. The optimum weld shape recommended is to provide shear leg ≤ 3 times the tension leg. A small variation in the side fillet connections has negligible effect on strength. In general, fillet welds are stronger in compression than in tension.

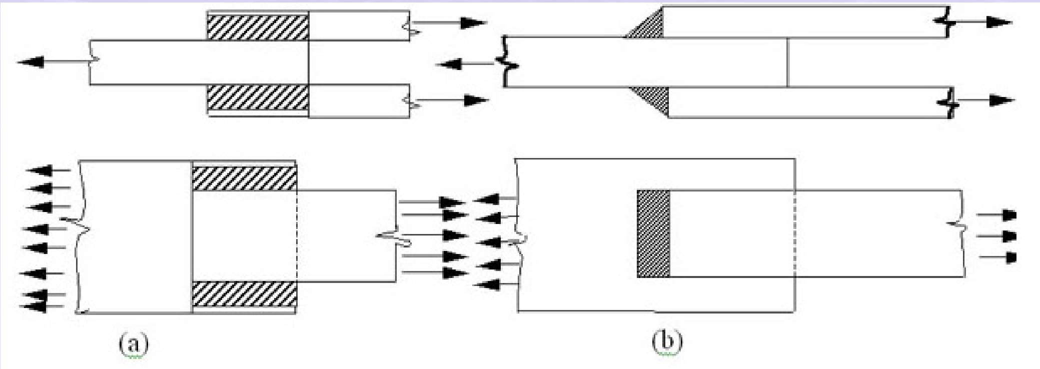


Fig.3.21 Fillet (a) side welds and (b) end welds

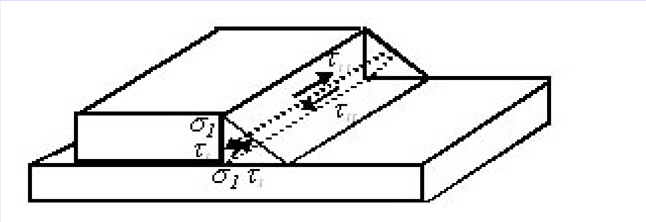


Fig.3.22 Average stress in the weld throat

A simple approach to design is to assume uniform fillet weld strength in all directions and to specify a certain throat stress value. The average throat thickness is obtained by dividing the applied loads summed up in vectorial form per unit length by the throat size.

This method is limited in usage to cases of pure shear, tension or compression (Fig.3.23). It cannot be used in cases where the load vector direction varies around weld group. For the simple method, the stress is taken as the vector sum of the force components acting in the weld divided by the throat area.

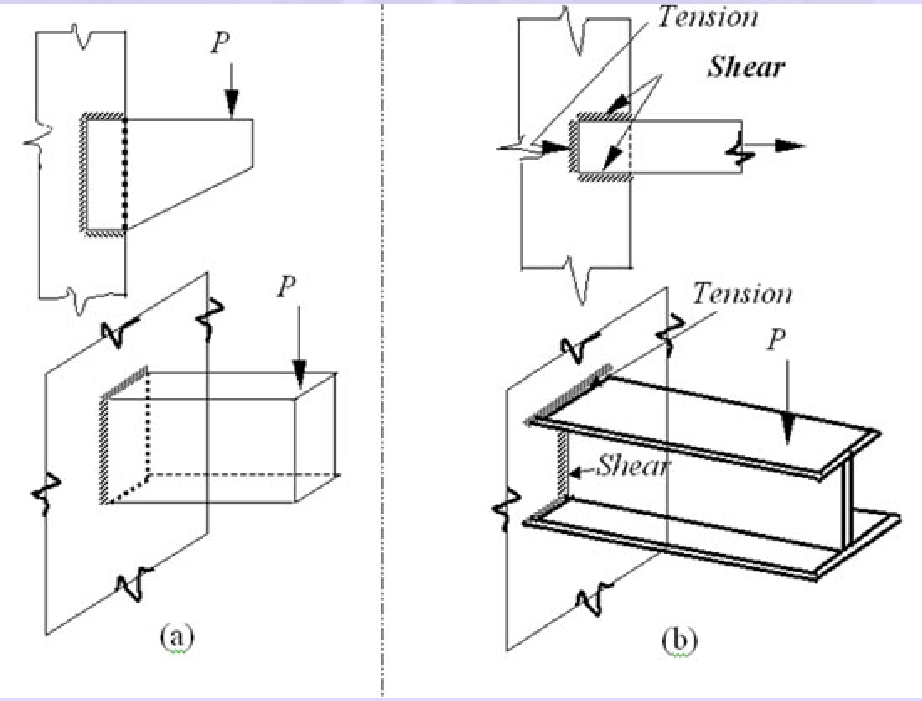


Fig.3.23 (a) connections with simple weld design, (b) connections with Direction- dependent weld design

Stresses Due to Individual forces - When subjected to either compressive or tensile or shear force alone, the stress in the weld is given by:

$$f_a \text{ or } q = \frac{P}{t_t\, l_w}$$

Where

f_a = calculated normal stress due to axial force in N/mm^2

q = shear stress in N/mm^2

P = force transmitted (axial force N or the shear force Q)

t_t = effective throat thickness of weld in mm

l_w= effective length of weld in mm

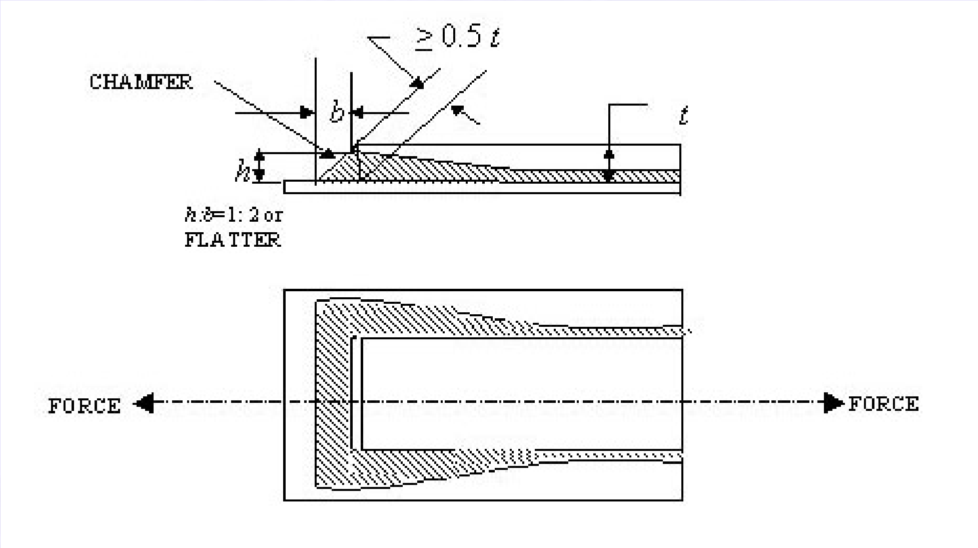


Fig. 3.24 End fillet weld normal to direction of force

The design strength of a fillet weld, *fwd*, shall be based on its throat area (Cl.10.5.7).

$$f_{wd} = f_{wn} / \gamma_{mw} \text{ in which } f_{wn} = f_u / \sqrt{3}$$

Where f_u = smaller of the ultimate stress of the weld and the parent metal and

γ_{mw} = partial safety factor (=1.25 for shop welds and = 1.5 for field welds)

The design strength shall be reduced appropriately for long joints as prescribed in the code.

The size of a normal fillet should be taken as the minimum leg size (Fig. 3.25)

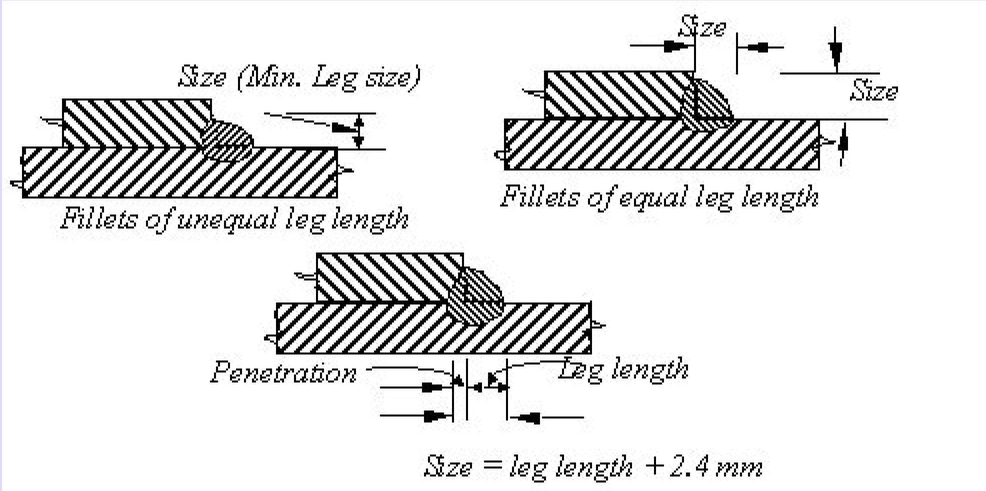


Fig. 3.25 Sizes of fillet welds

For a deep penetration weld, the depth of penetration should be a minimum of 2.4 mm. Then the size of the weld is minimum leg length plus 2.4 mm. The size of a fillet weld should not be less than 3 mm or more than the thickness of the thinner part joined. Minimum size requirement of fillet welds is given below in Table 3.4 (Cl.10.5.2.3). *Effective throat thickness* should not be less than 3 mm and should not exceed 0.7 t and 1.0 t under special circumstances, where't' is the thickness of thinner part.

Table 3.4 Minimum size of first run or of a single run fillet weld

Thickness of thicker part (mm)	Minimum size (mm)
$t \leq 10$	3
$10 < t \leq 20$	5
$20 < t \leq 32$	6
$32 < t \leq 50$	8 (First run)10 (Minimum size of fillet)

For stress calculations, the effective throat thickness should be taken as K times fillet size, where K is a constant. Values of K for different angles between tension fusion faces are given in Table 3.5 (Cl.10.5.3.2). Fillet welds are normally used for connecting parts whose fusion faces form angles between 60° and 120°. The actual length is taken as the length having the effective length plus twice the weld size. Minimum effective length should not be less than four times the weld size. When a fillet weld is provided to square edge of a part, the weld size should be at least 1.5 mm less than the edge thickness [Fig. 3.26 (*a*)]. For the rounded toe of a rolled section, the weld size should not exceed 3/4 thickness of the section at the toe [Fig. 3.26 (*b*)] (Cl.10.5.8.1).

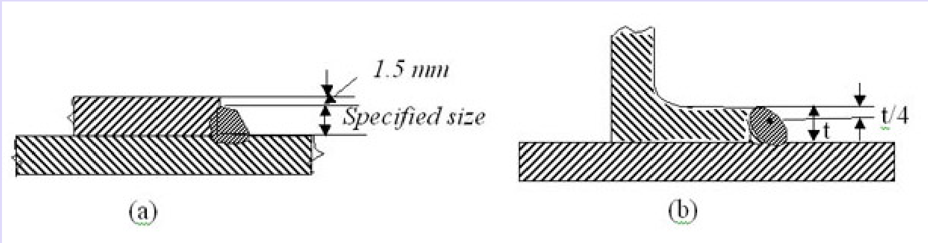


Fig.3.26 (a) Fillet welds on square edge of plate, (b) Fillet Welds on round toe of rolled section

Table 3.5. Value of K for different angles between fusion faces

Angle between fusion faces	60° - 90°	91°-100°	101°-106°	107°-113°	114°-120°
Constant K	0.70	0.65	0.60	0.55	0.50

Intermittent fillet welds may be provided where the strength required is less than that can be developed by a continuous fillet weld of the smallest allowable size for the parts joined. The length of intermediate welds should not be less than 4 times the weld size with a minimum of 40 mm. The clear spacing between the effective lengths of the intermittent welds should be less than or equal to 12 times the thickness of the thinner member in compression and 16 times in tension; in no case the length should exceed 20 cm. Chain intermittent welding is better than staggered intermittent welding. Intermittent fillet welds are not used in main members exposed to weather. For lap joints, the overlap should not be less than five times the thickness of the thinner part. For fillet welds to be used in slots and holes, the dimension of the slot or hole should comply with the following limits:

a) The width or diameter should not be less than three times the thickness or 25 mm whichever is greater

b) Corners at the enclosed ends or slots should be rounded with a radius not less than 1.5 times the thickness or 12 mm whichever is greater, and

c) The distance between the edge of the part and the edge of the slot or hole, or between adjacent slots or holes, should be not less than twice the thickness and not less than 25 mm for the holes.

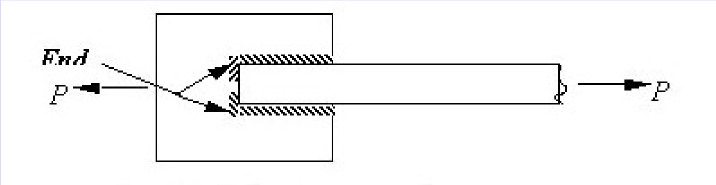


Fig. 3.27 End returns for side welds

The effective area of a plug weld is assumed as the nominal area of the whole in the plane of the *faying* surface. Plug welds are not designed to carry stresses. If two or more of the general types of weld (butt, fillet, plug or slots) are combined in a single joint, the effective capacity of each has to be calculated separately with reference to the axis of the group to determine the capacity of the welds.

The high stress concentration at ends of welds is minimised by providing welds around the ends as shown in Fig. 3.27. These are called *end returns*. Most designers neglect end returns in the effective length calculation of the weld. End returns are invariably provided for welded joints that are subject to eccentricity, impact or stress reversals. The end returns are provided for a distance not less than twice the size of the weld.

Design of plug and slot welds:

In certain instances, the lengths available for the normal longitudinal fillet welds may not be sufficient to resist the loads. In such a situation, the required strength may be built up by welding along the back of the channel at the edge of the plate if sufficient space is available. This is shown in Fig. 3.28 (a). Another way of developing the required strength is by providing slot or plug welds. Slot and plug welds [Fig. 3.28 (b)] are generally used along with fillet welds in lap joints. On certain occasions, plug welds are used to fill the holes that are temporarily made for erection bolts for beam and column connections. However, their strength may not be considered in the overall strength of the joint.

The limitations given in specifications for the maximum sizes of plug and slot welds are necessary to avoid large shrinkage, which might be caused around these welds when they exceed the specified sizes. The strength of a plug or slot weld is calculated by considering the allowable stress and its nominal area in the shearing plane. This area is usually referred to as the faying surface and is equal to the area of contact at the base of the slot or plug. The length of the slot weld can be obtained from the following relationship:

$$L = \frac{\text{Load}}{(\text{Width})\ \text{allowable stress}} \qquad \textbf{(3.15)}$$

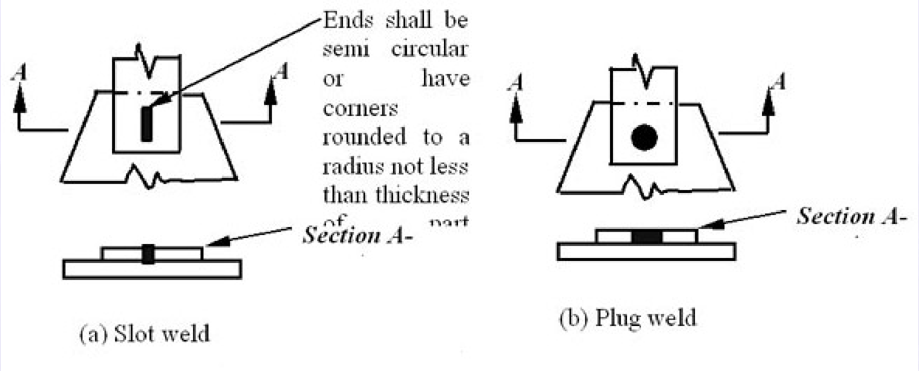


Fig. 3.28 Slot and plug welds

3.4 Analysis of bolt groups

In general, any group of bolts resisting a moment can be classified into either of two cases depending on whether the moment is acting in the shear plane or in a plane perpendicular to it. Both cases are described in this section.

3.4.1 Combined shear and moment in plane

Consider an eccentric connection carrying a load of P as shown in Fig. 3.29. The basic assumptions in the analysis are (1) deformations of plate elements are negligible, (2) the total shear is assumed to be shared equally by all bolts and (3) the equivalent moment at the geometric centre (point *O* in Fig. 3.29) of the bolt group, causes shear in any bolt proportional to the distance of the bolt from the point *O* acting perpendicular to the line joining the bolt centre to point *O* (radius vector).

Resolving the applied force *P* into its components P_x and P_y in *x* and *y*-directions respectively and denoting the corresponding force on any bolt *i* to these shear components by R_{xi} and R_{yi} and applying the equilibrium conditions we get the following:

$$R_{xi} = P_x/n \text{ and } R_{yi} = P_y/n \qquad \textbf{(3.16)}$$

Where *n* is the total number of bolts in the bolt group and R_{xi} and R_{yi} act in directions opposite to P_x and P_y respectively.

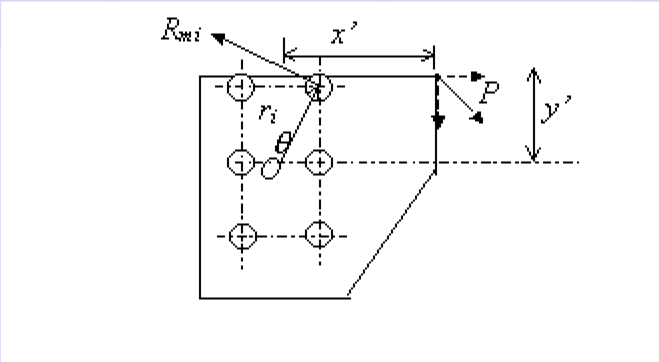


Fig. 29 Bolt group eccentrically loaded in shear

The moment of force *P* about the centre of the bolt group (point O) is given by

$$M = P_x\, y' + P_y\, x' \qquad \textbf{(3.17)}$$

where x' and y' denote the coordinates of the point of application of the force P with respect to the point *O*. The force in bolt *i,* denoted by R_{mi}, due to the moment *M* is proportional to its distance from point *O*, r_i, and perpendicular to

$$R_{mi} = k\, r_i \qquad \textbf{(3.18)}$$

Where, k is the constant of proportionality. The moment of R_{mi} about point *O* is

$$M_i = k\, r_i^2 \qquad \textbf{(3.19)}$$

Therefore the total moment of resistance of the bolt group is given by

$$MR = \Sigma\, k\, r_i^2 = k\, \Sigma\, r_i^2 \qquad \textbf{(3.20)}$$

For moment equilibrium, the moment of resistance should equal the applied moment and so *k* can be obtained as $k = M/\Sigma r_i^2$, which gives R_{mi} as

$$R_{mi} = M\, r_i / \Sigma\, r_i^2 \qquad \textbf{(3.21)}$$

Total shear force in the bolt R_i is the vector sum of R_{xi}, R_{yi} and R_{mi}

$$R2 = \sqrt{\left[\left(R_{xi} + R_{mi}\cos\theta_i\right)^2 + \left(R_{yi} + R_{mi}\sin\theta_i\right)^2\right]} \qquad \textbf{(3.22)}$$

After substituting for R_{xi}, R_{yi} and R_{mi} from equations (3.16) and (3.21) in (3.22), using $\cos\theta_i = x_i/r_i$ and $\sin\theta_i = y_i/r_i$ and simplifying we get

$$R_i = \sqrt{\left\{\left[\frac{P_x}{n} + \frac{My_i}{\sum\left(x_i^2 + y_i^2\right)}\right]^2 + \left[\frac{P_y}{n} + \frac{Mx_i}{\sum\left(x_i^2 + y_i^2\right)}\right]^2\right\}} \qquad \textbf{(3.23)}$$

The x_i and y_i co-ordinates should reflect the positive and negative values of the bolt location as appropriate.

3.4.2 Combined shear and moment out-of-plane

In the connection shown in Fig. 3.30, the bolts are subjected to combined shear and tension. The neutral axis may be assumed to be at a distance of one-sixth of the depth *d* above the bottom flange of the beam so as to account for the greater area in the compressed portions of the connection per unit depth.

The nominal tensile force in the bolts can be calculated assuming it to be proportional to the distance of the bolt from the neutral axis l_i in Fig. 3.30. If there exists a hard spot on the compressive load path such as a column web stiffener on the other side of the lower beam flange, the compressive force may be assumed to be acting at the mid-depth of the hard spot as shown in Fig. 3.30c. In such a case, the nominal tensile force in the bolts can be calculated in proportion to the distance of the bolt from the compressive force *($l_i = L_i$).*

$$T_i = kl_i \quad \text{where } k = \text{constant} \qquad \textbf{(3.24)}$$

$$M = \Sigma T_i L_i = k \Sigma l_i L_i \qquad \textbf{(3.25)}$$

$$T_i = Ml_i/\Sigma l_i L_i \qquad \textbf{(3.26)}$$

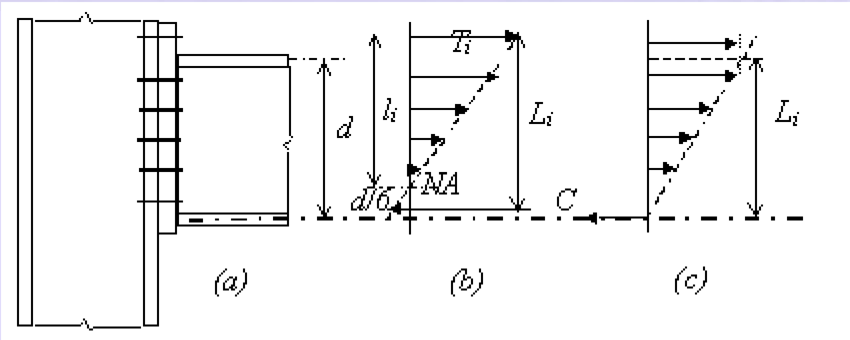


Fig. 30 Bolt group resisting out-of-plane moment

In the case of extended end plate connections, the top portion of the plate behaves as a T-stub symmetric about the tension flange. For calculating the bolt tensions in the rows immediately above and below the tension flange, l_i can be taken as the distance of the tension flange from the neutral axis to the line of action of the compressive force, as the case may be. If the end plate is thin, prying tension is likely to arise in addition to the nominal bolt tension calculated as above.

The shear can be assumed to share equally by all the bolts in the connection. Therefore, the top bolts will have to be checked for combined shear and tension as explained before.

3.5 Analysis of weld group

3.5.1 Eccentric welded connections

In some cases, eccentric loads may be applied to fillet welds causing either shear and torsion or shear and bending in the welds. Examples of such loading are shown in Fig. These two common cases are treated in this section.

Shear and torsion:

Considering the welded bracket shown in Fig. 3.31 (a), an assumption is made to the effect that the parts being joined are completely rigid and hence all the deformations occur in the weld. As seen from the figure, the weld is subjected to a combination of shear and torsion. The force caused by torsion is determined using the formula

$$F = T.s/J = (\text{Moment / Polar moment of inertia}) \qquad \textbf{\textit{(3.27)}}$$

Where, T is the tension, s is the distance from the centre of gravity of the weld to the point under consideration, and *J* is the polar moment of inertia of the weld. For convenience, the force can be decomposed into its vertical and horizontal components:

$$F_h = Tv/J \quad \text{and } f_v = Th/J \qquad \textbf{\textit{(3.28)}}$$

Where, v and h denote the vertical and horizontal components of the distance *s*. The stress due to shear force is calculated by the following expression

$$\tau = R/L \qquad \textbf{\textit{(3.29)}}$$

Where, τ is the shearing stress and *R* is the reaction and *L* is the total length of the weld. While designing a weld subjected to combined shear and torsion, it is a usual practice to assume a unit size weld and compute the stresses on a weld of unit length. From the maximum weld force per unit length the required size of the fillet weld can be calculated.

Shear and bending:

Welds, which are subjected to combined shear and bending, are shown in Fig. 3.31 (*b*). It is a common practice to treat the variation of shear stress as uniform if the welds are short. But, if the bending stress is calculated by the flexure formula, the shear stress variation for vertical welds will be parabolic with a maximum value equal to 1.5 times the average value. These bending and shear stress variations are shown in Fig. 3.32.

It may be observed here that the locations of maximum bending and shearing stresses are not the same. Hence, for design purposes the stresses need not be combined at a point. It is generally satisfactory if the weld is designed to withstand the maximum bending stress and the maximum shear stress separately. If the welds used are as shown in Fig. 3.33 it can be safely assumed that the web welds would carry all the of the shear and the flange welds all of the moment.

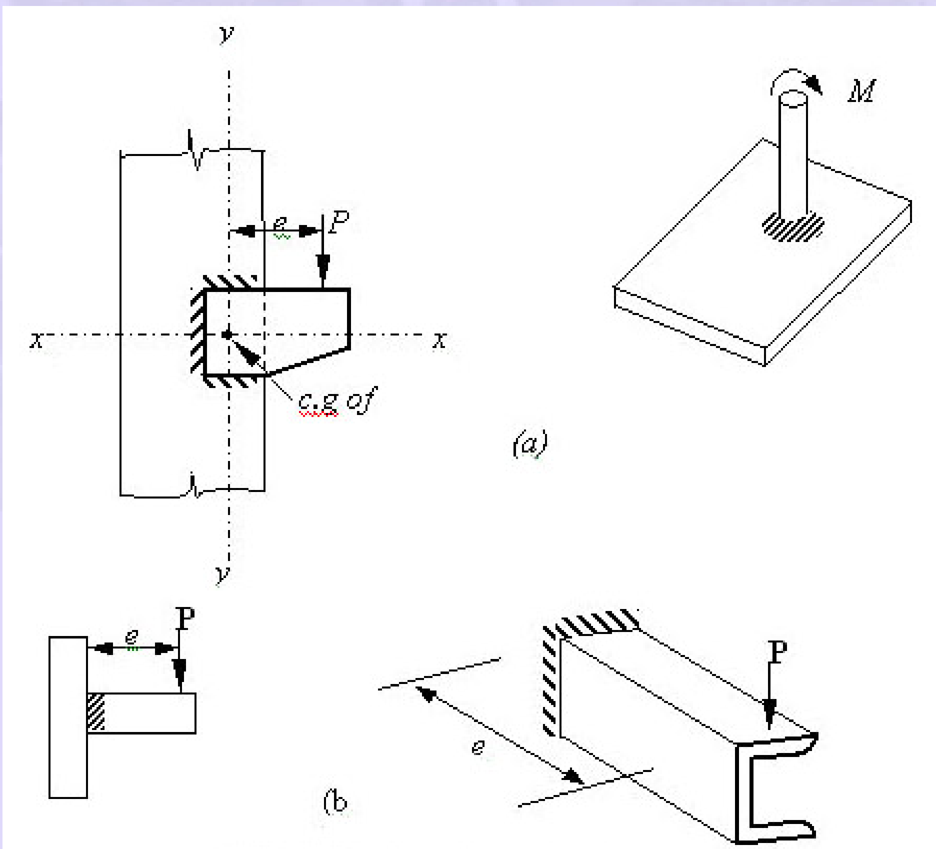


Fig. 3.31 (a) Welds subjected to shear and torsion,
(b) Welds subjected to shear and bending

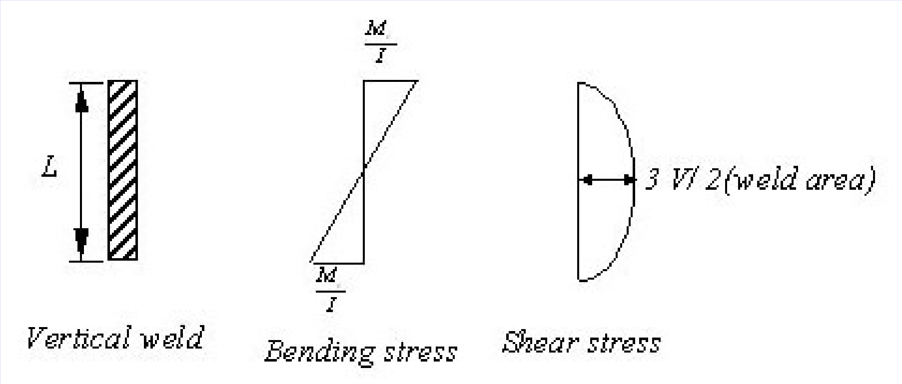


Fig. 3.32 Variation of bending and shear stress

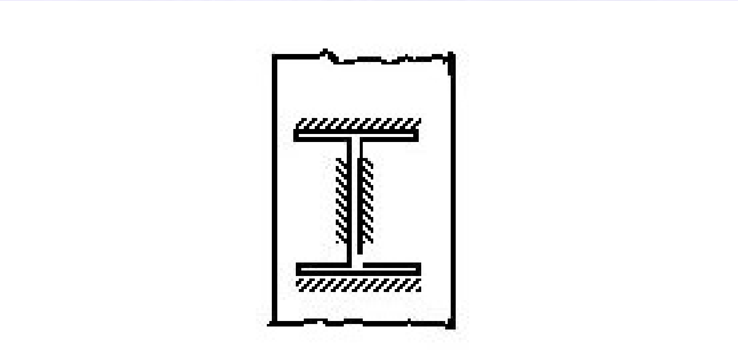


Fig.3.33 Weld provision for carrying shear and moment

When fillet welds are subjected to a combination of normal and shear stress, the equivalent stress f_e shall satisfy the following

$$f_e = \sqrt{f_a^{\,2} + 3q^2} \leq \frac{f_u}{\sqrt{3}\gamma_{mw}} \qquad \textbf{(3.30)}$$

Where, f_a is the normal stresses, compression or tension, due to axial force or bending moment and q is the shear stress due to shear force or tension.

However, check for the combination of stresses need not be done:

i) for side fillet welds joining cover plates and flange plates, and

ii) for fillet welds where sum of normal and shear stresses does not exceed *fwd.*

Similarly, the check for the combination of stresses in butt welds need not be done if:

i) butt welds are axially loaded, and

ii) in single and double bevel welds the sum of normal and shear stresses does not exceed the design normal stress, and the shear stress does not exceed 50 percent of the design shear stress.

Combined bearing, bending and shear:

Where bearing stress, f_{br} is combined with bending (tensile or compressive) and shear stresses under the most unfavorable conditions of loading, the equivalent stress, f_e, shall be obtained from the following formula

$$f_e = \sqrt{f_b^2 + f_{br}^2 + f_b \, f_{br} + 3q^2} \qquad \textbf{\textit{(3.31)}}$$

Where, f_e is the equivalent stress; f_b = calculated stress due to bending in N/mm^2; f_{br} is the calculated stress due to bearing in N/mm^2 and q = shear stress in N/mm^2. However, the equivalent stress so calculated shall not exceed the values allowed for the parent metal.

3.6 Beam and column splices

It is often required to join structural members along their length due to the available length of sections being limited and also due to transportation and erection constraints. Such joints are called splices. Splices have to be designed so as to transmit all the member forces and at the same time provide sufficient stiffness and ease in erection. Splices are usually located away from critical sections. In members subjected to instability, the splice should be preferably located near the point of lateral restraint else the splice may have to be designed for additional forces arising due to instability effects. In all cases, the requirements of the code should be satisfied.

3.6.1 Beam splices

Beam splices typically resist large bending moments and shear forces. If a rolled section beam splice is located away from the point of maximum moment, it is usually assumed that the flange splice carries all the moment and the web splice carries the shear. Such an assumption simplifies the splice design considerably. Where such simplification is not possible, as in the case of a plate girder, the total moment is divided between the flange and the web in accordance with the stress distribution. The web connection is then designed to resist its share of moment and shear (Cl.G.3).

A typical bolted splice plate connection is shown in Fig. 3.34 (*a*). To avoid deformation associated with slip before bearing in bearing bolts, HSFG bolts should be used. Usually double-splice plates are more economical because they require less number of bolts. However, for rolled steel sections with flange widths less than 200 mm, single splice plates may be used in the flange. End-plate connections may also be used as beam splices [Fig. 3.34(*b*)] although they are more flexible.

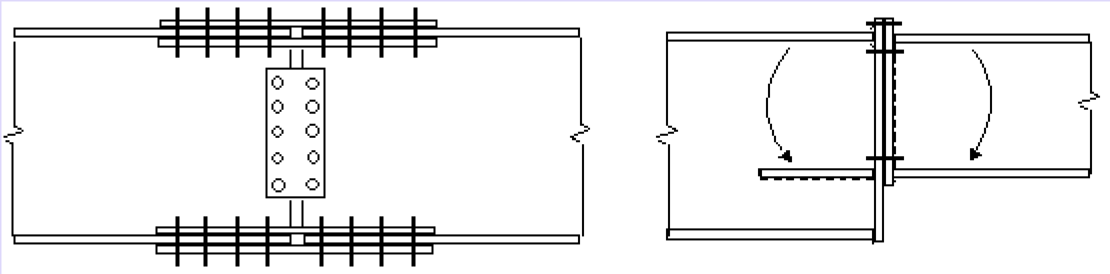


Fig. 3.34 Bolted beam splice: (a) Conventional splice (b) End-plate splice

3.6.2 Column splice

Column splices can be of two types. In the bearing type, the faces of the two columns are prepared to butt against each other and thus transmit the load by physical bearing. In such cases only a nominal connection needs to be provided to keep the columns aligned. However, this type of splice cannot be used if the column sections are not prepared by grinding, if the columns are of different sizes, if the column carries moment or if continuity is required. In such cases, HSFG bolts will have to be used and the cost of splice increases. When connecting columns of different sizes, end plates or packing plates should be provided similar to the beam splice shown in Fig. 3.34(*b*) (Cl.G.3).

3.7 Summary

Different types of bolted connections were described and classified. The bearing and friction grip bolts were introduced and their installation procedures described. The force transfer mechanisms were explained and the failure modes and corresponding strength calculations were given. This will help in the design of simple bolted connections as in the worked examples. Simple analysis methods for bolt groups resisting in-plane and out-of-plane moments were described. Beam and column splices as well as various types of beam-to-column connections were described and their general behaviour as well as points to be kept in mind during their design was explained.

3.8 References

1) IS 800-2005 (Draft) 'Code of Practice for general construction in steel', Bureau of Indian Standards, New Delhi.

2) IS 812-1957 Glossary of terms relating to welding and cutting of metals

3) IS 813-1986 Scheme of symbols for welding

4) IS 814-1991 Covered electrodes for manual metal arc welding of carbon and carbon manganese steel (fifth revision)

5) IS 1367-1992 (Parts 1 to 18) Technical supply conditions for threaded steel fasteners

6) IS 1395-1982 Low and medium alloy steel covered electrodes for manual metal arc welding (third revision)

7) IS 3640-1982 Specification for Hexagon fit bolts (first revision)

8) IS 3757-1985 Specification for high strength structural bolts (second revision)

9) IS 4000-1992 Code of practice for high strength bolts in steel structures (first revision)

10) IS 6610-1972 Specifications for heavy washers for steel structures

11) IS 6623-1985 Specifications for high strength structural nuts (first revision)

12) IS 6639-1972 Specifications for hexagonal bolts for steel structures

13) Teaching resources for structural Steel Design (Volume 1 to 3), INSDAG publication, Calcutta2000.

14) Owens. G.W and Cheal. B.D., (1989): 'Structural Steelwork Connections', Butterworths.

15) Owens G.W and Knowles P.R., (1994): 'Steel Designers Manual', The Steel Construction Institute, Blackwell Scientific Publications, ELBS 5th edition.

16) Geschwindner L.F. et al (1994): 'Load and Resistance Factor Design of Steel Structures', Prentice Hall, Englewood Cliffs, New Jersey.

Design Example 1:

Design a Lap joint between plates 100? 8 so as to transmit a factored load of 100 kN using black bolts of 12mm diameter and grade 4.6. The plates are made of steel of grade ST-42-S.

Solution:

1) Strength Calculations:

Nominal diameter of bolt d= 12 mm

For grade 4.6 bolt, f_u =40 kgf / mm^2=392.4 MPa, $_{mb}$ =1.25

Assuming threads in the shear plane, n_n=1, n_s=0

Shear Area of one bolt A_{nb} = 0.8 A_{sb} = 0.8 x 113.1 = 90.5 mm^2

Design shear strength per bolt $V_{nsb} = f_u A_{nb} / \gamma_{mb} \sqrt{3}$ = 16.4 kN (Cl. 10.3.2)

Design bearing strength per bolt V_{npb} = 2.5 d t f_u

=2.5 x 12 x 8 x 392.4 x 10^{-3} =75.2 kN (Cl. 10.3.3)

Therefore, bolt value = 16.4 kN

No. of bolts required = 100 / 16.4 = 6.1 say 7 bolts

2) Detailing:

Minimum pitch = 2.5 d = 30 mm (Cl. 10.2.1)

Minimum edge distance = 1.4 D = 16.8 mm say 20 mm (Cl. 10.2.3)

Provide 8 bolts as shown in Fig. E1.

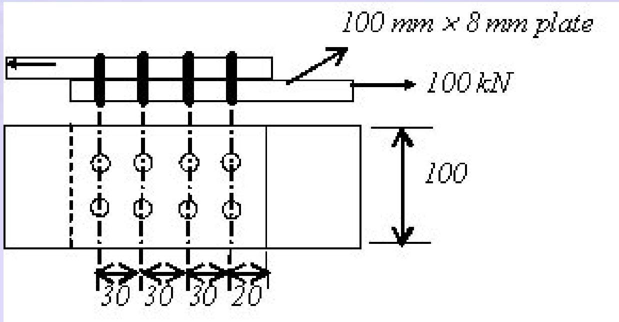


Fig. E1

Design Example 2:

Design a hanger joint along with an end plate to carry a downward load of 2T = 330 kN. Use end plate size 240 mm x 160 mm and appropriate thickness and 2 nos of M25 Gr.8.8 HSFG bolts (fo = 565 MPa).

Solution

Assume 10mm fillet weld between the hanger plate and the end plate

Distance from center line of bolt to toe of fillet weld l_v = 60 mm

1) For minimum thickness design, M = T l_v / 2 = 165 x 60 / 2 = 4950 N-m

$$\therefore t_{\min} = \sqrt{\frac{1.15 \times 4 \times 4950 \times 10^3}{236 \times 160}} = 24.56 \text{ say } 25\,mm$$

$$\mathrm{Mp} = \mathrm{Zp.fy} = \frac{\mathrm{Wt2}}{4} . \frac{\mathrm{fy}}{\gamma \mathrm{m0}}$$

$$\mathrm{t} = \sqrt{4\mathrm{Mp}\,{}^{\gamma \mathrm{m0}}\!/_{\mathrm{fy}}\ \mathrm{x\ w}}$$

2) Check for prying forces distance' l_e' from center line of bolt to prying force is the minimum of edge distance or 1.1t

$\sqrt{\ }$ (βp_o / f_y) = 1.1 x 25 $\sqrt{\ }$ (2 x 565 / 236) = 60mm (Cl. 10.4.7)

l_e = 40 mm

prying force Q= M / l_e = 4950 / 40= 123.75 kN

bolt load = 165 + 123.75=288.75 kN (Cl. 10.4.5)

tension capacity of 25 mm dia HSFG bolt = $0.9F_uA_{nb}/\gamma mb$ = 222 kN << 288.75

Load carrying Capacity << Required load Capacity

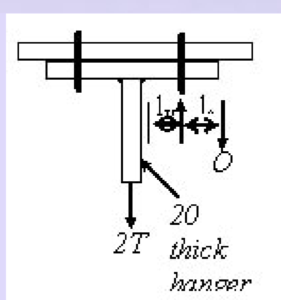


Fig E2

In order to reduce the load on bolt to a value less than the bolt capacity, a thicker end plate will have to be used.

Allowable prying force Q = 222 - 165 = 57 kN

Trying a 36 mm thick end plate gives l_e = 40 mm as before

Moment at toe of weld = T l_v - Q l_e = 165 x 60 – 57 x 40 = 7620 N-m

Moment capacity = (236 / 1.10) (160 x 36^2/4) x 10^{-3} = 11122 N-m > 7620 OK

Minimum prying force

$$Q = \frac{l_v}{2l_e}\left[T - \frac{\beta\gamma p_o b_e t^4}{27 l_e l_v^2}\right] = \frac{60}{2\times 40}\left[165 - \frac{2\times 1.5\times 0.565\times 160\times 36^4}{27\times 40\times 60^2}\right] \quad \text{(Cl. 10.4.7)}$$

$$= 36 \text{ kN} < 57 \text{ kN safe!}$$

Therefore, 36 mm end plate needs to be used to avoid significant prying action.

4. TENSION MEMBERS

4.1 Introduction

Tension members are linear members in which axial forces act so as to elongate (stretch) the member. A rope, for example, is a tension member. Tension members carry loads most efficiently, since the entire cross section is subjected to uniform stress. Unlike compression members, they do not fail by buckling (see chapter on compression members). Ties of trusses [Fig 4.1(a)], suspenders of cable stayed and suspension bridges [Fig.4.1(b)], suspenders of buildings systems hung from a central core [Fig.4.1(c)] (such buildings are used in earthquake prone zones as a way of minimising inertia forces on the structure), and sag rods of roof purlins [Fig4.1(d)] are other examples of tension members.

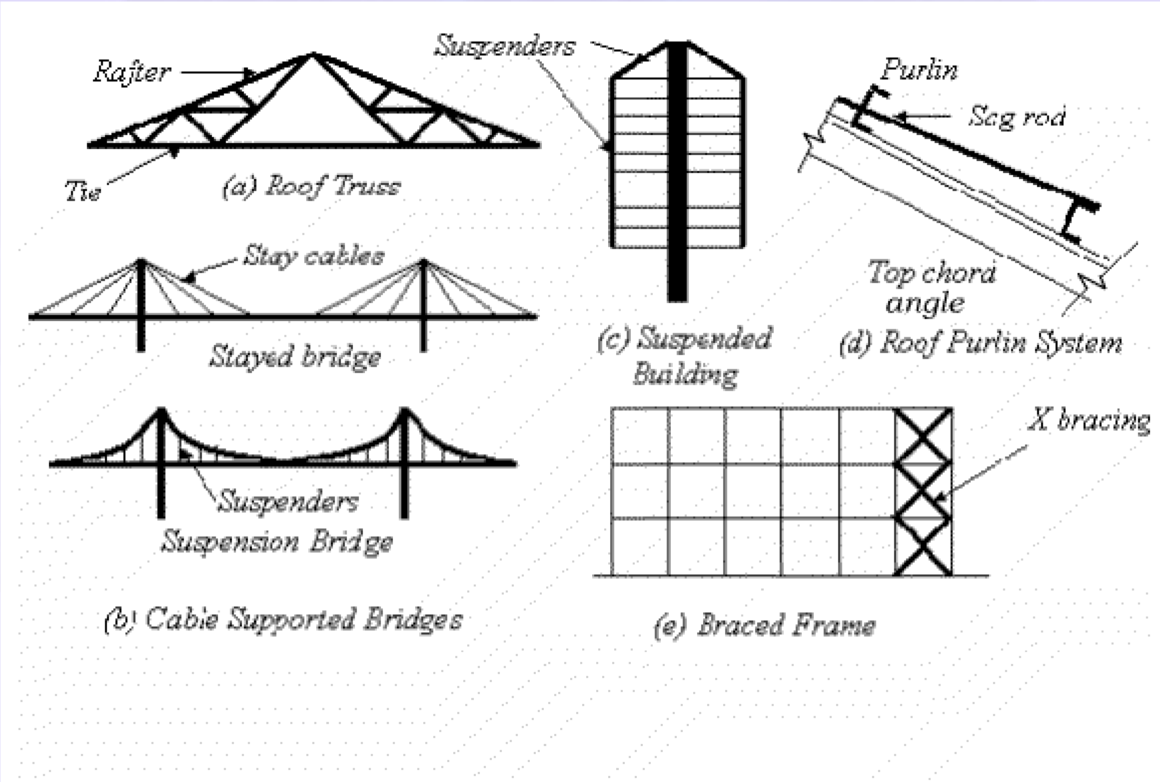


Fig 4.1 Tension members in structures

Tension members are also encountered as bracings used for the lateral load resistance. In X type bracings [Fig.4.1(e)] the member which is under tension, due to lateral load acting in one direction, undergoes compressive force, when the direction of the lateral load is changed and vice versa. Hence, such members may have to be designed to resist tensile and compressive forces.

The tension members can have a variety of cross sections. The single angle and double angle sections [Fig4.2 (a)] are used in light roof trusses as in industrial buildings. The tension members in bridge trusses are made of channels or I sections, acting individually or built-up [Figs.4.2(c) and 2(d)]. The circular rods [Fig.4.2 (d)] are used in bracings designed to resist loads in tension only. They buckle at very low compression and are not considered effective. Steel wire ropes [Fig.4.2 (e)] are used as suspenders in the cable suspended bridges and as main stays in the cable-stayed bridges.

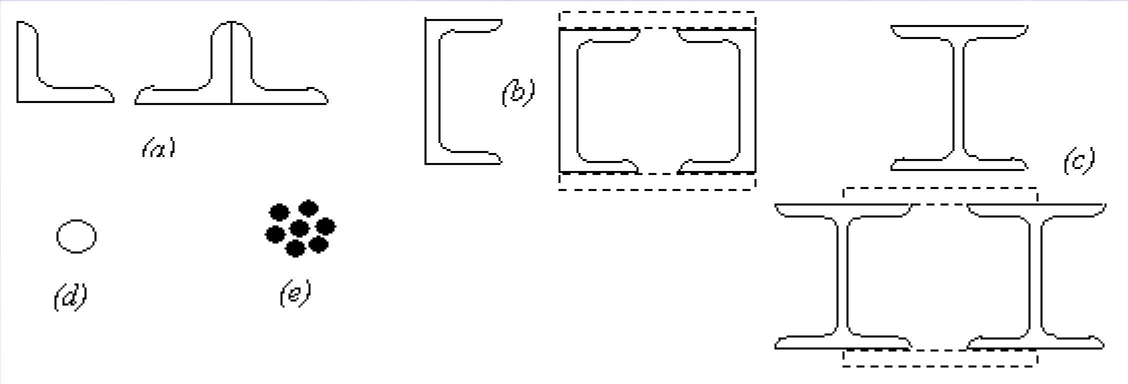


Fig 4.2 Cross sections of tension members

4.2 Behaviour of tension members

Since axially loaded tension members are subjected to uniform tensile stress, their load deformation behaviour (Fig.4.3) is similar to the corresponding basic material stress strain behaviour. Mild steel members (IS: 2062 & IS: 226) exhibit an elastic range (a-b) ending at yielding (b). This is followed by yield plateau (b-c). In the Yield Plateau the load remains constant as the elongation increases to nearly ten times the yield strain. Under further stretching the material shows a smaller increase in tension with elongation (c-d), compared to the elastic range. This range is referred to as the strain hardening range. After reaching the ultimate load (d), the loading decreases as the elongation increases (d-e) until rupture (e). High strength steel tension members do not exhibit a well-defined yield point and a yield plateau (Fig.4.3). The 0.2% offset load, T, as shown in Fig.4.3 is usually taken as the yield point in such cases.

Load-elongation of tension member to view click here

Fig. 4.3 Load – elongation of tension members

4.2.1 Design strength due to yielding of gross section

Although steel tension members can sustain loads up to the ultimate load without failure, the elongation of the members at this load would be nearly 10-15% of the original length and the structure supported by the member would become unserviceable. Hence, in the design of tension members, the yield load is usually taken as the limiting load. The corresponding design strength in member under axial tension is given by (C1.62)

$$T_d = f_y A / \gamma_{mO} \qquad \textbf{\textit{(4.1)}}$$

Where, f_y is the yield strength of the material (in MPa), A is the gross area of cross section in mm^2 and γ_{mO} is the partial safety factor for failure in tension by yielding. The value of γ_{mO} according to IS: 800 is 1.10.

4.2.2 Design strength due to rupture of critical section

Frequently plates under tension have bolt holes. The tensile stress in a plate at the cross section of a hole is not uniformly distributed in the Tension Member: Behaviour of Tension Members elastic range, but exhibits stress concentration adjacent to the hole [Fig 4.4(a)]. The ratio of the maximum elastic stress adjacent to the hole to the average stress on the net cross section is referred to as the Stress Concentration Factor. This factor is in the range of 2 to 3, depending upon the ratio of the diameter of the hole to the width of the plate normal to the direction of stress.

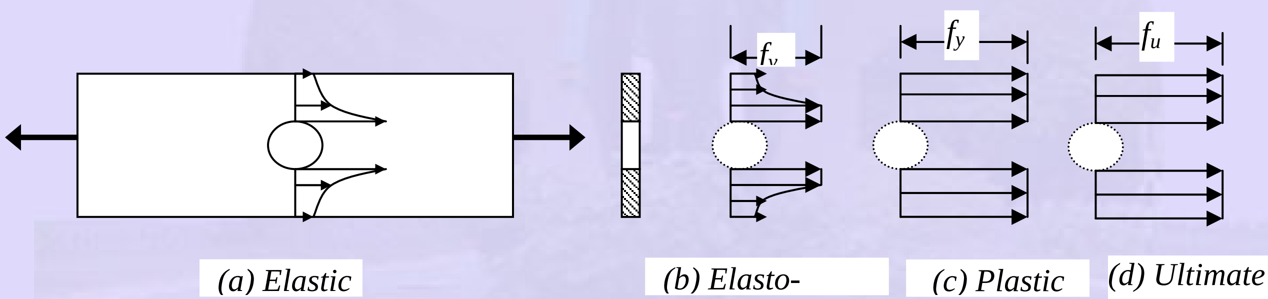


Fig. 4.4 Stress distribution at a hole in a plate under tension

In statically loaded tension members with a hole, the point adjacent to the hole reaches yield stress, f_y, first. On further loading, the stress at that point remains constant at the yield stress and the section plastifies progressively away from the hole [Fig.4.4(b)], until the entire net section at the hole reaches the yield stress, f_y, [Fig.4.4(c)]. Finally, the rupture (tension failure) of the member occurs when the entire net cross section reaches the ultimate stress, f_u, [Fig.4.4 (d)]. Since only a small length of the member adjacent to the smallest cross section at the holes would stretch a lot at the ultimate stress, and the overall member elongation need not be large, as long as the stresses in the gross section is below the yield stress. Hence, the design strength as governed by net cross-section at the hole, T_{dn}, is given by (C1.6.3)

$$P_{tn} = 0.9 f_u A_n / \gamma_{m1} \qquad \textbf{\textit{(4.2)}}$$

Where, f_u is the ultimate stress of the material, A_n is the net area of the cross section after deductions for the hole [Fig.4.4 (b)] and γ_{m1} is the partial safety factor against ultimate tension failure by rupture ($\gamma_{m1} = 1.25$). Similarly threaded rods subjected to tension could fail by rupture at the root of the threaded region and hence net area, A_n, is the root area of the threaded section (Fig.4.5).

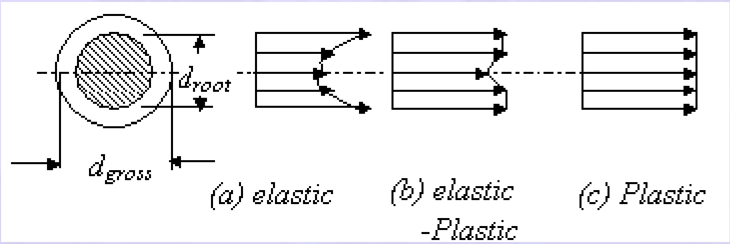


Fig 4.5 Stress in a threaded rod

The lower value of the design tension capacities, as given by Eqn.4.1 and 4.2, governs the design strength of a plate with holes.

Frequently, plates have more than one hole for the purpose of making connections. These holes are usually made in a staggered arrangement [Fig.4.6 (a)]. Let us consider the two extreme arrangements of two bolt holes in a plate, as shown in Fig.4.6 (b) & 4.6(c). In the case of the arrangement shown in Fig.4.6 (b), the gross area is reduced by two bolt holes to obtain the net area. Whereas, in arrangement shown in Fig.4.6c, deduction of only one hole is necessary, while evaluating the net area of the cross section. Obviously the change in the net area from the case shown in Fig.4.6(c) to Fig.4.6 (b) has to be gradual. As the pitch length (the centre to centre distance between holes along the direction of the stress) p, is decreased, the critical cross section at some stage changes from straight section [Fig.4.6(c)] to the staggered section 1-2-3-4 [Fig.4.6(d)]. At this stage, the net area is decreased by two bolt holes along the staggered section, but is increased due to the inclined leg (2-3) of the staggered section. The net effective area of the staggered section 1-2-3-4 is given by

$$A_n = \left(b - 2d + p^2 / 4g\right)t \qquad \textbf{\textit{(4.3)}}$$

Where, the variables are as defined in Fig.4.6 (a). In Eqn.4.3 the increase of net effective area due to inclined section is empirical and is based on test results. It can be seen from Eqn.4.3 that as the pitch distance, p, increases and the gauge distance, g, decreases, the net effective area corresponding to the staggered section increases and becomes greater than the net area corresponding to single bolt hole. This occurs when

$$p^2 / 4g > d \qquad \textbf{\textit{(4.4)}}$$

When multiple holes are arranged in a staggered fashion in a plate as shown in Fig.4.6 (a), the net area corresponding to the staggered section in general is given by

$$A_{net} = \left(b - nd + \sum \frac{p^2}{4g}\right)t \qquad \textbf{\textit{(4.5)}}$$

Where, n is the number of bolt holes in the staggered section [n = 7 for the zigzag section in Fig.4.6 (a)] and the summation over $p^2/4g$ is carried over all inclined legs of the section [equal to n-1 = 6 in Fig.4.6 (a)].

Normally, net areas of different staggered and straight sections have to be evaluated to obtain the minimum net area to be used in calculating the design strength in tension.

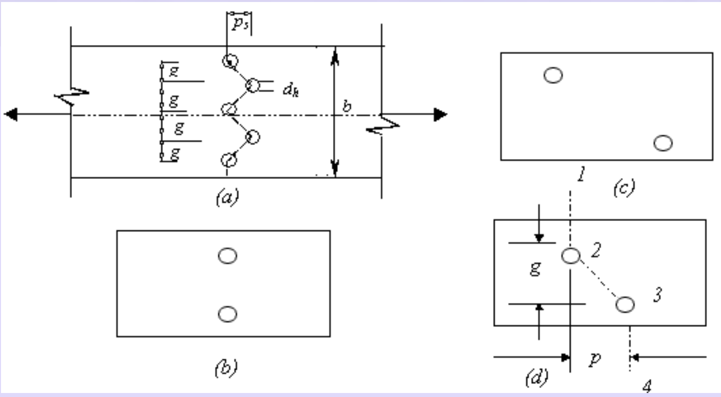


Fig 4.6 Plates with bolt hole under tension

4.2.3 Design strength due to block shear

A tension member may fail along end connection due to block shear as shown in Fig.4.7. The corresponding design strength can be evaluated using the following equations. The block shear strength T_{db}, at an end connection is taken as the smaller of (C1.64)

$$T_{db} = \left(A_{vg} f_y / \left(\sqrt{3}\,\gamma_{m0}\right) + f_u\, A_{tn} / \gamma_{m1}\right) \qquad \textbf{(4.6)}$$

or

$$T_{db} = \left(f_u\, A_{vn} / \left(\sqrt{3}\,\gamma_{m1}\right) + f_y\, A_{tg} / \gamma_{m0}\right) \qquad \textbf{(4.7)}$$

Where, A_{vg}, A_{vn} = minimum gross and net area in shear along a line of transmitted force, respectively (1-2 and 4 –3 as shown in Fig 4.6 and 1-2 as shown in Fig 4.7), A_{tg}, A_{tn} = minimum gross and net area in tension from the hole to the toe of the angle or next last row of bolt in plates, perpendicular to the line of force, respectively (2-3) as shown in Fig 4.7 and f_u, f_y = ultimate and yield stress of the material respectively

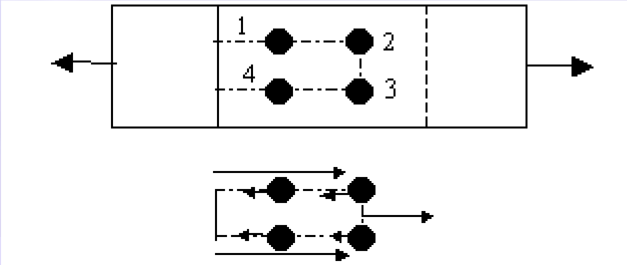


Fig 4.7 Block shearing failure plates

4.2.4 Angles under tension

Angles are extensively used as tension members in trusses and bracings. Angles, if axially loaded through centroid, could be designed as in the case of plates.

However, usually angles are connected to gusset plates by bolting or welding only one of the two legs (Fig. 4.8). This leads to eccentric tension in the member, causing non-uniform distribution of stress over the cross section. Further, since the load is applied by connecting only one leg of the member there is a shear lag locally at the end connections.

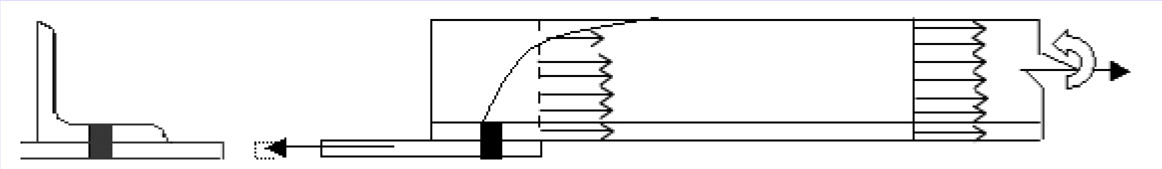


Fig 4.8 Angles eccentrically loaded through gussets

Kulak and Wu (1997) have reported, based on an experimental study, the results on the tensile strength of single and double angle members. Summary of their findings is:

- The effect of the gusset thickness, and hence the out of plane stiffness of the end connection, on the ultimate tensile strength is not significant.
- The thickness of the angle has no significant influence on the member strength.
- The effects of shear lag, and hence the strength reduction, is higher when the ratio of the area of the outstanding leg to the total area of cross-section increases.
- When the length of the connection (the number of bolts in end connections) increases, the tensile strength increases up to 4 bolts and the effect of further increase in the number of bolts, on the tensile strength of the member is not significant. This is due to the connection restraint to member bending caused by the end eccentric connection.

- Even double angles connected on opposite sides of a gusset plate experience the effect of shear lag.

Based on the test results, Kulak and Wu (1997) found that the shear lag due to connection through one leg only causes at the ultimate stage the stress in the outstanding leg to be closer only to yield stress even though the stress at the net section of the connected leg may have reached ultimate stress. They have suggested an equation for evaluating the tensile strength of angles connected by one leg, which accounts for various factors that significantly influence the strength. In order to simplify calculations, this formula has suggested that the stress in the outstanding leg be limited to f_y (the yield stress) and the connected sections having holes to be limited to f_u (the ultimate stress).

The strength of an angle connected by one leg as governed by tearing at the net section is given by (C1.6.3.3)

$$T_{tn} = \left(A_{nc} f_u / \gamma_{m1} + \beta A_0 f_y / \gamma_{m0} \right) \qquad \textbf{\textit{(4.8)}}$$

Where, f_y and f_u are the yield and ultimate stress of the material, respectively. An_{nc} and A_o, are the net area of the connected leg and the gross area of the outstanding leg, respectively. The partial safety factors $\gamma_{m0} = 1.10$ and $\gamma_{m1} = 1.25$ β accounts for the end fastener restraint effect and is given by

$$\beta = 1.4 - 0.035\left(w / t\right)\left(f_u / f_y\right)\left(b_s / L\right) \qquad \textbf{\textit{(4.9)}}$$

Where w and b_s are as shown in Fig 4.9. L = Length of the end connection, i.e., distance between the outermost bolts in the joint along the length direction or length of the weld along the length direction

Alternatively, the tearing strength of net section may be taken as

$$T_{dn} = \alpha A_n f_u / \gamma_{m1} \qquad \textbf{\textit{(4.10)}}$$

Where, $\alpha = 0.6$ for one or two bolts, 0.7 for three bolts and 0.8 for four or more bolts in the end connection or equivalent weld length, A_n = net area of the total cross section, A_{nc}= net area of connected leg, A_{go}= gross area of outstanding leg, t = thickness of the leg.

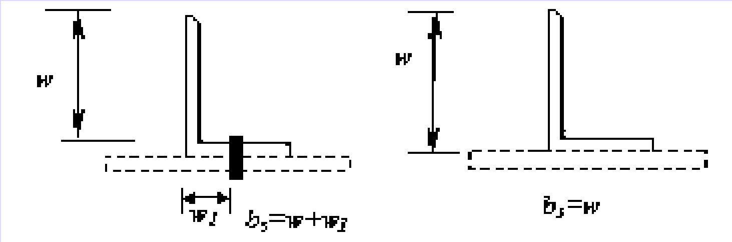


Fig 4.9 Angles with ended connections

$\beta = 1.0$, if the number of fasteners is ≤ 4,

$\beta = 0.75$ if the number of fasteners = 3 and

"Above is not recommended in code anywhere" $\beta = 0.5$, if number of fasteners = 1 or 2.

In case of welded connection, $\beta = 1.0$

The strength η as governed by yielding of gross section and block shear may be calculated as explained for the plate. The minimum of the above strengths will govern the design.

The efficiency, of an angle tension member is calculated as given below:

$$\eta = F_d / \left(A_g f_y / \gamma_{m0}\right) \quad \textit{(4.11)}$$

Depending upon the type of end connection and the configuration of the built-up member, the efficiency may vary between 0.85 and 1.0. The higher value of efficiency is obtained in the case of double angles on the opposite sides of the gusset connected at the ends by welding and the lower value is usual in the bolted single angle tension members. In the case of threaded members the efficiency is around 0.85.

In order to increase the efficiency of the outstanding leg in single angles and to decrease the length of the end connections, some times a short length angle at the ends are connected to the gusset and the outstanding leg of the main angle directly, as shown in Fig.4.10. Such angles are referred to as lug angles. The design of such connections should confirm to the codal provisions given in C1.10.12.

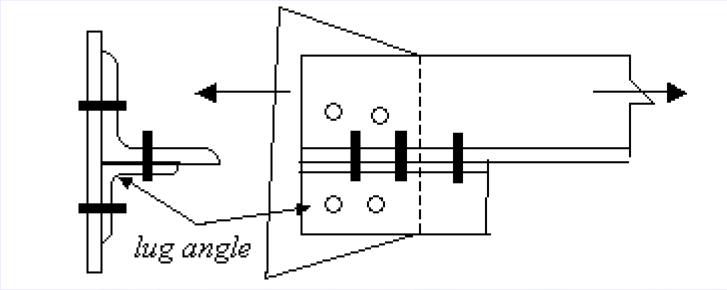


Fig 4.10 Tension member with lug

4.3 Design of tension members

In the design of a tension member, the design tensile force is given and the type of member and the size of the member have to be arrived at. The type of member is usually dictated by the location where the member is used. In the case of roof trusses, for example, angles or pipes are commonly used. Depending upon the span of the truss, the location of the member in the truss and the force in the member either single angle or double angles may be used in roof trusses. Single angle is common in the web members and the double angles are common in rafter and tie members of a roof truss.

Plate tension members are used to suspend pipes and building floors. Rods are also used as suspenders and as sag rods of roof purlins. Steel wires are used as suspender cables in bridges and buildings. Pipes are used in roof trusses on aesthetic considerations, in spite of fabrication difficulty and the higher cost of such tubular trusses. Built-up members made of angles, channels and plates are used as heavy tension members, encountered in bridge trusses.

4.3.1 Trial and error design process

The design process is iterative, involving choice of a trial section and analysis of its capacity. This process is discussed in this section. Initially, the net effective area required is calculated from the design tension and the ultimate strength of the material as given below.

$$A_n = F_t / \left(f_u / \gamma_{m1}\right) \qquad \textbf{\textit{(4.12)}}$$

Using the net area required, the gross area required is calculated, allowing for some assumed number and size of bolt holes in plates, or assumed efficiency index in the case of angles and threader rods. The gross area required is also checked against that required from the yield strength of the gross sections as given below.

$$A_g = T_d / \left(f_y / \gamma_{m0} \right) \qquad \textit{(4.13)}$$

A suitable trial section is chosen from the steel section handbook to meet the gross area required. The bolt holes are laid out appropriately in the member and the member is analyses to obtain the actual design strength of the trial section. The design strength of the trial section is evaluated using Eqs.4.1 to 4.5 in the case of plates and threaded bars and using Eqs.4.6 in the case of angle ties. If the actual design strength is smaller than or too large compared to the design force, a new trial section is chosen and the analysis is repeated until a satisfactory design is obtained.

4.3.2 Stiffness requirement

The tension members, in addition to meeting the design strength requirement, frequently have to be checked for adequate stiffness. This is done to ensure that the member does not sag too much during service due to self-weight or the eccentricity of end plate connections. The IS: 800 imposes the following limitations on the slenderness ratio of members subjected to tension:

(a) In the case of members that are normally under tension but may experience compression due to stress reversal caused by wind / earthquake loading $l / r \leq 250$.

(b) In the case of members that are designed for tension but may experience stress reversal for which it is not designed (as in X bracings). $l / r \leq 350$

(c) In the case of members subjected to tension only. $l / r \leq 400$

In the case of rods used as a tension member in X bracings, the slenderness ratio limitation need not be checked for if they are pre-tensioned by using a turnbuckle or other such arrangement.

4.4 Summary

The important factors to be considered while evaluating the tensile strength are the reduction in strength due to bolt holes and due to eccentric application of loads through gusset plates attached to one of the elements. The yield strength of the gross area or the ultimate strength of the net area may govern the tensile strength. The effect of connecting the end gusset plate to only one of the elements of the cross section can be empirically accounted for by the reduction in the effectiveness of the out standing leg, while calculating the net effective area. The iterative method has to be used in the design of tension members.

4.5 References

1. IS 800-2005 (Draft) ‘Code of practice for general construction in steel’, Bureau of Indian Standards, New Delhi.

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3. Dowling P.J., Knowles P and Owens GW., ‘ Structural Steel Design’ , The Steel construction Institute, 1988.

4. Owens GW and Knowles PR, ‘Steel Designers Manual’, fifth edition, Blackwell science 1992

Structural steel design project

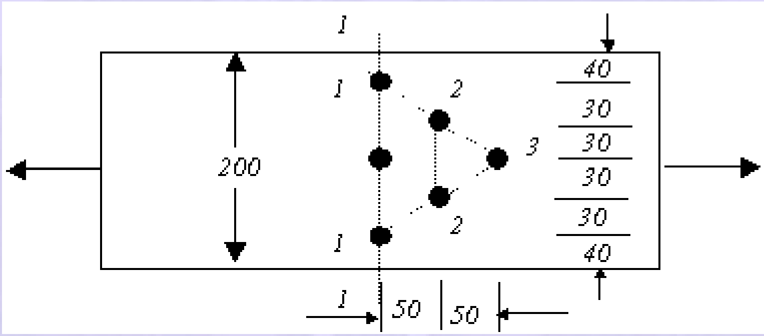
Worked example 1

Problem 1:

Determine the desig tensile strength of the plate (200 X 10 mm) with the holes as shown below, if the yield strength and the ultimate strength of the steel used are 250 MPa and 420 MPa and 20 mm diameter bolts are used.

f_y = 250 MPa

f_u = 420 MPa



Calculation of net area, A_{net}:

A_n Results you need, click here

P_t is lesser of

$$(i)\ A_g f_y / \gamma M_0 = \frac{200*10*250/1.15}{1000} = 434.8\,kN$$

$$(ii)\ 0.9 A_n f_u / \gamma M_1 = \frac{0.9*1342*420/1.25}{1000} = 405.8\,kN$$

$$P_t = 405.8\ kN$$

Efficiency of the plate with holes = $\frac{P_t}{A_g f_y / yM_0} = \frac{409.8}{434.8} = 0.93$

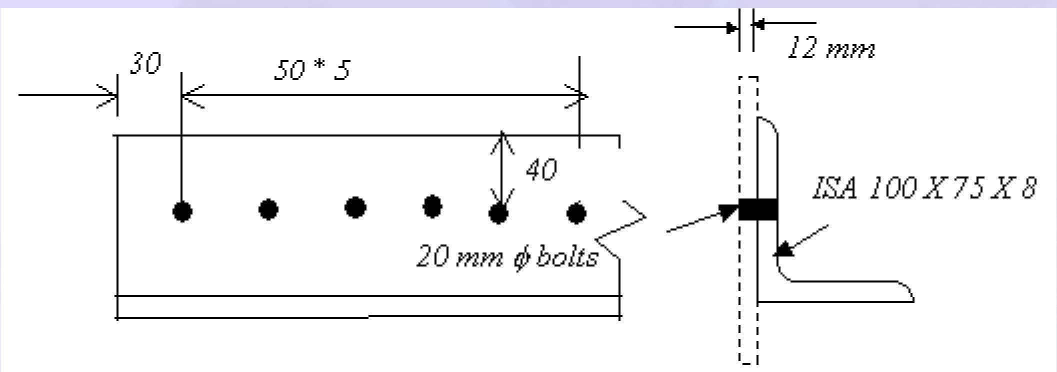
Structural steel design project

Worked example 2

Problem 2:

Analysis of single angle tension members

A single unequal angle 100 X 75 X 8 mm is connected to a 12 mm thick gusset plate at the ends with 6 nos. 20 mm diameter bolts to transfer tension. Determine the design tensile strength of the angle. (a) if the gusset is connected to the 100 mm leg, (b) if the gusset is connected to the 75 mm leg, (c) if two such angles are connected to the same side of the gusset through the 100 mm leg. (d) if two such angles are connected to the opposite sides of the gusset through 100 mm leg.



a) The 100mm leg bolted to the gusset :

A_{nc} = (100 - 8/2 - 21.5) *8 = 596 mm^2.

A_o = (75 - 8/2) * 8 = 568.mm^2

A_g = ((100-8/2) + (75 – 8/2)) * 8 =1336 mm^2

Strength as governed by tearing of net section:

Since the number of bolts = 4; $\beta = 1.0$

$$P_t = A_{nc} f_u / \gamma_{m1} + \beta A_0 f_y / \gamma_{m0}$$

= 596 * 420/1.25 + 1.0 * 568 * 250 / 1.15

= 323734 N (or) 323.7 kN

Strength as governed by yielding of gross section:

$$P_t = A_g f_y / \gamma_{m0}$$

$$= 1336 * 250 / 1.15 = 290435 \text{ N (or) } 290.4 \text{ kN}$$

Block shear strength

V_g – Grass "shearing"

t_n – Tearing net

$$P_v = \left(0.62\, A_{vg}\, f_y / \gamma_{m0} + A_{tn}\, f_u / \gamma_{m1}\right)$$ (Shear yield + tensile fracture)

= 0.62 * (5 *50 +30)* 8 * 250/1.15 + (40-21.5/2) * 8 * 420/1.25

= 380537 N = 380.5 kN

or

$$P_v = \left(0.62\, A_{vn}\, f_u / \gamma_{m1} + A_{tg}\, f_y / \gamma_{m0}\right)$$ (Shear fracture + tensile yield)

= (0.62 (5 * 50 + 30 –5.5 * 21.5) * 8 * 420 / 1.25 + 40 * 8 * 250/ 1.15)

= 339131 N = 339.1

The design tensile strength of the member = 290.4 kN

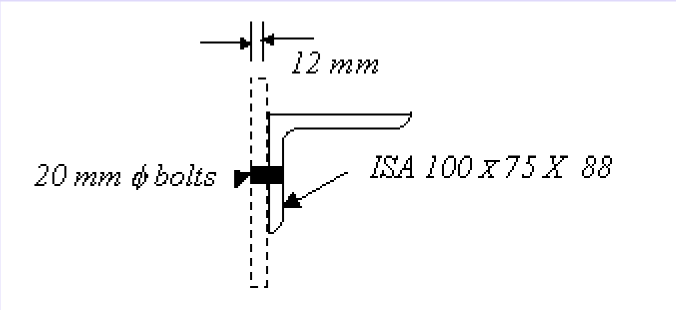
The efficiency of the tension member, is given by

$$\eta = \frac{P_t}{A_g f_y} = \frac{290.4 * 1000}{(100 + 75 - 8) * 8 * 250/1.15} = 1.0$$

b) The 75 mm leg is bolted to the gusset:

A_{nc} = (75 - 8/2 - 21.5) * 8 = 396 mm^2

A_o = (100 - 8/2) * 8 = 768 mm^2



Strength as governed by tearing of net section:

Since the number of bolts = 6, $\beta = 1.0$

$$P_t = A_{nc} f_u / \gamma_{m1} + \beta A_0 f_y / \gamma_{m0}$$

= 396 * 420/1.25 + 1.0 * 768 *250 / 1.15

= 300123 N (or) 300.1 kN

Strength as governed by yielding of gross section:

$$P_t = A_g f_y / \gamma_{m0}$$

= 1336 * 250 / 1.15 = **290435 N (or) 290.4 kN**

Block shear strength:

$$P_v \leq \left(0.62 A_{vg} f_y / \gamma_{m0} + A_{tn} f_u / \gamma_{m1}\right)$$

= 0.62 * (5 *50 +30)* 8 * 250/1.15 + (35-21.5/2) * 8 * 420/1.25

= 367097 N = **367.1 kN**

$$P_v \leq \left(0.62 A_{vn} f_u / \gamma_{m1} + A_{tg} f_y / \gamma_{m0}\right)$$

= (0.62 (5 * 50 + 30 –5.5 *x 21.5) * 8 * 420 / 1.25 + 35 *8 * 250/ 1.15

= 330435 N = **330.4 kN**

The design tensile strength of the member = **290.4 kN**

Even though the tearing strength of the net section is reduced, the yielding of the gross section still governs the design strength.

The efficiency of the tension member is as before 1.0

Note: The design tension strength is more some times if the longer leg of an unequal angle is connected to the gusset (when the tearing strength of the net section governs the design strength).

An understanding about the range of values for the section efficiency, η, is useful to arrive at the trial size of angle members in design problems.

(c & d)The double angle strength would be twice single angle strength as obtained above in case (a)

P_t = 2 * 290.4 = 580.8 kN

1.8 Summary

Steel, with its high strength to weight ratio and ductility is the most suitable material for modern construction. The physical properties of steel are a function of its metallurgy and manufacturing process. Rolled steel sections of standard dimensions are available in the market for use in construction. However, it is important to design and protect steel structures against corrosion, fire and fatigue and guidelines for this are available in the code IS 800 and other references.

1.9 References

1. IS 800-2005 (Draft) ‘Code of practice for general construction in steel’ , Bureau of Indian Standards, New Delhi.
2. IS 801-1975 Code of practice for use of cold-formed light gauge steel structural members in general building construction (first revision), Bureau of Indian Standards, New Delhi.
3. IS 1024-1999 Code of practice for use of welding in bridges and structures subject to dynamic loading (second revision), Bureau of Indian Standards, New Delhi.
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8. IS 1477-2000 (Part1) Code of practice for painting of ferrous metals in buildings: Pre-treatment (second revision), Bureau of Indian Standards, New Delhi.
9. IS 1477-1971 (Part2) Code of practice for painting of ferrous metals in buildings: painting (first revision), Bureau of Indian Standards, New Delhi.
10. SP7-2003 Part 4: Fire and life safety, Bureau of Indian Standards, New Delhi.
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13. Dowling P.J., Knowles P and Owens GW., ‘ Structural Steel Design’ , The Steel construction Institute, 1988.
14. Owens GW and Knowles PR, ‘Steel Designers Manual’, fifth edition, Blackwell science 1992

5.10 Summary

In the previous sections, the behaviour of practical columns subjected to axial compressive loading was discussed and the following conclusions were drawn.

· Very short columns subjected to axial compression fail by yielding. Very long columns fail by buckling in the Euler mode.

· Practical columns generally fail by inelastic buckling and do not conform to the assumptions made in Euler theory. They do not normally remain linearly elastic upto failure unless they are very slender

· Slenderness ratio (l/r) and material yield stress (f_y) are dominant factors affecting the ultimate strengths of axially loaded columns.

· The compressive strengths of practical columns are significantly affected by (i) the initial imperfection (ii) eccentricity of loading (iii) residual stresses and (iv) lack of distinct yield point and strain hardening. Ultimate load tests on practical columns reveal a scatter band of results shown in Fig. 5.19. A lower bound curve of the type shown therein can be employed for design purposes.

5.11 Concluding remarks

The elastic buckling of an ideally straight column pin ended at both ends and subjected to axial compression was considered. The elastic buckling load was shown to be dependent on the slenderness ratio (/r) of the column. Factors affecting the column strengths (viz. initial imperfection, eccentricity of loading, residual stresses and lack of well-defined elastic limit) were all individually considered. Finally a generalized column strength curve (taking account of all these factors) has been suggested, as the basis of column design curves employed in Design Practices. The concept of **"effective length"** of the column has been described, which could be used as the basis of design of columns with differing boundary conditions.

The phenomenon of Elastic Torsional and Torsional-flexural buckling of a perfect column were discussed conceptually. The instability effects due to torsional buckling of slender sections are explained and discussed.

Design of columns using multiple column curves as given in the code; was discussed. Built-up fabricated members frequently employed (when rolled sections are found inadequate) were discussed in detail. Design guidance is provided for laced/battened columns. Steps in the design of axially loaded column were listed.

5. COMPRESSION MEMBERS

5.1 Introduction

Column, top chords of trusses, diagonals and bracing members are all examples of compression members. Columns are usually thought of as straight compression members whose lengths are considerably greater than their cross-sectional dimensions.

An initially straight strut or column, compressed by gradually increasing equal and opposite axial forces at the ends is considered first. Columns and struts are termed **“long”** or **“*short*”** depending on their proneness to buckling. If the strut is **“short”**, the applied forces will cause a compressive strain, which results in the shortening of the strut in the direction of the applied forces. Under incremental loading, this shortening continues until the column yields or **"squashes"**. However, if the strut is **“long”**, similar axial shortening is observed only at the initial stages of incremental loading. Thereafter, as the applied forces are increased in magnitude, the strut becomes **“unstable”** and develops a deformation in a direction normal to the loading axis and its axis is no longer straight. (See Fig.5.1). The strut is said to have “buckled”.

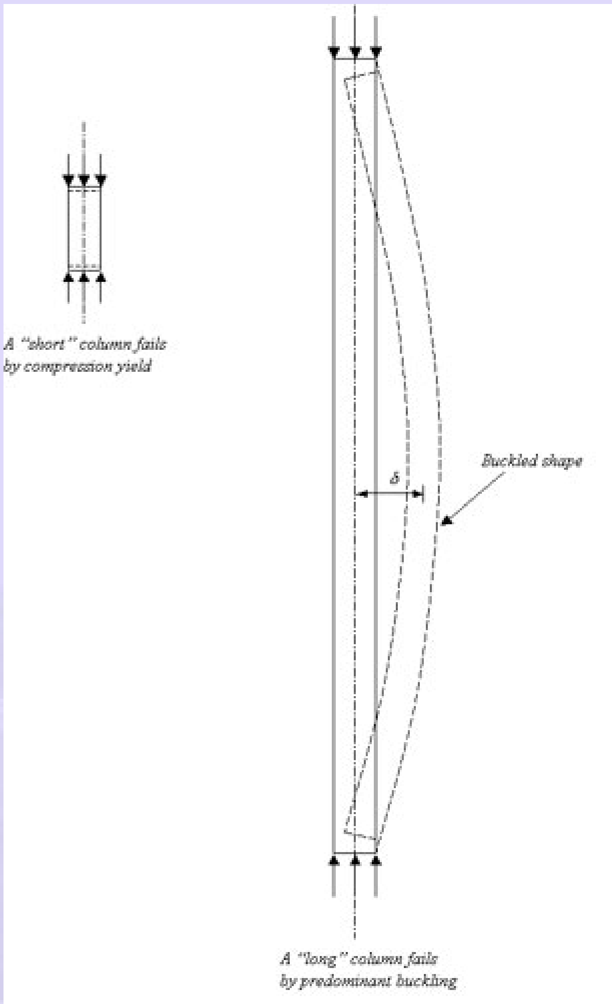


Fig 5.1 'short' vs 'long' columns

Short Columns **Long Columns**

Buckling behaviour is thus characterized by large deformations developed in a direction (or plane) normal to that of the loading that produces it. When the applied loading is increased, the buckling deformation also increases. Buckling occurs mainly in members subjected to compressive forces. If the member has high bending stiffness, its buckling resistance is high. Also, when the member length is increased, the buckling resistance is decreased. Thus the buckling resistance is high when the member is short or **"stocky"** (i.e. the member has a high bending stiffness and is short) conversely, the buckling resistance is low when the member is long or **"*slender*"**.

Structural steel has high yield strength and ultimate strength compared with other construction materials. Hence compression members made of steel tend to be slender compared with reinforced concrete or prestressed concrete compression members. Buckling is of particular interest while employing slender steel members. Members fabricated from steel plating or sheeting and subjected to compressive stresses also experience local buckling of the plate elements. This chapter introduces buckling in the context of axially compressed struts and identifies the factors governing the buckling behaviour. Both global and local buckling is instability phenomena and should be avoided by an adequate margin of safety.

Traditionally, the design of compression members was based on Euler analysis of ideal columns which gives an upper band to the buckling load. However, practical columns are far from ideal and buckle at much lower loads. The first significant step in the design procedures for such columns was the use of Perry Robertsons curves. Modern codes advocate the use of multiple-column curves for design. Although these design procedures are more accurate in predicting the buckling load of practical columns, Euler's theory helps in the understanding of the behaviour of slender columns and is reviewed in the following sections.

5.2 Elastic buckling of an ideal column or struct with pinned end

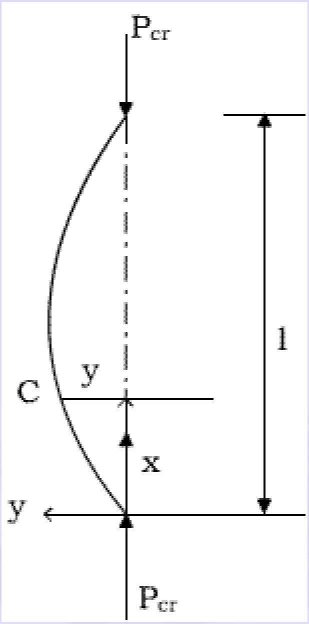


Fig 5.2 Buckling of pin-ended column

The classical Euler analysis of the elastic behaviour of an idealized, pin-ended, uniform strut makes the following assumptions.

- The material is homogeneous and linearly elastic (i.e. it obeys Hooke's Law).
- The strut is perfectly straight and there are no imperfections.
- The loading is applied at the centroid of the cross section at the ends.

We will assume that the member is able to bend about one of the principal axes. (See Fig. 5.2). Initially, the strut will remain straight for all values of P, but at a particular value $P = P_{cr}$, it buckles. Let the buckling deformation at a section distant x from the end B be y.

The bending moment at this section = $P_{cr}.y$

The differential equation governing the deformation can be obtained by considering moment equilibrium about point C as

$$P_{cr}\, y + EI\frac{d^2y}{dx^2} = 0 \qquad \textbf{\textit{(5.1)}}$$

The general solution for this differential equation is given by

$$y = A_1 \mathrm{Cos}\, x\sqrt{\frac{P_{cr}}{E1}} + B_1 \mathrm{Sin}\, x\sqrt{\frac{P_{cr}}{E1}} \qquad \textbf{\textit{(5.2)}}$$

Where A_1 and B_1 are constants.

Since $y = 0$ when x = 0, $A_1 = 0$.

Also y = 0 when x = L gives

$$B_1 \mathrm{Sin}\, L\sqrt{\frac{P_{cr}}{E1}} = 0$$

Thus, either $B_1 = 0$ or $B_1 \mathrm{Sin}\, L\sqrt{\frac{P_{cr}}{E1}} = 0$

$B_1 = 0$ means $y = 0$ for all values of x (i.e. the column remains straight).

Alternatively

This equation is satisfied only when

$$L\sqrt{\frac{P_{cr}}{E1}} = 0, \pi, 2\pi, \ldots\ldots\ldots.$$

$$\text{This gives} \qquad \textbf{\textit{(5.3)}}$$

$$p_{cr} = \frac{\pi^2 EI}{L^2}, \frac{4\pi^2 EI}{L^2}, \ldots\ldots\ldots \frac{n^2\pi^2 EI}{L^2}$$

Where n is any integer.

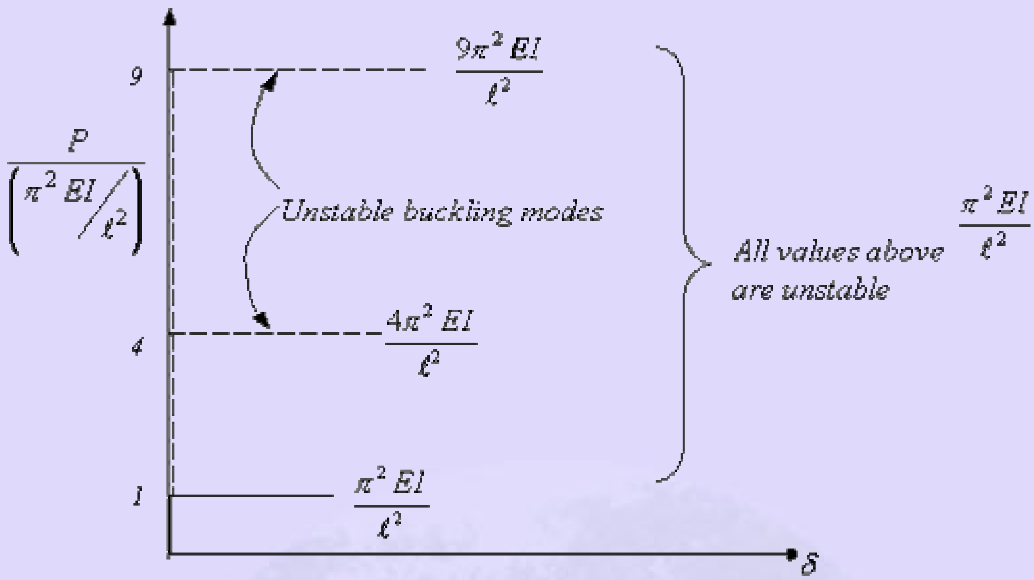


Fig 5.3 Buckling load Vs centre lateral deflection relationship

While there are several buckling modes each corresponding to n = 1, 2, 3… (See Fig. 5.3) the lowest stable buckling mode corresponds to n = 1.

The lowest value of the critical load (i.e. the load causing buckling) is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \textbf{\textit{(5.4)}}$$

Thus the Euler buckling analysis for a **“straight"** strut; will lead to the following conclusions:

1. The strut can remain straight for any value of P.
2. Under incremental loading, when P reaches a value of $P_{cr} = \pi^2 EI / L^2$ the strut can buckle in the shape of a half-sine wave; the amplitude of this buckling deflection is indeterminate.

3. At higher values of the loads given by $n^2\pi^2 EI / L^2$ other sinusoidal buckled shapes (n half waves) are possible. However, it is possible to show that the column will be in unstable equilibrium for all values of $P > \pi^2 EI / L^2$ whether it be straight or buckled. *This means that the slightest disturbance will cause the column to deflect away from its*

original position. Elastic Instability may be defined in general terms as a condition in which the structure has no tendency to return to its initial position when slightly disturbed, even when the material is assumed to have an infinitely large yield stress. Thus $P_{cr} = \pi^2EI / L^2$ represents the maximum load that the strut can usefully support.

It is often convenient to study the onset of elastic buckling in terms of the mean applied compressive stress (rather than the force). The mean compressive stress at buckling, f_{cr} , is given by

$$f_{cr} = \frac{p_{cr}}{A} = \frac{\pi^2}{AL^2} = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 E}{\lambda^2} \qquad \textbf{\textit{(5.5)}}$$

Where *A* = area of cross section of the strut, *r* = radius of gyration of the cross section, ($I = Ar^2$) and $\lambda = L / r$

λ = the slenderness ratio of the column defined by

The equation $f_{cr} = (\pi^2E) / \lambda^2$, implies that the critical stress of a column is inversely proportional to the square of the slenderness ratio of the column (see Fig. 5.4).

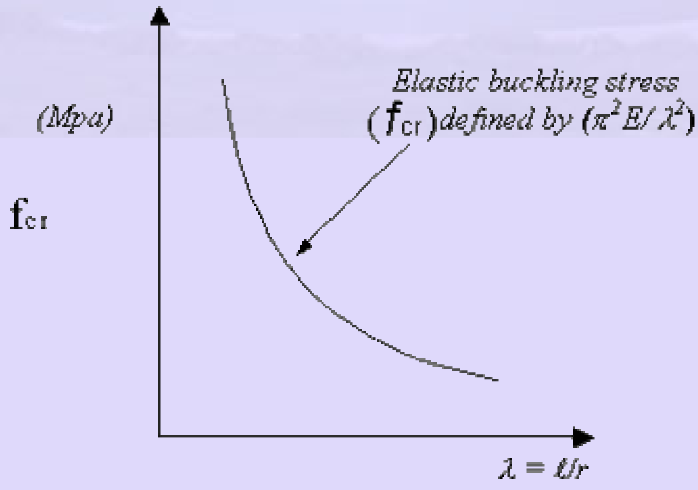


Fig 5.4 Euler buckling relation between f_{cr} and λ

5.3 Strength curve for all ideal strut

We will assume that the stress-strain relationship of the material of the column is as shown in Fig. 5.5. Such a strut under compression can therefore resist only a maximum force given by f_y.A, when plastic squashing failure would occur by the plastic yielding of the entire cross section; this means that the stress at failure of a column can never exceed f_y , shown by $A\text{-}A^1$ in Fig. 5.6(a).

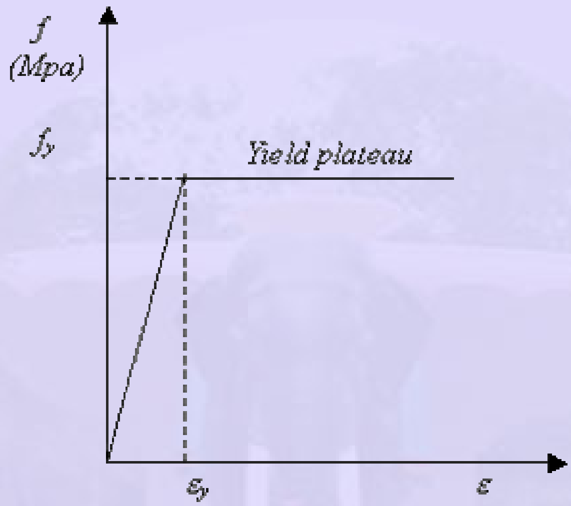


Fig 5.5 Idealised elastic-plastic relationship for steel

From Fig. 5.4, it is obvious that the column would fail by buckling at a stress given by (π^2 EI / λ^2)

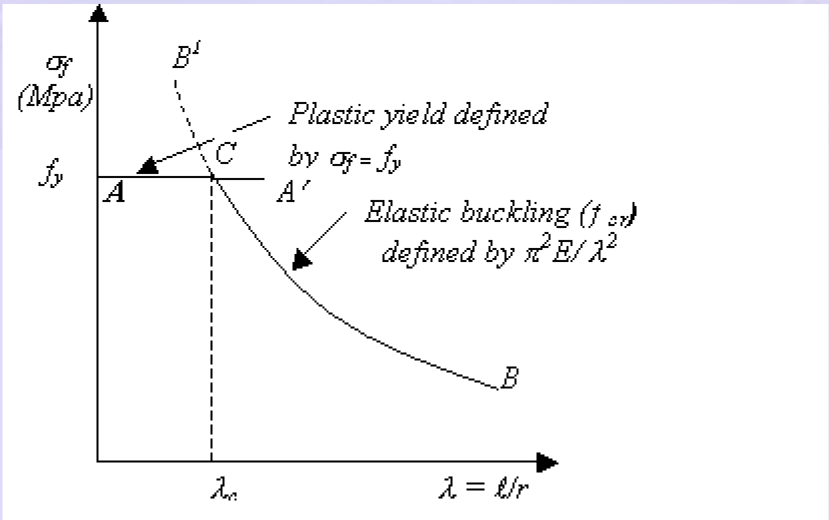


Fig 5.6(a) Strength curve for an axially loaded initially straight pin-ended column

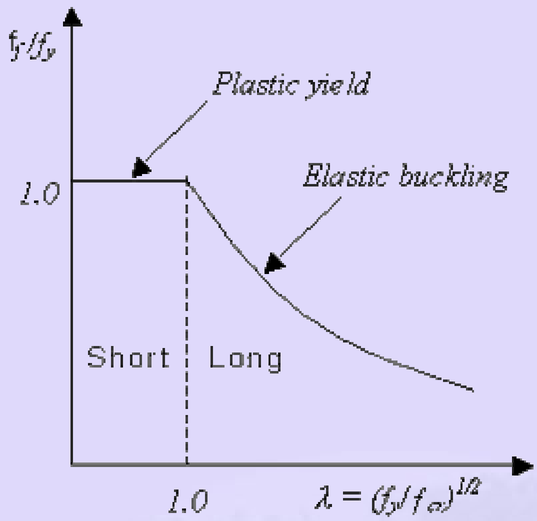


Fig 5.6(b) Strength curve in a non-dimensional form

This is indicated by B-B[1] in Fig. 5.6(a), which combines the two types of behaviour just described. The two curves intersect at C. Obviously the column will fail when the axial compressive stress equals or exceeds the values defined by ACB. In the region AC, where the slenderness values are low, the column fails by yielding. In the region *CB*, the failure will be triggered by buckling. The changeover from yielding to buckling failure occurs at the point C, defined by a slenderness ratio given by λ_c and is evaluated from

$$f_y = \frac{\pi^2 E}{\lambda_c^2}$$
$$\lambda_c = \pi\sqrt{\frac{E}{f_y}} \qquad \textbf{(5.6)}$$

Plots of the type Fig. 5.6(a) are sometimes presented in a non-dimensional form as illustrated in Fig. 5.6(b). Here (f_f / f_y) is plotted against a generalized slenderness given by

$$\lambda_g = \lambda / \lambda_c = \sqrt{\frac{f_y}{f_{cr}}} \qquad \textbf{(5.7)}$$

This single plot can be employed to define the strength of all axially loaded, initially straight columns irrespective of their E and f_y values. The change over from plastic yield to elastic critical buckling failure occurs when $\lambda_g = 1$ (i.e when $f_y = \sigma_{cr}$), the corresponding slenderness ratio is $\pi \sqrt{E / f_y}$. This slenderness ratio demarcates short and long columns.

5.4 Strength of compression members in practice

The highly idealized straight form assumed for the struts considered so far cannot be achieved in practice. Members are never perfectly straight and they can never be loaded exactly at the centroid of the cross section. Deviations from the ideal elastic plastic behaviour defined by Fig. 5 are encountered due to strain hardening at high strains and the absence of clearly defined yield point in some steel. Moreover, residual stresses locked-in during the process of rolling also provide an added complexity.

Thus the three components, which contribute to a reduction in the actual strength of columns (compared with the predictions from the "ideal" column curve) are

(i) Initial imperfection or initial bow.

(ii) Eccentricity of application of loads.

(iii) Residual stresses locked into the cross section.

5.4.1 The effect of initial out-of-straightness

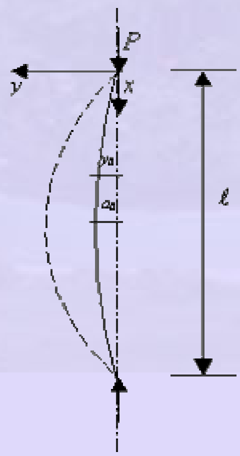


Fig 5.7 Pin -ended strut with initial imperfection

A pin-ended strut having an initial imperfection and acted upon by a gradually increasing axial load is shown in Fig 5.7. As soon as the load is applied, the member experiences a bending moment at every cross section, which in turn causes a bending deformation. For simplicity of calculations, it is usual to assume the initial shape of the column defined by

$$y_0 = a_0 \sin \frac{\pi x}{l} \qquad \textbf{(5.8)}$$

where a_o is the maximum imperfection at the centre, where x = l / 2. Other initial shapes are, of course, possible, but the half sine-wave assumed above corresponding to the lowest mode shape, represents the greatest influence on the actual behaviour, and hence is adequate.

Provided the material remains elastic, it is possible to show that the applied force, P, enhances the initial deflection at every point along the length of the column by a multiplier factor, given by

$$MF = \frac{1}{1 - \left(\frac{P}{P_{cr}}\right)} \qquad \textbf{(5.9)}$$

The deflection will tend to infinity, as P is tends to P_{cr} as shown by curve-A, in Fig. 5.8. However the column will fail at a lower load P_f when the deflection becomes large enough. The corresponding stress is denoted as f_f

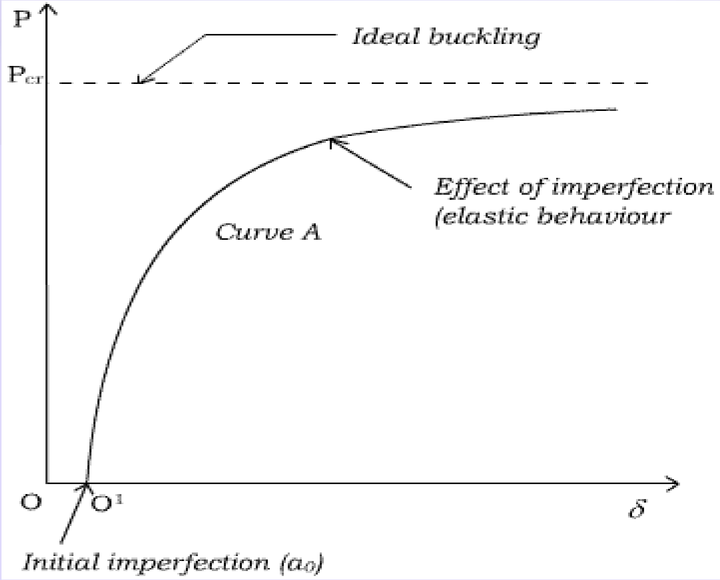


Fig 5.8 Theoretical and actual load deflection response of a strut with initial imperfection

If a large number of imperfect columns are tested to failure, and the data points representing the values of the mean stress at failure plotted against the slenderness (λ) values, the resulting lower bound curve would be similar to the curve shown in Fig. 5.9.

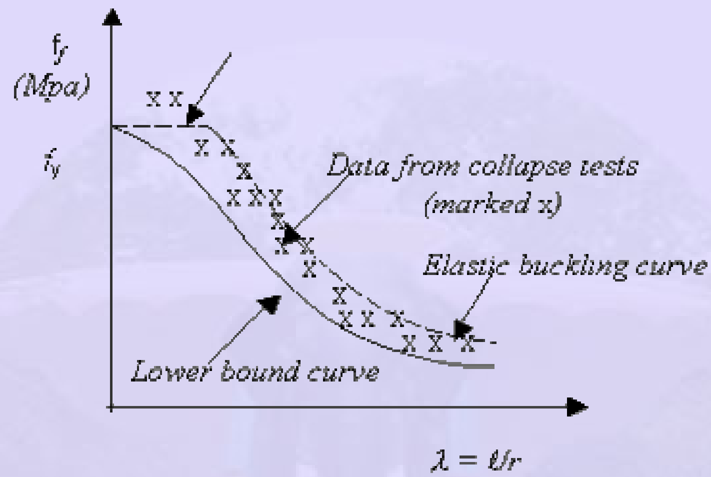


Fig 5.9 Strength curves for strut with initial imperfection

For very stocky members, the initial out of straightness – which is more of a function of length than of cross sectional dimensions – has a very negligible effect and the failure is at plastic squash load. For a very slender member, the lower bound curve is close to the elastic critical stress (f_{cr}) curve. At intermediate values of slenderness the effect of initial out of straightness is very marked and the lower bound curve is significantly below the f_y line and f_{cr} line.

5.4.2 The effect of eccentricity of applied loading

As has already been pointed out, it is impossible to ensure that the load is applied at the exact centroid of the column. Fig. 5.10 shows a straight column with a small eccentricity (e) in the applied loading. The applied load (P) induces a bending moment (P.e) at every cross section. This would cause the column to deflect laterally, in a manner similar to the initially deformed member discussed previously. Once again the greatest compressive stress will occur at the concave face of the column at a section midway along its length. The load-deflection response for purely elastic and elastic-plastic behaviour is similar to those described in Fig. 5.8 except that the deflection is zero at zero load.

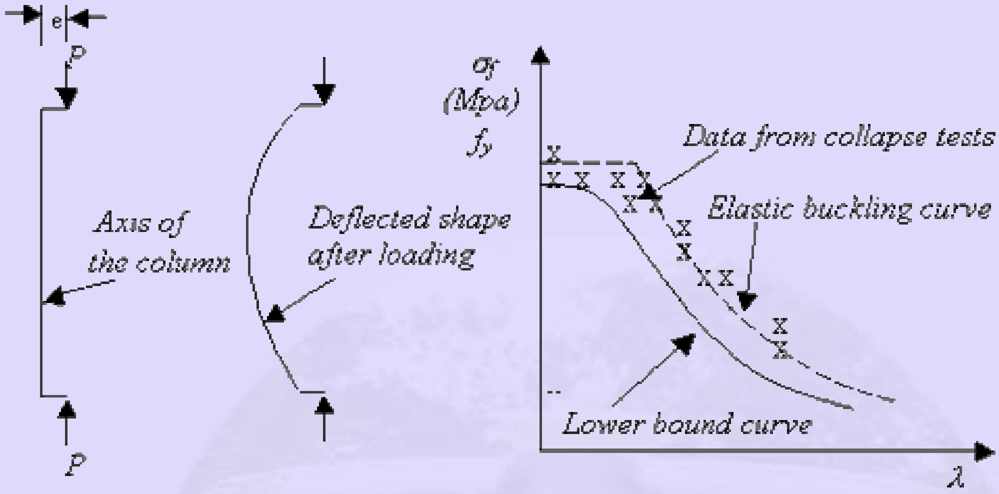


Fig 5.10 Strength curves for *eccentrically loaded columns*

The form of the lower bound strength curve obtained by allowing for eccentricity is shown in Fig. 5.10. The only difference between this curve and that given in Fig. 5.9 is that the load carrying capacity is reduced (for stocky members) even for low values of λ .

5.4.3 The effect of residual stress

As a consequence of the differential heating and cooling in the rolling and forming processes, there will always be inherent residual stresses. A simple explanation for this phenomenon follows. Consider a billet during the rolling process when it is shaped into an I section. As the hot billet shown in Fig. 5.11(a) is passed successively through a series of rollers, the shapes shown in 5.11(b), (*c*) and (d) are gradually obtained. The outstands (b-b) cool off earlier, before the thicker inner elements (a-a) cool down.

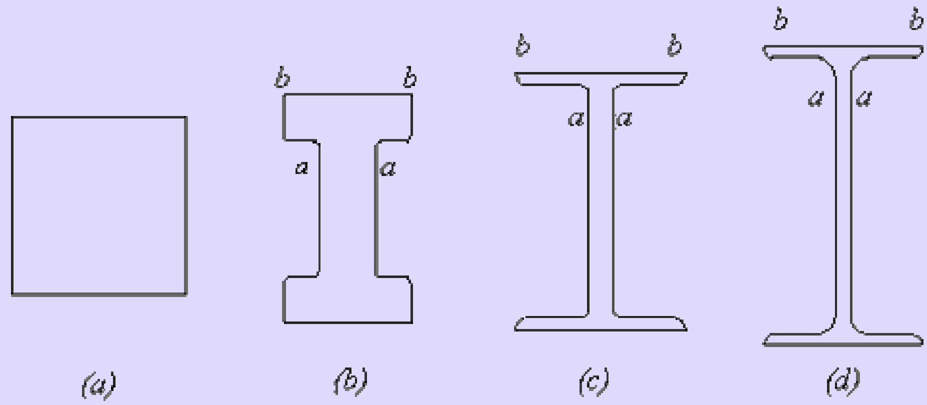


Fig 5.11 Various stages of rolling a steel grider

As one part of the cross section (b-b) cools off, it tends to shrink first but continues to remain an integral part of the rest of the cross section. Eventually the thicker element (a) also cools off and shrinks. As these elements remain composite with the edge elements, the differential shrinkage induces compression at the outer edges (b). But as the cross section is in equilibrium – these stresses have to be balanced by tensile stresses at inner location (*a*). These stress called residual stresses, can sometimes be very high and reach upto yield stress.

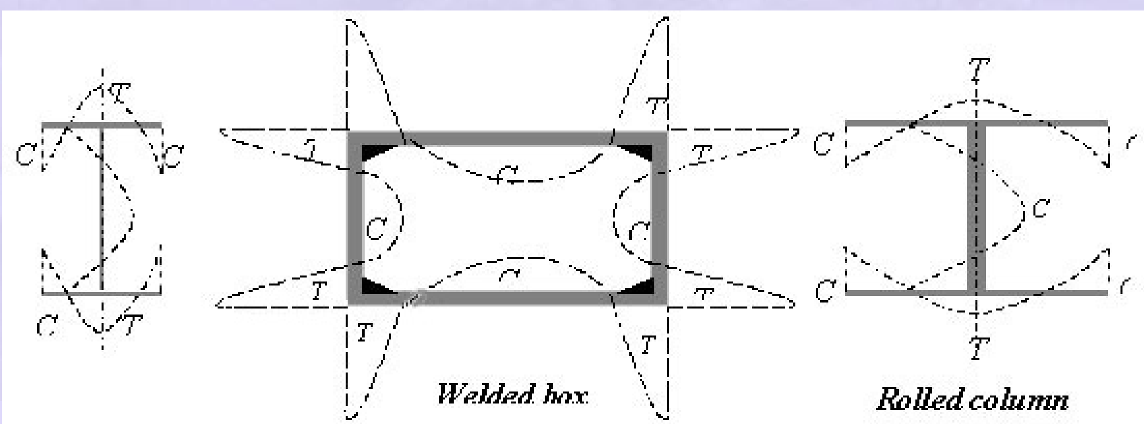


Fig. 5.12 Distribution of residual stresses

Consider a short compression member (called a "stub column", having a residual stress distribution as shown in Fig. 5.12. When this cross section is subjected to an applied uniform compressive stress (f_a) the stress distribution across the cross section can be obtained by superposing the applied stress over the residual stress f_r, provided the total stress nowhere reaches yield, the section continues to deform elastically. Under incremental loading, the flange tips will yield first when $[(f_a + f_r) = f_y]$. Under further loading, yielding will spread inwards and eventually the web will also yield. When $f_a = f_y$, the entire section will have yielded and the column will get squashed.

Only in a very stocky column (i.e. one with a very low slenderness) the residual stress causes premature yielding in the manner just described. The mean stress at failure will be f_y, i.e. failure load is not affected by the residual stress. A very slender strut will fail by buckling, i.e. $f_{cr} << f_y$. For struts having intermediate slenderness, the premature yielding at the tips reduces the effective bending stiffness of the column; in this case, the column will buckle elastically at a load below the elastic critical load and the plastic squash load. The column strength curve will thus be as shown in Fig. 5.13.

Notice the difference between the buckling strength and the plastic squash load is most pronounced when

$\lambda = l / r = \pi (E / f_y)^{1/2}$

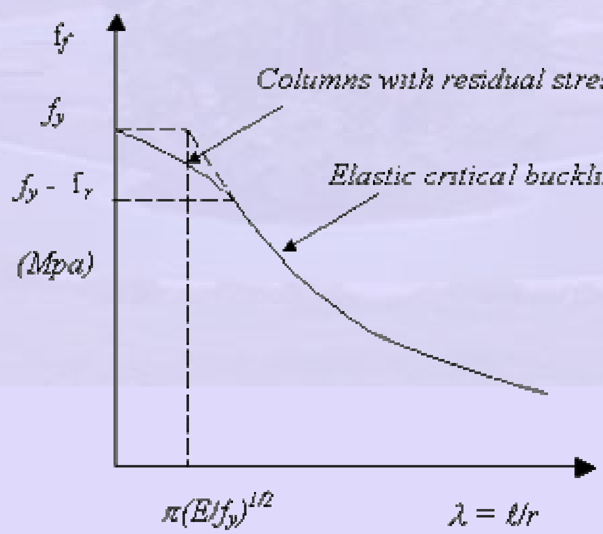


Fig 5.13 Strength curve for columns having residual stress

5.4.4 The effect of strain-hardening and the absence of clearly defined yield point

If the material of the column shows strain hardering after an yield platean, the onset of first yield will not be affected, but the collapse load may be increased. Designers tend to ignore the effect of strain hardening which in fact provides an additional margin of safety.

High strength steels generally have stress-strain curves without a clear yield point. At stresses above the limit of proportionality (f_p), the material behaviour is non linear and on unloading and reloading the material is linear-elastic. Most high strength structural steels have an ultimate stress beyond which the curve becomes more or less horizontal. Some steels do not have a plastic plateau and exhibit strain-hardening throughout the inelastic range. In such cases, the yield stress is generally taken as the 0.2% proof stress, for purposes of computation.

5.4.5 The effect of all features taken together

In practice, a loaded column may experience most, if not all, of the effects listed above i.e. out of straightness, eccentricity of loading, residual stresses and lack of clearly defined yield point and strain hardening occurring simultaneously. Only strain hardening tends to raise the column strengths, particularly at low slenderness values. All other effects lower the column strength values for all or part of the slenderness ratio range.

When all the effects are put together, the resulting column strength curve is generally of the form shown in Fig. 5.14. The beneficial effect of strain hardening at low slenderness values is generally more than adequate to provide compensation for any loss of strength due to small, accidental eccentricities in loading. Although the column strength can exceed the value obtained from the yield strength (f_y), for purposes of structural design, the column strength curve is generally considered as having a cut off at f_y, to avoid large plastic compressive deformation. Since it is impossible to quantify the variations in geometric imperfections, accidental eccentricity, residual stresses and material properties, it is impossible to calculate with certainty, the greatest reduction in strength they might produce in practice. Thus for design purposes, it may be impossible to draw a true lower bound column strength curve. A commonly employed method is to construct a curve on the basis of specified survival probability. (For example, over 98% of the columns to which the column curve relates, can be expected - on a statistical basis – to survive at applied loads equal to those given by the curve). All design codes provide column curves based on this philosophy. Thus a lower band curve (Fig 5.14) or a family of such curves is used in design.

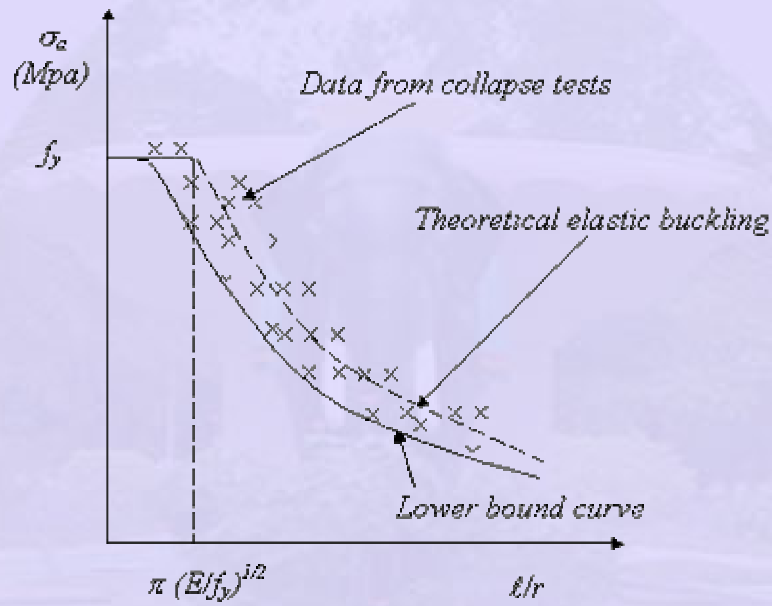


Fig. 5.14 Column strength curves for struts used in practice

5.5 The concepts of effective lengths

So far, the discussion in this chapter has been centred around pin-ended columns. The boundary conditions of a column may, however, be idealized in one the following ways

- Both the ends pin jointed (i.e. the case considered before)
- Both ends fixed.
- One end fixed and the other end pinned.
- One end fixed and the other end free.

By setting up the corresponding differential equations, expressions for the critical loads as given below are obtained and the corresponding buckled shapes are given in Fig. 18.

Both ends fixed: $$p_{cr} = \frac{4\pi^2 EI}{L^2} = \frac{\pi^2 E}{\left[\left(L/2\right)r\right]^2}$$

One end fixed and the other end pinned: $$p_{cr} = \frac{2\pi^2 EI}{L^2} = \frac{\pi^2 E}{\left[\left(L/\sqrt{2}\right)r\right]^2}$$

One end fixed and the other end free: $$p_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 E}{\left[\left(2L\right)/r\right]^2}$$

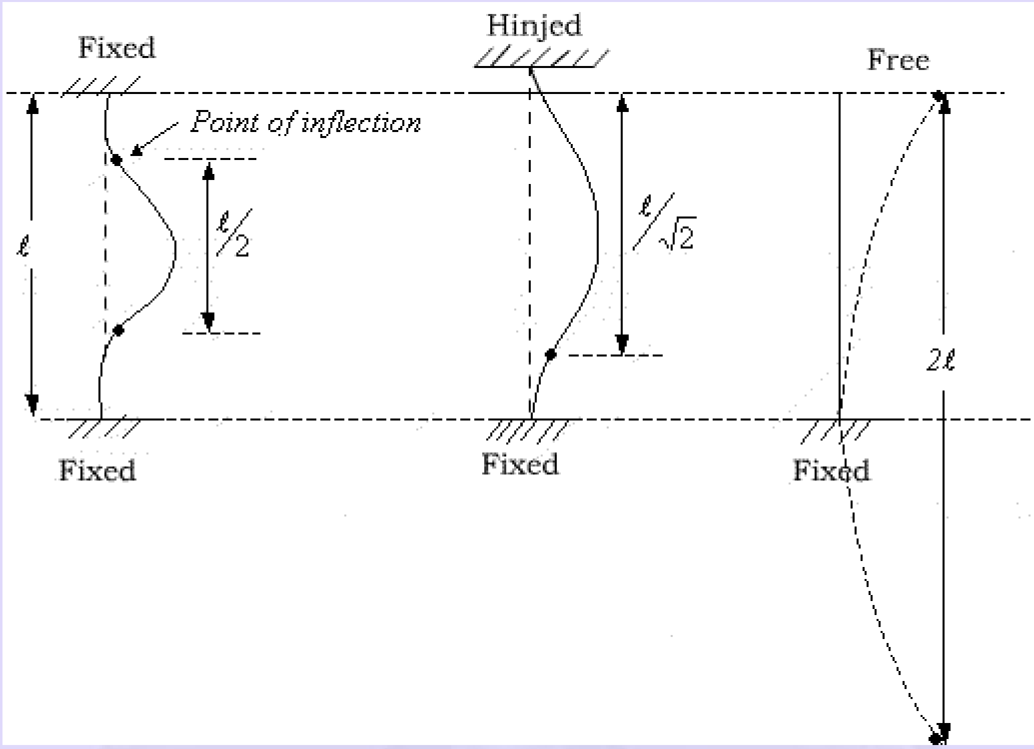


Fig 5.15 Buckled mode for different end connections

Using the column, pinned at both ends as the basis of comparison, the critical load in all the above cases can be obtained by employing the concept of **"effective length"**, L_e.

It is easily verified that the calculated effective length for the various end conditions are as in Table 5.1

Table 5.1 Effective length of compression members

Boundary conditions	Theory	Code value (Cl.7.2.2)
Both ends pin ended	1.0L	1.0L
Both ends fixed	0.5L	0.65L
One end fixed and the other end pinned	0.707L	0.8L
One end fixed, and the other free to sway	1.2L	1.2L
One end fixed and the other end free	2.0L	2.0L

It can be seen that the effective length corresponds to the distance between the points of inflection in the buckled mode. The effective column length can be defined as the length of an equivalent pin-ended column having the same load-carrying capacity as the member under consideration. The smaller the effective length of a particular column, the smaller its danger of lateral buckling and the greater its load carrying capacity. It must be recognized that column ends in practice are neither perfectly fixed nor perfectly hinged. The designer may have to interpolate between the theoretical values given above, to obtain a sensible approximation to actual restraint conditions. Effective lengths prescribed by the code are also given in Table 5.1.

5.5.1 Effective lengths in different planes

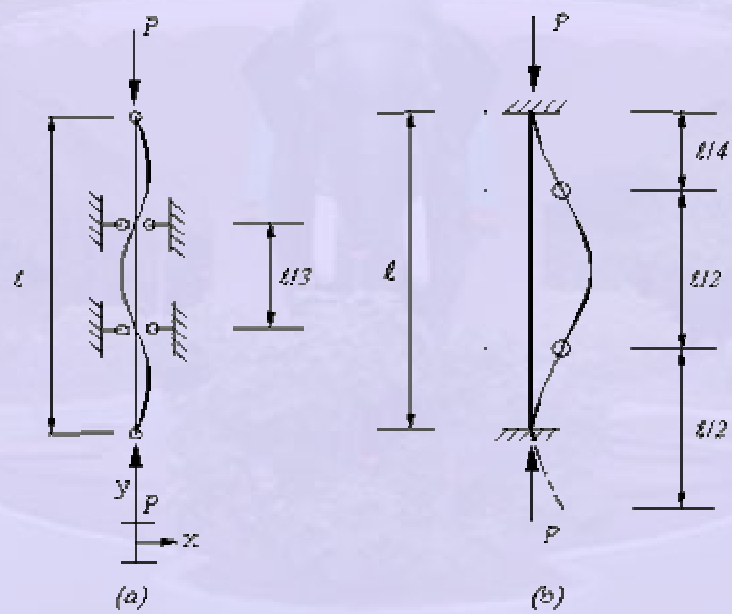


Fig 5.16 Columns with different effective length L

The restraint against buckling may be different for buckling about the two column axes. For example, if a column of solid rectangular section were to be connected to the support with a single bolt at either end, it will be like a hinged-hinged column with L_e equal to the distance between the bolts. However, in the perpendicular plane, the column cannot rotate without bending the bolts and will be liked a fixed-fixed column with L_e equal to half the distance between the bolts. Fig 5.16(a) shows a pin-ended column of I section braced about the minor axis against lateral movement (but not rotationally restrained) at spacing L / 3. The minor axis buckling mode would be with an effective pin-ended column length $(L_e)_y$ of L / 3. If there was no major axis bracing the effective length for buckling about the major axis $(L_e)_x$ would remain as L. Therefore, the design slenderness about the major and minor axis would be L / r_x and (L / *3*)r_y, respectively. Generally $\mathbf{r_x < 3r_y}$ for all I sections, hence the major axis slenderness (L / r_x) would be greater, giving the lower value of critical load, and failure would occur by major axis buckling. Anyway, checks should be carried out about both the axes.

5.5.2 Effective lengths of columns in frames

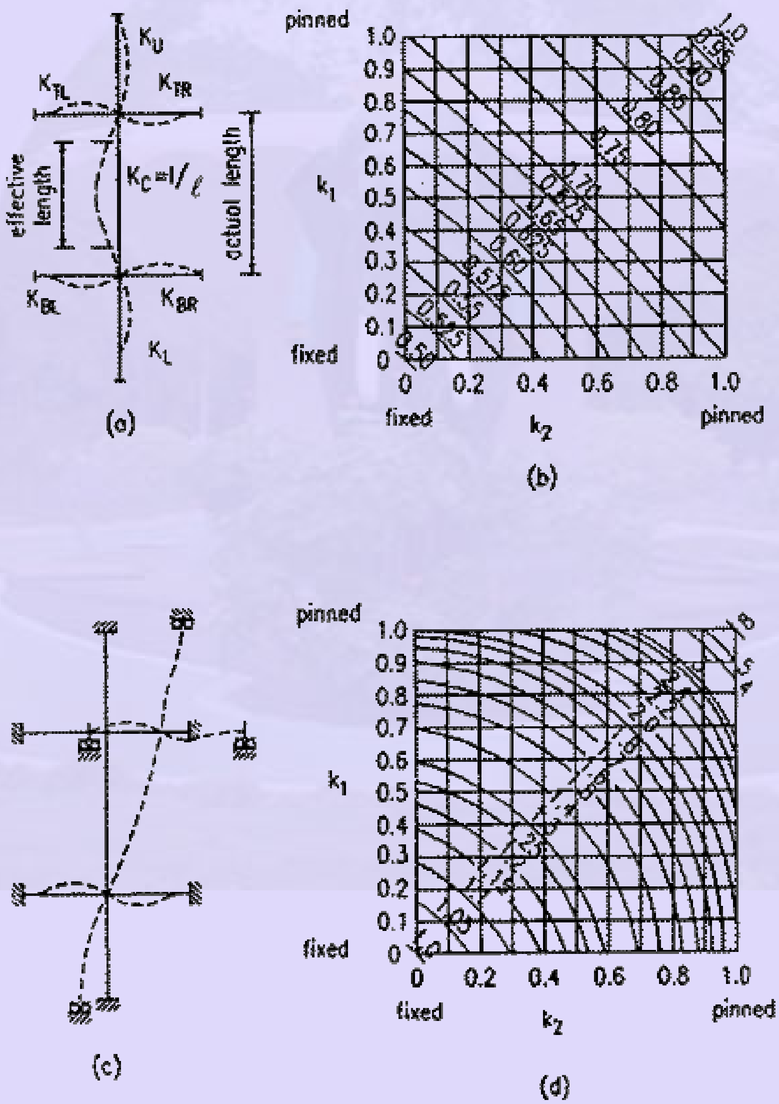
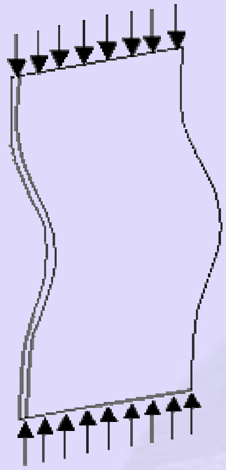


Fig 5.17 Limited frames and corresponding effective length charts of IS: 800(draft)

(a) Limited frame and (b) effective length ratio (k3 = ∞), for non-sway frames. (c) Limited frames and (d) effective length ratios (without partial bracing, k3 = 0), for sway frames

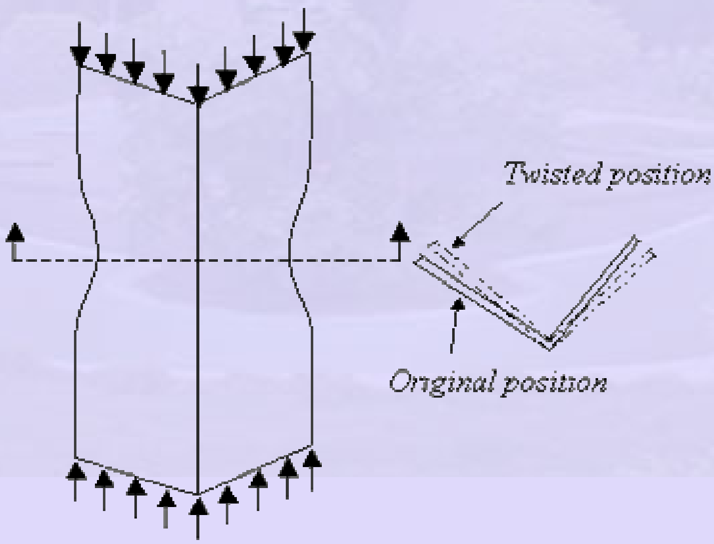
For compression members in rigid-jointed frames the effective length is directly related to the restraint provided by all the surrounding members. In a frame the interaction of all the members occurs because of the frame buckling as a whole rather than column buckling. For individually design purposes, the behaviour of a limited region of the frame is considered. The limited frame comprises the column under consideration and each immediately adjacent member treated as if it were fixed at the far end. The effective length of the critical column is then obtained from a chart which is entered with two coefficients k_1, and k_2, the values of which depends upon the stiffnesses of the surrounding members k_u, k_{TL} etc. Two different cases are considered viz. columns in non-sway frames and columns in sway frames. All these cases as well as effective length charts are shown in Fig.5.17. For the non-sway columns, the effective lengths will vary from 0.5 to 1.0 depending on the values of k_1 and k_2, while for the sway columns, the variation will be between 1.0 and α. These end points correspond to cases of: (1) rotationally fixed ends with no sway and rotationally free ends with no sway; (2) rotationally fixed ends with free sway and rotationally free ends with free sway. The equations for calculating k_1, and k_2, are given in the code (Cl.7.2.2).

5.6 Torsional and torsional-flexural buckling of columns



Show the buckling figure click here

Fig 5.18(a) Plate with unsupported edges



Folded plate twist click here

Fig 5.18 (b) Folded plate twist under axial load

We have so far considered the flexural buckling of a column in which the member deforms by bending in the plane of one of the principal axes. The same form of buckling will be seen in an initially flat wide plate, loaded along its two ends, the two remaining edges being unrestrained. [See Fig. 5.18 (a)]

On the other hand, if the plate is folded at right angles along the vertical centre-line, the resulting angle section has a significantly enhanced bending stiffness. Under a uniform axial compression, the two unsupported edges tend to wave in the Euler type buckles. At the fold, the amplitude of the buckle is virtually zero. A horizontal cross-section at mid height of the strut shows that the cross-section rotates relative to the ends. This mode of buckling is essentially torsional in nature and is initiated by the lack of support at the free longitudinal edges. This case illustrates buckling in torsion, due to the low resistance to twisting (polar moment of inertia) of the member.

Thus the column curves of the type discussed before are only satisfactory for predicting the mean stress at collapse, when the strut buckles by bending in a plane of symmetry of the cross section, referred to as **"flexural buckling"**. Members with low torsional stiffness (eg. angles, tees etc made of thin walled members) will undergo torsional buckling before flexural buckling. Cruciform sections are generally prone to torsional buckling before flexural buckling. Singly symmetric or un-symmetric cross sections may undergo combined twisting about the shear centre and a translation of the shear centre. This is known as **"torsional – flexural buckling"**.

Thus a singly symmetric section such as an equal angle or a channel can buckle either by flexure in the plane of symmetry or by a combination of flexure and torsion. All centrally loaded columns have three distinct buckling loads, at least one of which corresponds to torsional or torsional - flexural mode in a doubly symmetric section. Flexural buckling load about the weak axis is almost always the lowest. Hence, we disregard the torsional buckling load in doubly symmetric sections. In non-symmetric sections, buckling will be always in torsional – flexural mode regardless of its shape and dimensions. However, non-symmetric sections are rarely used.

Thin-walled open sections, such as angles and channels, can buckle by bending or by a combination of bending and twisting. Which of these two modes is critical depends on the shape and dimensions of the cross-section. Hence, torsional-flexural buckling must be considered in their design. This is normally done by calculating an equivalent slenderness ratio and using the same column strength curve as for flexural buckling.

5.7 Design strength

Based on the studies of Ayrton & Perry (1886), the compressive strength of the column can be obtained from the following equation.

$$\left(f_y - f_c\right)\left(f_e - f_c\right) = \eta . f_e . f_c \qquad \textit{(5.10)}$$

Where, f_y = yield stress, f_c = compressive strength, f_e = Euler buckling stress, λ = Slenderness ratio (l/r) and η = a parameter allowing for the effects of lack of straightness and eccentricity of loading and can be expressed as aλ where α is a function of the shape of the cross section. Since Robertson evaluated the mean values of α for many sections, the design method is termed "Perry-Robertson method".

Equation (5.8) will result in column strength values lower than f_y even in very low slenderness cases as indicated by the Robertson's curve in Fig. 5.19. By modifying the slenderness, λ to $(\lambda - \lambda_o)$, a plateau to the design curve can be introduced for low slenderness values. This has the effect of shifting the curve to the right by a value equal to λ_o. The value of λ_o may be taken as $0.2(\pi\sqrt{E/f_y})$. Thus, the elastic critical stress can be calculated as $f_e = \pi^2 E/(\lambda - \lambda_o)^2$. Note that calculations for f_e is not needed when $\lambda \leq \lambda_e$ as the column would fail by squashing at f_y.

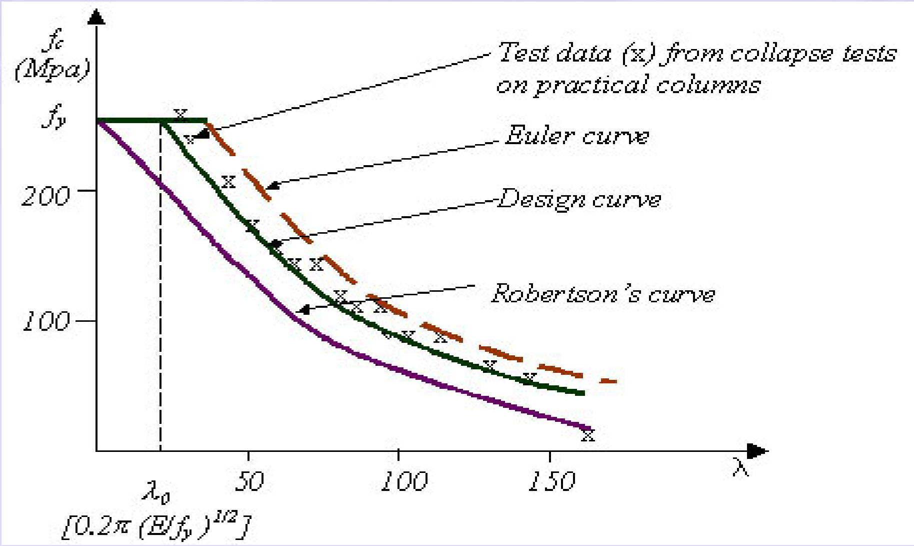


Fig 5.19 Column strength curves

5.7.1 Design Strength as per the Code

Common hot rolled and built-up steel members, used for carrying axial compression, usually fail by flexural buckling. The buckling strength of these members is affected by residual stresses, initial bow and accidental eccentricities of load. To account for all these factors, strength of members subjected to axial compression are given by multiple design curves corresponding to buckling class a, b, c, or d, as given below. The design compressive strength of a member is given by (Cl.7.1)

$$P_d = A_e\ f_{cd} \qquad \textbf{\textit{(5.11)}}$$

Where, A_e = effective sectional area and

f_{cd} = design stress in compression, obtained as per the following equation:

$$fcd = \frac{f_y/\gamma_{m0}}{\phi + \left[\phi^2 - \lambda^2{}_m\right]^{0.3}} = \quad X\ f_y/\gamma_{m0} \le f_y/\gamma_{m0} \qquad \textbf{\textit{(5.12)}}$$

Where

$\phi = 0.5[1+\alpha\ (\lambda - 0.2) + \lambda^2]$

$\neq$ f = 0.5[1=a (1-0.2) + 1^2]

λ_* = non-dimensional effective slenderness ratio

$$= \sqrt{f_y/f_u} = \sqrt{f_y\left(\frac{L_e}{r}\right)^2 \Big/ \pi^2 E} \qquad \textbf{\textit{(5.13)}}$$

f_{cc} = Euler buckling stress = $\pi^2E/(KL/r)^2$

L_e/r = effective slenderness ratio, ratio of effective length L_e, to appropriate radius of gyration, r

α = imperfection factor given in Table 5.2

χ = stress reduction factor

c = stress reduction factor

$l_{m0} \neq (\gamma_{m0})$

λ_{mo} = partial safety factor for material strength = 1.1

Table 5.2 Imperfection factor, α

Buckling Class	a	b	c	d
α	0.21	0.34	0.49	0.76

The classifications of different sections under different buckling class a, b, c or d, is given in Table 5.3 of the Code. Note that thicker sections and welded sections which are likely to have more residual stresses are assigned lower buckling classes. The curves corresponding to different buckling class are presented in non-dimensional form, in Fig 5.20. The selection of an appropriate curve is based on cross section and suggested curves are listed in Table 5.3. Although both hot rolled sections and welded sections have lock-in residual stresses, the distribution and magnitude differ significantly. Residual stresses due to welding are very high and can be of greater consequence in reducing the ultimate capacity of compression members.

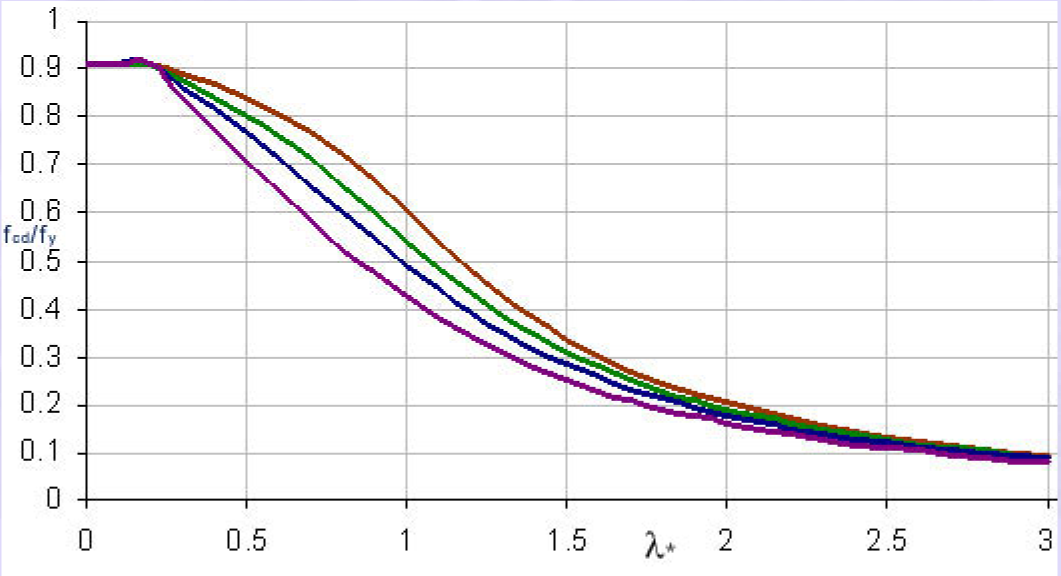
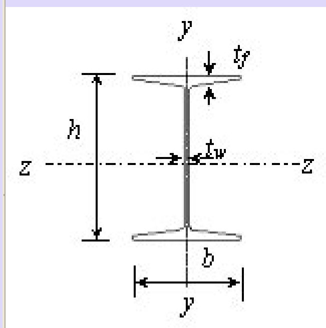
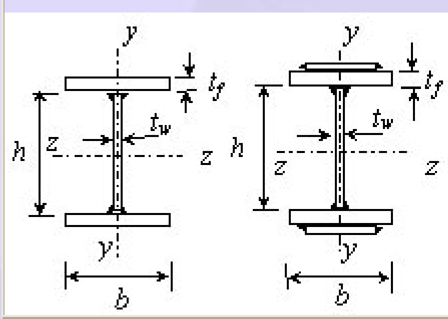
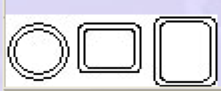
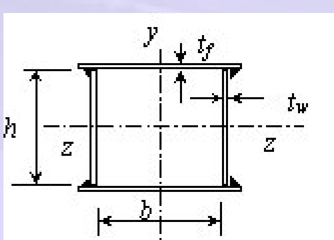
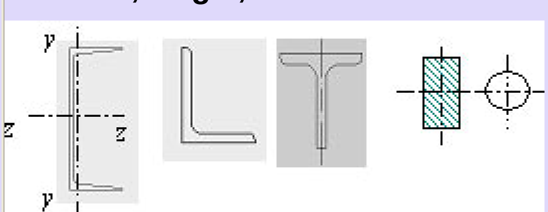
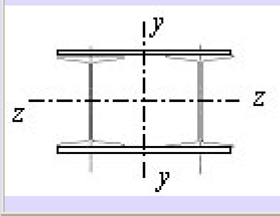


Fig. 5.20 Column Buckling Curves

Table5.3 Buckling class of cross sections (Section 7.1.2.2)

Cross Section	Limits	Buckling about axis	Buckling Class
Rolled I-Sections 	$h/b > 1.2$: $t_f \le 40$ mm	z-z y-y	a b
	40 mm< $t_f \le 100$ mm	z-z y-y	b c
	$h/b \le 1.2$: $t_f \le 100$ mm	z-z y-y	b c
	$t_f > 100$ mm	z-z y-y	d d
Welded I-Section 	$t_f \le 40$ mm	z-z y-y	b c
	$t_f > 40$ mm	z-z y-y	c d
Hollow Section 	Hot rolled	Any	a
	Cold formed	Any	b
Welded Box Section 	Generally (Except as below)	Any	b
	Thick welds and $b/t_f < 30$ $h/t_w < 30$	z-z y-y	c c
Channel, Angle, T and Solid Sections 		Any	c

Built-up Member 	Any	c

5.7..2 Design of angle compression members

When angles are loaded in compression through their centroid, they can be designed as per the procedure described above using curve c. However, angles are usually loaded eccentrically by connecting one of its legs either to a gusset or to an adjacent member. Such angles will buckle in flexural-torsional mode in which there will be significant twisting of the member. Such twisting may be facilitated by the flexibility of the gusset plate and the other members connected to it. To simplify the design, the code considers only two cases – gusset fixed and gusset hinged. The other parameter which will influence the strength of the angle strut is its width-thickness ratio of either leg. Thus, to account for the reduction in strength due to flexural-torsional mode, the code gives an equivalent slenderness ratio as a function of the overall slenderness ratio and the width-thickness ratio. In general, the equivalent slenderness ratio will be less than or equal to the slenderness ratio for flexural buckling λ_{vv}

The flexural torsional buckling strength of single angle loaded in compression through one of its legs may be evaluated using the equivalent slenderness ratio, λ_{eq}, as given below (Cl. 7.5.1.2)

$$\lambda_{eq} = \sqrt{k_1 + k_2\lambda^2_{w} + k^3\lambda^2_{\phi}} \qquad \textbf{\textit{(5.14)}}$$

Where

k_1, k_2, k_3 = constants depending upon the end condition, as given in Table 5.4,

$$\lambda_w = \frac{\left(\frac{l}{r_w}\right)}{\varepsilon\sqrt{\frac{\pi^2 E}{250}}} \quad \text{and} \quad \lambda_\phi = \frac{(b_1 + b_2)}{\varepsilon\sqrt{\frac{\pi^2 E}{250}} \times 2t} \qquad \textbf{\textit{(5.15 a,b)}}$$

Where

l = centre to centre length of the supporting member

r_{vv} = radius of gyration about the minor axis

b_1, b_2 = width of the two legs of the angle

t = thickness of the leg

ε = yield stress ratio $(250/f_y)^{0.5}$

Table 5.4 Constants k_1, k_2 and k_3
(Section 7.5.1.2)

No. of bolts at the each end connection	Gusset/Connecting member Fixity†	k_1	k_2	k_3
≥ 2	Fixed	0.20	0.35	20
	Hinged	0.70	0.60	5
1	Fixed	0.75	0.35	20
	Hinged	1.25	0.50	60

Stiffeness of in-plane rotational restraint provided to the gusset/connecting member. For partial restraint, the λ_{eq} can be interpolated between the λ_{eq} results for fixed and hinged cases.

For double angle discontinuous struts, connected back to back, on opposite sides of the gusset or a section, by not less than two bolts or rivets in line along the angles at each end, or by the equivalent in welding, the load may be regarded as applied axially (Cl. 7.5.2). The effective length, L_e, in the plane of end gusset can be taken as between 0.7 and 0.85 times the distance between intersections, depending on the degree of the restraint provided. The effective length, L_e, in the plane perpendicular to that of the end gusset, shall be taken as equal to the distance between centers of intersections.

5.8 Types of column sections

5.8.1 Rolled Steel Sections

Some of the sections employed as compression members are shown in Fig. 5.21. Single angles [Fig 5.21(a)] are satisfactory for bracings and for light trusses. Top chord members of roof trusses are usually made up of double angles back-to-back [Fig 5.21(b)]. The pair of angles used, has to be connected together, so they will act as one unit. Welds may be used at intervals – with a spacer bar between the connecting legs. Alternately “stitch bolts”, washers and “ring fills” are placed between the angles to keep them at the proper distance apart (e.g. to enable a gusset to be connected). Such connections are called tack connections and the terms tack welding or tacks bolting are used.

When welded roof trusses are required, there is no need for gusset plates and T sections [Fig 5.21(c)] can be employed as compression members.

Single channels or C-sections [Fig. 5.21(d)] are generally not satisfactory for use in compression, because of the low value of radius of gyration in the weak direction. They can be used if they could be supported in a suitable way in the weak direction.

Circular hollow sections [Fig. 5.21(e)] are perhaps the most efficient as they have equal values of radius of gyration about every axis. But connecting them is difficult but satisfactory methods have been evolved in recent years for their use in tall buildings.

The next best in terms of structural efficiency will be the square hollow sections (SHS) and rectangular hollow sections, [Fig. 5.21(f)] both of which are increasingly becoming popular in tall buildings, as they are easily fabricated and erected. Welded tubes of circular, rectangular or square sections are very satisfactory for use as columns in a long series of windows and as short columns in walkways and covered warehouses. For many structural applications the weight of hollow sections required would be only 50% of that required for open profiles like I or C sections.

When the available sections are not suitable, a suitable section may be built-up either by welding or by lacing or battening two sections separated by a suitable distance.

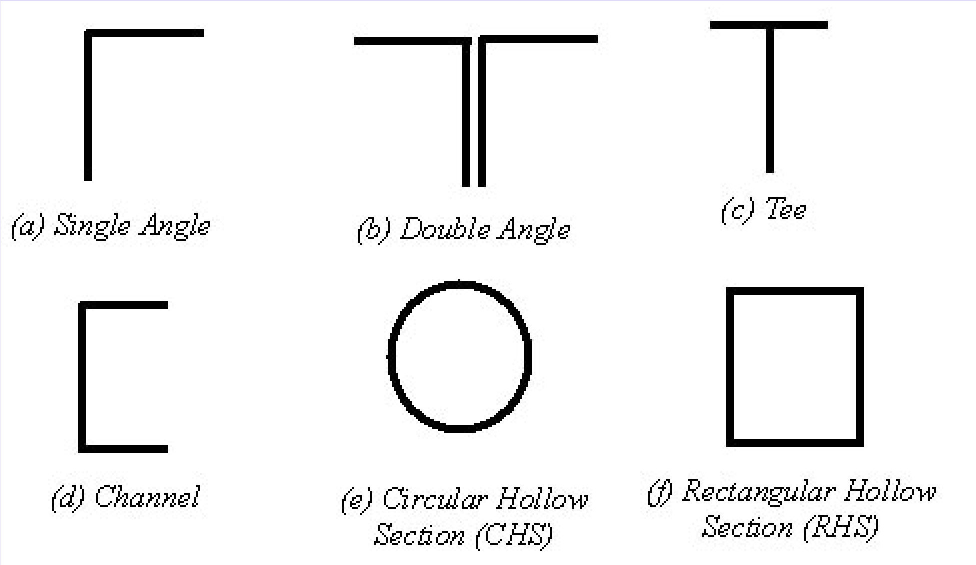


Fig 5.21: Cross Section Shapes for Rolled Steel Compression Members

5.8.2 Built-up column or fabricated compression members

Compression members composed of two angles, channels, or tees back-to-back in contact or separated by a small distance shall be connected together by tack riveting, tack bolting or tack welding so that the individual sections do not buckle between the tacks before the whole member buckles (Cl. 7.8). Special types of columns called Laced and Battened columns are discussed later in this chapter.

When compression members are required for large structures like bridges, it will be necessary to use built-up sections. They are particularly useful when loads are heavy and members are long (e.g. top chords of Bridge Trusses). Built up sections [illustrated in Fig. 5.22(a) and 5.22(b)] are popular in India when heavy loads are encountered. The cross section consists of two channel sections connected on their open sides with some type of lacing or latticing (dotted lines) to hold the parts together and ensure that they act together as one unit. The ends of these members are connected with "batten plates" which tie the ends together. Box sections of the type shown in Fig. 5.22(a) or 5.22(b) are sometimes connected by such solid plates either at intervals (battened) or continuously along the length.

A pair of channels connected by cover plates on one side and latticing on the other is sometimes used as top chords of bridge trusses. The gussets at joints can be conveniently connected to the inside of the channels. Plated I sections or built-up I sections are used when the available rolled I sections do not have sufficient strengths to resist column loads [Fig 5.22(c)]. Flange plates or channels may be used in combination with rolled sections to enhance the load resistance of the commonly available sections, which are directly welded or bolted to each other. The lateral dimension of the column is generally chosen at around 1/10 to 1/15 of the height of the column. For purposes of detailing the connection between the flange cover plates or the outer rolled sections to the flanges of the main rolled section, it is customary to design the fasteners for a transverse shear force equal to 2.5% of the compressive load of the column. Columns with open webs may be classified as laced columns or battened columns. In Fig. 5.23, the two channel sections of the column are connected together by batten plates or laces which are shown by dotted lines. A typical lacing or batten plate is shown in Fig. 5.23

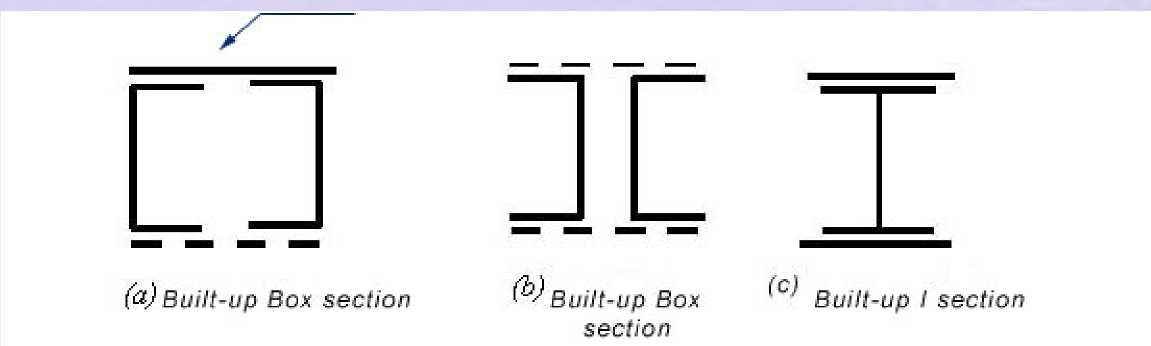


Fig 5.22: Cross Section Shapes for Built - up or fabricated Compression Members

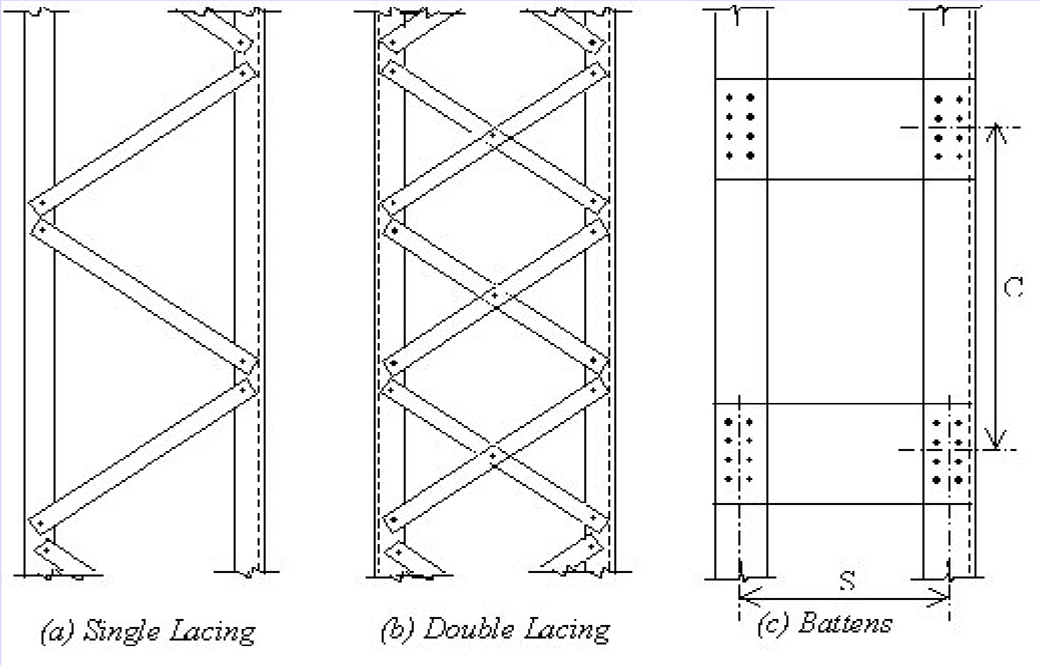


Fig. 5.23 Built-up column members

The Code gives simple guidelines for the design of laced (Cl.7.6) and battened columns (Cl.7.7). One of the guidelines is that such columns should where practicable, have a radius of gyration about the axis perpendicular to the plane of lacing not less than the radius of gyration about the axis parallel to the plane of lacing (Cl. 7.6.1.1). To account for the inherent flexibility of laced and battened columns, the Code suggests that the effective slenderness ratio be taken as 5 and 10% respectively more than the calculated values (Cl.7.7.1.4). All columns should be tied at the ends by tie plates or end battens to ensure a satisfactory performance.

The two channel constituents of a laced column, shown in Fig. 5.23(a) and 5.23(b) have a tendency to buckle independently. Lacing provides a tying force to ensure that the channels do not do so. The load that these tying forces cause is generally assumed to cause a shearing force equal to 2.5% of axial load on the column (Cl. 7.6.2.1). (Additionally if the columns are subjected to moments or lateral loading the lacing should be designed for the additional bending moment and shear). To prevent local buckling of unsupported lengths between the two constituent lattice points (or between two battens), the slenderness ratio of individual components should be less than 50 or 0.7 of the slenderness ratio of the built up column (whichever is less).

In laced columns, the lacing should be symmetrical in any two opposing faces to avoid torsion. Lacings and battens are not combined in the same column. The inclination of lacing bars from the axis of the column should not be less than 40^o nor more than 70^o. (Cl. 7.6.5) The slenderness ratio of the lacing bars should not exceed 145 (Cl. 7.6.3). The effective length of lacing bars is the length between bolts for single lacing and 0.7 of this length for double lacing. The width of the lacing bar should be at least 3 times the diameter of the bolt (Cl. 7.6.3). Thickness of lacing bars should be at least $1/40^{th}$ of the length between bolts for single lacing and 1/60 of this length for double lacing (both for welded and bolted connections) (Cl. 7.6.4).

In battened columns, the Battens plates at their ends shall be riveted or welded to the main components so as to resist simultaneously a shear $V_b = V_t\ C/N\ S$ along the column axis and a moment $M = V_t\ C\ /\ 2\ N$ at each connection (Cl.7.7). where, V_t = the transverse shear force; C = the distance between centre-to-centre of battens, longitudinally; N = the number of parallel planes of battens (usually 2); S = the minimum transverse distance between the centroids of the bolt group/welding connecting the batten to the main member.

When plates are used for battens, the end battens shall have an effective depth not less than the perpendicular distance between the centroids of the main members. The intermediate battens shall have an effective depth of not less than three quarters of this distance, but in no case shall the effective depth of any batten be less than twice the width of one member, in the plane of the battens. The thickness of batten or the tie plates shall be not less than one fiftieth of the distance between the innermost connecting lines of rivets or welds, perpendicular to the main member.

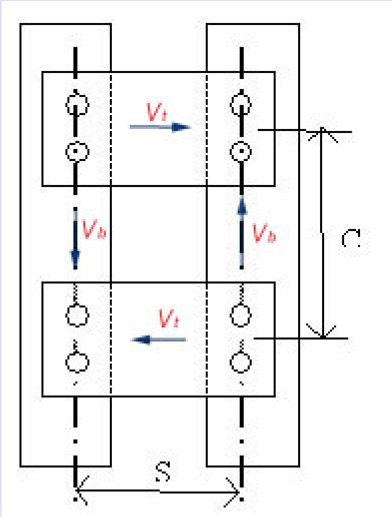


Fig. 5.24 Battened Column

5.9 Steps in the design of axially loaded columns

The procedure for the design of an axially compressed column is as follows:

(i) Assume a suitable trial section and classify the section in accordance with the classification in chapter.

(ii) Arrive at the effective length of the column by suitably considering the end conditions.

(iii) Calculate the slenderness ratios (λ values) in both minor and major axes direction and also calculate λ_o using the formula given below:

$$\lambda_0 = 0.2\pi\sqrt{E/f_y}$$

(iv) Calculate f_{cd} values along both major and minor axes from equation 12

(v) Compute the load that the compression member can resist ($p_d = A_c f_{cd}$)

(vi) Calculate the factored applied load and check whether the column is safe against the given loading. The most economical but safe section can be arrived at by trial and error, i.e. repeating the above process.

The following values are suggested for initial choice of members:

(i) Single angle size: 1/30 of the length of the strut (L / r ~ 150)

(ii) Double angle size: 1/35 of the length of strut (L / r ~ 100-120)

(iii) Circular hollow sections diameter = 1/40 length (L / r ~ 100)

6. BEAMS

6.1 Introduction

One of the frequently used structural members is a beam whose main function is to transfer load principally by means of flexural or bending action. In a structural framework, it forms the main horizontal member spanning between adjacent columns or as a secondary member transmitting floor loading to the main beams. Normally only bending effects are predominant in a beam except in special cases such as crane girders, where effects of torsion in addition to bending have to be specifically considered.

The type of responses of a beam subjected to simple uniaxial bending are shown in Table 6.1. The response in a particular case depends upon the proportions of the beam, the form of the applied loading and the type of support provided. In addition to satisfying various strength limits as given in the Table, the beam should also not deflect too much under the working loads i.e. it has to satisfy the serviceability limit state also.

Recently, IS: 800, the structural steel code has been revised and the limit state method of design has been adopted in tune with other international codes of practice such as BS, EURO, and AISC. This chapter attempts to throw light on the provisions for bending members in this code.

6.2 Limit state design of beams

In the working stress or allowable stress method of design, the emphasis is on limiting a particular stress in a component to a fraction of the specified strength of the material of the component. The magnitude of the factor for a structural action depends upon the degree of safety required. Further, elastic behaviour of the material is assumed. The main objection to the permissible stress method is that the stress safety factor relating the permissible stress to the strength of the material is not usually the same as the ratio of the strength to the design load. Thus it does not give the degree of safety based on collapse load.

In the limit state method, both collapse condition and serviceability condition are considered. In this method, the structure has to be designed to withstand safely all loads and deformations likely to occur on it throughout its life. Designs should ensure that the structure does not become unfit for the use for which it is required. The state at which the unfitness occurs is called a limit state. Special features of limit state design method are:

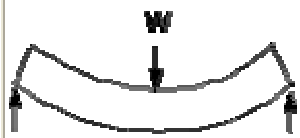
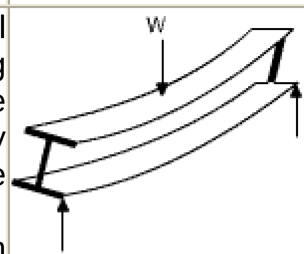
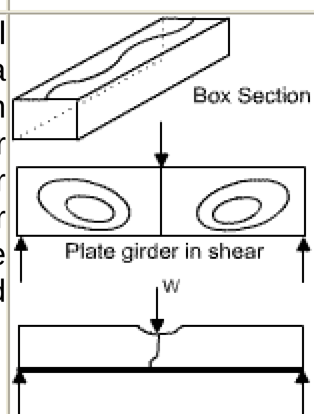
- It is possible to take into account a number of limit states depending upon the particular instance

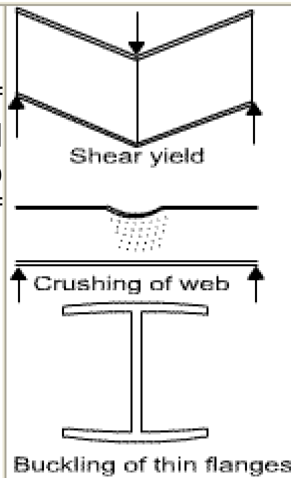
- This method is more general in comparison to the working stress method. In this method, different safety factors can be applied to different limit states, which is more rational than applying one common factor (load factor) as in the plastic design method.

- This concept of design is appropriate for the design of structures since any new knowledge of the structural behaviour, loading and materials can be readily incorporated.

The limit state design method is essentially based on the concept of probability. Its basic feature is to consider the possibility and probability of the collapse load. In this respect, it is necessary to consider the possibility of reduced strength and increased load.

Table 6.1 Main failure modes of hot-rolled beams

Category	Mode		Comments
1	Excessive bending triggering collapse		This is the basic failure mode provided (1) the beam is prevented from buckling laterally,(2) the component elements are at least compact, so that they do not buckle locally. Such "stocky" beams will collapse by plastic hinge formation.
2	Lateral torsional buckling of long beams which are not suitably braced in the lateral direction.(i.e. "un restrained" beams)		Failure occurs by a combination of lateral deflection and twist. The proportions of the beam, support conditions and the way the load is applied are all factors, which affect failure by lateral torsional buckling.
3	Failure by local buckling of a flange in compression or web due to shear or web under compression due to concentrated loads		Unlikely for hot rolled sections, which are generally stocky. Fabricated box sections may require flange stiffening to prevent premature collapse. Web stiffening may be required for plate girders to prevent shear buckling. Load bearing stiffeners are sometimes needed under point loads to resist web buckling.

4	Local failure by (1) shear yield of web (2) local crushing of web (3) buckling of thin flanges.	 	Shear yield can only occur in very short spans and suitable web stiffeners will have to be designed. Local crushing is possible when concentrated loads act on unstiffened thin webs. Suitable stiffeners can be designed. This is a problem only when very wide flanges are employed. Welding of additional flange plates will reduce the plate b / t ratio and thus flange buckling failure can be avoided.

The object of design is to keep an acceptable level the probability of any limit state not being exceeded. This is achieved by taking account of the variation in strength and properties of materials to be used and the variations in the loads to be supported by the structure, by using the characteristic values of the strength of materials as well as the loads to be applied. The deviations from the characteristic values in the actual structures are allowed by using their design values. The characteristic values should be based on statistical evidence where necessary data are available; where such data are not available they should be based on an appraisal of experience. The design values are derived from the characteristic values through the use of partial safety factors, one for material strengths and the other for loads and load effects.

6.3 Behaviour of steel beams

Laterally stable steel beams can fail only by (a) Flexure (b) Shear or (c) Bearing, assuming the local buckling of slender components does not occur. These three conditions are the criteria for limit state design of steel beams. Steel beams would also become unserviceable due to excessive deflection and this is classified as a limit state of serviceability.

The factored design moment, M at any section, in a beam due to external actions shall satisfy

$$M \leq M_d$$

Where M_d = design bending strength of the section

6.3.1 Design strength in bending (Flexure)

The behaviour of members subjected to bending demonstrated in Fig 6.1

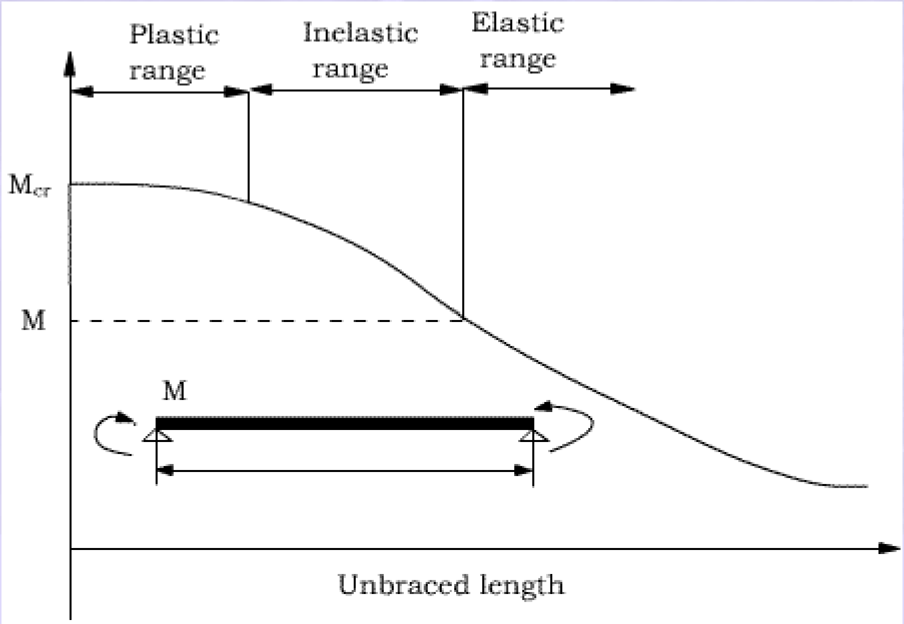


Fig 6.1 Beam buckling behaviour

This behaviour can be classified under two parts:

- When the beam is adequately supported against lateral buckling, the beam failure occurs by yielding of the material at the point of maximum moment. The beam is thus capable of reaching its plastic moment capacity under the applied loads. Thus the design strength is governed by yield stress and the beam is classified as laterally supported beam.

- Beams have much greater strength and stiffness while bending about the major axis. Unless they are braced against lateral deflection and twisting, they are vulnerable to failure by lateral torsional buckling prior to the attainment of their full in-plane plastic moment capacity. Such beams are classified as laterally supported beam.

Beams which fail by flexual yielding

Type1: Those which are laterally supported

The design bending strength of beams, adequately supported against buckling (laterally supported beams) is governed by yielding. The bending strength of a laterally braced compact section is the plastic moment $M_{p.}$ If the shape has a large shape factor (ratio of plastic moment to the moment corresponding to the onset of yielding at the extreme fiber), significant inelastic deformation may occur at service load, if the section is permitted to reach M_p at factored load. The limit of $1.5M_y$ at factored load will control the amount of inelastic deformation for sections with shape factors greater than 1.5. This provision is not intended to limit the plastic moment of a hybrid section with a web yield stress lower than the flange yield stress. Yielding in the web does not result in significant inelastic deformations.

Type2: Those which are laterally shift

Lateral-torsional buckling cannot occur, if the moment of inertia about the bending axis is equal to or less than the moment of inertia out of plane. Thus, for shapes bent about the minor axis and shapes with $I_z = I_y$ such as square or circular shapes, the limit state of lateral-torsional buckling is not applicable and yielding controls provided the section is compact.

6.3.1.1 Laterally supported beam

When the lateral support to the compression flange is adequate, the lateral buckling of the beam is prevented and the section flexual strength of the beam can be developed. The strength of I-sections depends upon the width to thickness ratio of the compression flange. When the width to thickness ratio is sufficiently small, the beam can be fully plastified and reach the plastic moment, such section are classified as compact sections. Howeverprovided the section can also sustain the moment during the additional plastic hinge rotation till the failure mechanism is formed. Such sections are referred to as plastic sections. When the compression flange width to thickness ratio is larger, the compression flange may buckle locally before the complete plastification of the section occurs and the plastic moment is reached. Such sections are referred to as non-compact sections. When the width to thickness ratio of the compression flange is sufficiently large, local buckling of compression flange may occur even before extreme fibre yields. Such sections are referred to as slender sections.

The flexural behaviour of such beams is presented in Fig. 6.2. The section classified as slender cannot attain the first yield moment, because of a premature local buckling of the web or flange. The next curve represents the beam classified as 'semi-compact' in which, extreme fibre stress in the beam attains yield stress but the beam may fail by local buckling before further plastic redistribution of stress can take place towards the neutral axis of the beam. The factored design moment is calculated as per Section **8.2** of the code.

The curve shown as 'compact beam' in which the entire section, both compression and tension portion of the beam, attains yield stress. Because of this plastic redistribution of stress, the member o attains its plastic moment capacity (M_p) but fails by local buckling before developing plastic mechanism by sufficient plastic hinge rotation. The moment capacity of such a section can be calculated by provisions given in Section **8.2.1.2**. This provision is for the moment capacity with low shear load.

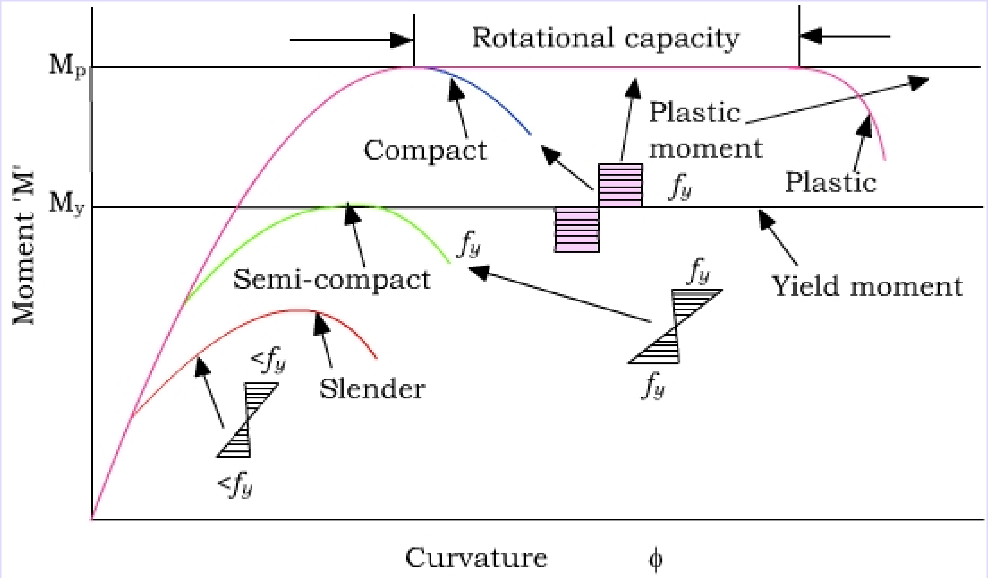


Fig 6.2 Flexual member performance using section classification

Low shear load is referred to the factored design shear force that does not exceed $0.6V_d$, where V_d is the design shear strength of cross section as explained in **8.2.1.2** of the code.

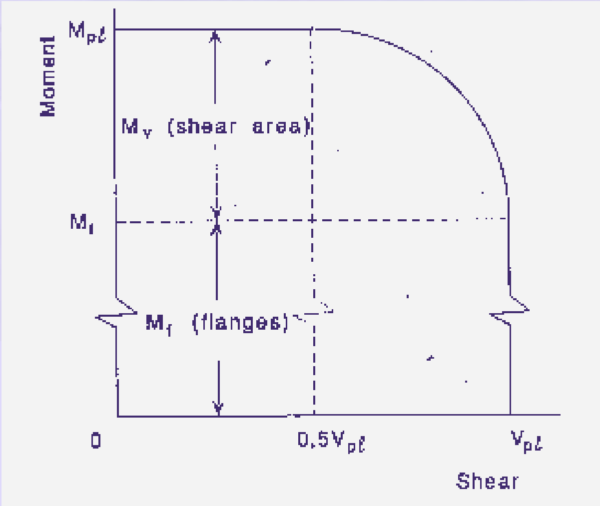


Fig.6.3 Interaction of high shear and bending moment

6.3.1.1.1 Holes in the tension zone

The fastener holes in the tension flange need not be allowed for provided that for the tension flange the condition as given in **8.2.1.4** of the code is satisfied. The presence of holes in the tension flange of a beam due to connections may lead to reduction in the bending capacity of the beam.

6.3.1.1.2 Shear lag effects

The simple theory of bending is based on the assumption that plane sections remain plane after bending. But, the presence of shear strains causes the section to warp. Its effect in the flanges is to modify the bending stresses obtained by the simple theory, producing higher stresses near the junction of a web and lower stresses at points away from it (Fig. 6.4). This effect is called ‘shear lag’. This effect is minimal in rolled sections, which have narrow and thick flanges and more pronounced in plate girders or box sections having wide thin flanges when they are subjected to high shear forces, especially in the vicinity of concentrated loads. The provision with regard to shear lag effects is given in **8.2.1.5.**

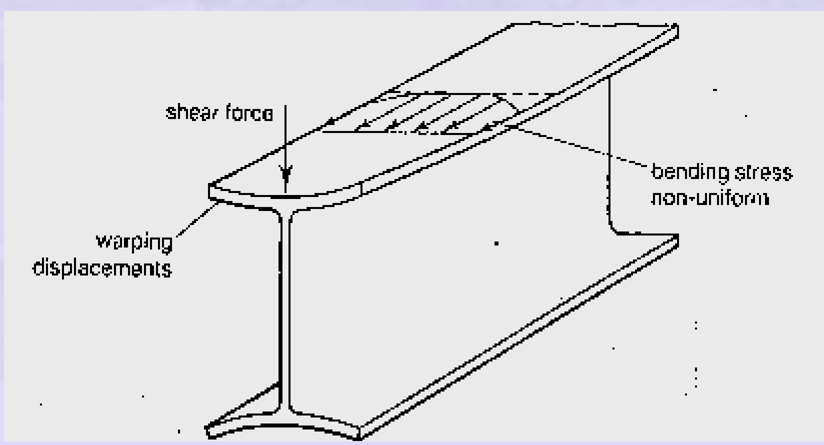


Fig.6.4 Shear Lag effects

6.3.2 Laterally unsupported beams

Under increasing transverse loads, a beam should attain its full plastic moment capacity. This type of behaviour in a laterally supported beam has been covered in

Section **8.2.1**. Two important assumptions have been made therein to achieve the ideal beam behaviour.

They are:

- The compression flange of the beam is restrained from moving laterally; and
- Any form of local buckling is prevented

A beam experiencing bending about major axis and its compression flange not restrained against buckling may not attain its material capacity. If the laterally unrestrained length of the compression flange of the beam is relatively long then a phenomenon known as lateral buckling or lateral torsional buckling of the beam may take place and the beam would fail well before it can attain its full moment capacity. This phenomenon has close similarity with the Euler buckling of columns triggering collapse before attaining its squash load (full compressive yield load).

6.3.2.1 Lateral-torsional buckling of beams

Lateral-torsional buckling is a limit-state of structural usefulness where the deformation of a beam changes from predominantly in-plane deflection to a combination of lateral deflection and twisting while the load capacity remains first constant, before dropping off due to large deflections. The analytical aspects of determining the lateral-torsional buckling strength are quite complex, and close form solutions exist only for the simplest cases.

The various factors affecting the lateral-torsional buckling strength are:

- Distance between lateral supports to the compression flange.
- Restraints at the ends and at intermediate support locations (boundary conditions).
- Type and position of the loads.
- Moment gradient along the length.
- Type of cross-section.

- Non-prismatic nature of the member.
- Material properties.
- Magnitude and distribution of residual stresses.
- Initial imperfections of geometry and loading.

They are discussed here briefly:

The distance between lateral braces has considerable influence on the lateral torsional buckling of the beams.

The restraints such as warping restraint, twisting restraint, and lateral deflection restraint tend to increase the load carrying capacity.

If concentrated loads are present in between lateral restraints, they affect the load carrying capacity. If this concentrated load applications point is above shear centre of the cross-section, then it has a destabilizing effect. On the other hand, if it is below shear centre, then it has stabilizing effect.

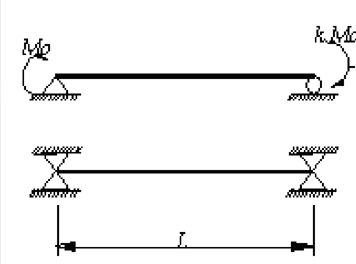
For a beam with a particular maximum moment-if the variation of this moment is non-uniform along the length (Fig. 6.5) the load carrying capacity is more than the beam with same maximum moment uniform along its length.

If the section is symmetric only about the weak axis (bending plane), its load carrying capacity is less than doubly symmetric sections. For doubly symmetric sections, the torque-component due to compressive stresses exactly balances that due to the tensile stresses. However, in a mono-symmetric beam there is an imbalance and the resistant torque causes a change in the effective torsional stiffeners, because the shear centre and centroid are not in one horizontal plane. This is known as "Wagner Effect".

If the beam is non-prismatic within the lateral supports and has reduced width of flange at lesser moment section the lateral buckling strength decreases.

The effect of residual stresses is to reduce the lateral buckling capacity. If the compression flange is wider than tension flange lateral buckling strength increases and if the tension flange is wider than compression flange, lateral buckling strength decreases. The residual stresses and hence its effect is more in welded beams as compared to that of rolled beams.

The initial imperfections in geometry tend to reduce the load carrying capacity.



$$\alpha_y = n.\frac{EI_y}{L_b}(1-\frac{P_m}{P_y})$$

Fig 6.5 Beam subjected to Non-uniform moment

The design buckling (Bending) resistance moment of laterally unsupported beams are calculated as per Section **8.2.2** of the code.

If the non-dimensional slenderness $\lambda_{LT} \leq 0.4$, no allowance for lateral-torsional buckling is necessary. Appendix **F** of the code gives the method of calculating $M_{cr,}$, the elastic lateral torsional buckling moment for difficult beam sections, considering loading and a support condition as well as for non-prismatic members.

6.3.3 Effective length of compression flanges

The lateral restraints provided by the simply supported condition assumption in the basic case, is the lowest and therefore the M_{cr} is also the lowest. It is possible, by other restraint conditions, to obtain higher values of M_{cr}, for the same structural section, which would result in better utilisation of the section and thus, saving in weight of material. As lateral buckling involves three kinds of deformation, namely, lateral bending, twisting and warping, it is feasible to think of various types of end conditions. But the supports should either completely prevent or offer no resistance to each type of deformation. Solutions for partial restraint conditions are complicated. The effect of various types of support conditions is taken into account by way of a parameter called effective length.

For the beam with simply supported end conditions and no intermediate lateral restraint the effective length is equal to the actual length between the supports, when a greater amount of lateral and torsional restraints is provided at support. When the effective length is less than the actual length and alternatively the length becomes more when there is less restraint. The effective length factor would indirectly account for the increased lateral and torsional rigidities by the restraints.

6.3.4 Shear

Let us take the case of an 'I' beam subjected to the maximum shear force (at the support of a simply supported beam). The external shear '*V*' varies along the longitudinal axis 'x' of the beam with bending moment as V=*d*M/*d*x . While the beam is in the elastic stage, the internal shear stresses τ, which resist the external shear, V, can be written as,

$$\tau = \frac{VQ}{It}$$

where

V = shear force at the section

I = moment of inertia of the entire cross section about the neutral axis

Q = moment about neutral axis of the area that is beyond the fibre at which τ is calculated and '*t*' is the thickness of the portion at which τ is calculated.

The above Equation is plotted in Fig. 6.6, which represents shear stresses in the elastic range. It is seen from the figure that the web carries a significant proportion of shear force and the shear stress distribution over the web area is nearly uniform. Hence, for the purpose of design, we can assume without much error that the average shear stress as

$$\tau_{av} = \frac{V}{t_w d_w}$$

where

t_w = thickness of the web

d_w = depth of the web

The nominal shear yielding strength of webs is based on the Von Mises yield criterion, which states that for an un-reinforced web of a beam, whose width to thickness ratio is comparatively small (so that web-buckling failure is avoided), the shear strength may be taken as

$$\tau_y = \frac{f_y}{\sqrt{3}} = 0.58 f_y$$

where

f_y = yield stress.

The shear capacity of rolled beams V_c can be calculated as

$$V_c \approx 0.6 f_y t_w d_w$$

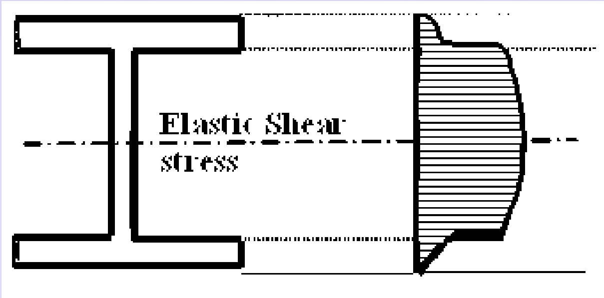


Fig 6.6 Elastic shear stresses

When the shear capacity of the beam is exceeded, the 'shear failure' occurs by excessive shear yielding of the gross area of the webs as shown in Fig 6.7. Shear yielding is very rare in rolled steel beams.

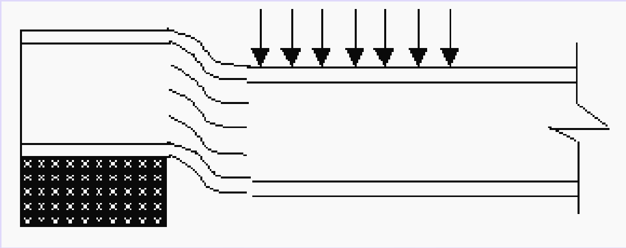


Fig 6.7 Shear yielding near support

The factored design shear force *V* in the beam should be less than the design shear strength of web. The shear area of different sections and different axes of bending are given in Section **8.4.1.1**

6.3.4.1 Resistance to shear buckling

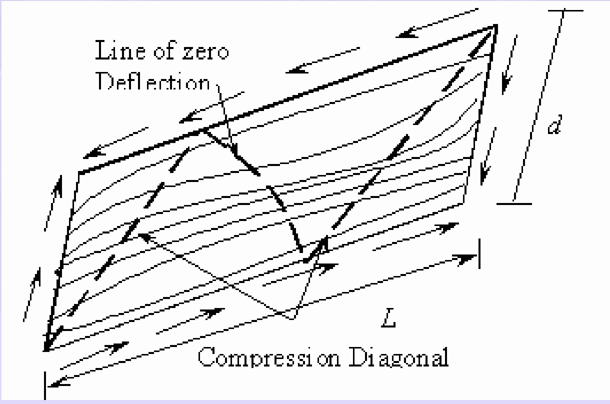


Fig 6.8 Buckling of a girder web in shear

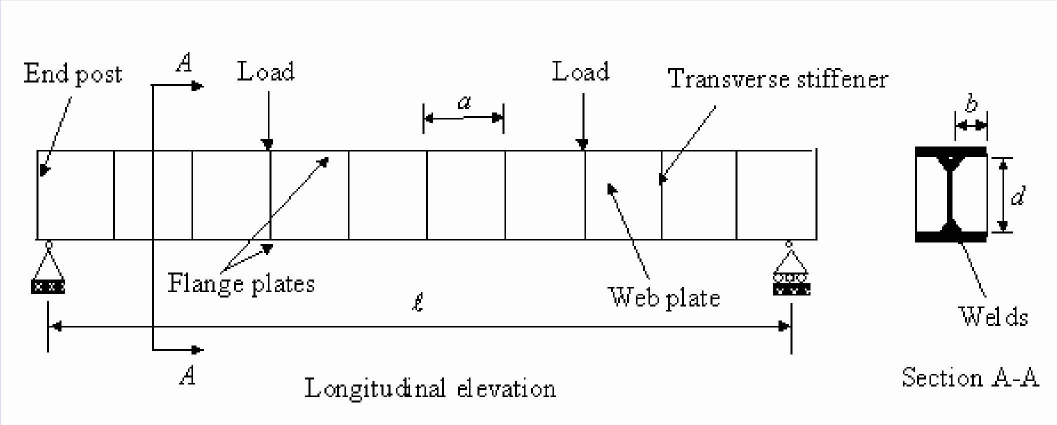


Fig 6.9 A typical plate girder

The girder webs will normally be subjected to some combination of shear and bending stresses. The most severe condition in terms of web buckling is normally the pure shear case. It follows that it is those regions adjacent to supports or the vicinity of point loads, which generally control the design. Shear buckling occurs largely as a result of the compressive stresses acting diagonally within the web, as shown in Fig.6. 8 with the number of waves tending to increase with an increase in the panel aspect ratio c */ d.*

When d $/t_w \leq 67\varepsilon$ where $\varepsilon = (250 / f_y)^{0.5}$ the web plate will not buckle because the shear stress τ is less than critical buckling stress 'τ_{cr}'. The design in such cases is similar to the rolled beams here. Consider plate girders having thin webs with $d/t_w > 67\varepsilon$. In the design of these webs, shear buckling should be considered. In a general way, we may have an un-stiffened web, a web stiffened by transverse stiffeners (Fig. 6.9) and a web stiffened by both transverse and longitudinal stiffeners (Fig. 6.10)

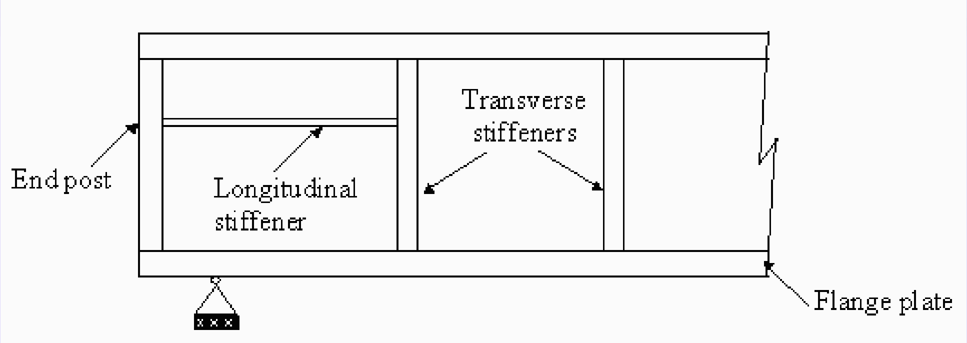


Fig 6.10 End panel strengthened by longitudinal stiffener

6.3.4.2 Shear buckling design methods

The webs, designed either with or without stiffeners and governed by buckling may be evaluated by using two methods

1. Simple Post Critical Method
2. Tension Field Method

6.3.4.2.1 Simple post critical method

It is a simplified version of a method for calculating the post-buckled member stress. The web possesses considerable post-buckling strength reserve and is shown in Fig. 6.11.

When a web plate is subjected to shear, we can visualize the structural behaviour by considering the effect of complementary shear stresses generating diagonal tension and diagonal compression. Consider an element E in equilibrium inside a square web plate with shear stress *q.* The requirements of equilibrium result in the generation of complementary shear stresses as shown in Fig.6.12. This result in the element being subjected to principal compression along the direction AC and tension along the direction BD. As the applied loading is incrementally enhanced, with corresponding increases in *q,* very soon, the plate will buckle along the direction of compression diagonal AC.

The plate will lose its capacity to any further increase in compressive stress. The corresponding shear stress in the plate is the "critical shear stress" τ_{cr}. The value of τ_{cr} can be determined from classical stability theory, if the boundary conditions of the plate are known. As the true boundary conditions of the plate girder web are difficult to establish due to restraints offered by flanges and stiffeners we may conservatively assume them to be simply supported. The critical shear stress in such a case is given by

$$\tau_{cr} = \frac{k_r \pi^2 E}{12\left(1 = \mu^2\right)\left(d / t_w\right)^2}$$

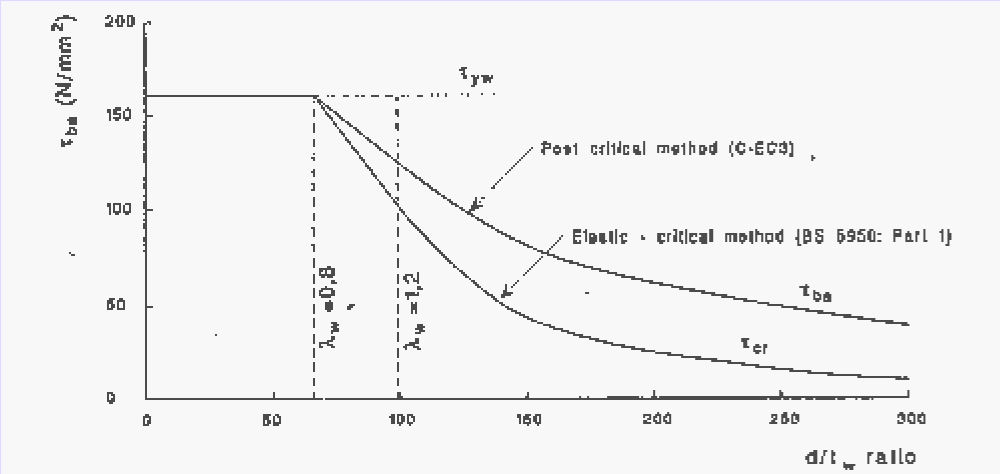


Fig. 6.11 Postbuckling reserve strength of web

k_v = 5.35 When the transverse stiffeners is provided at the support,

k_v=4.0 +5.35(c/d)^2 for c/d < 1.0 i.e., for webs with closely spaced transverse stiffeners.

k_v=5.35 +4(c/d)^2 for c/d ≥1.0 for wide panels.

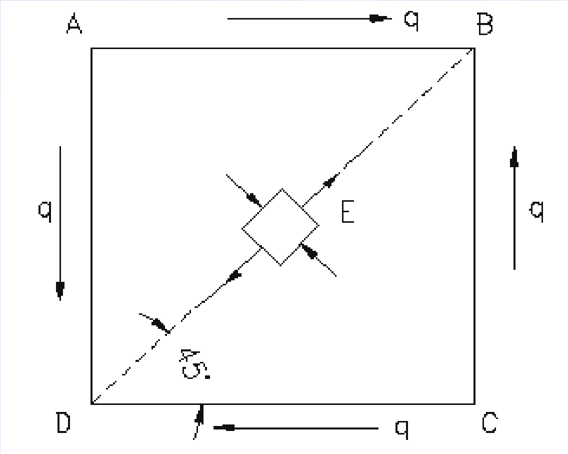


Fig 6.12 Unbuckled shear panel

When the value of (d/t) is sufficiently low (d/t < 85) τ_{cr} increases above the value of yield shear stress, and the web will yield under shear before buckling.

Based on this theory, the code gives the following values for τ_{cr} for webs, which are not too slender (Section 8.4.2.2a). The values depend on the slenderness parameter λ_w as defined in the code.

6.3.4.2.2. Tension field methods

Design of plate girders with intermediate stiffeners, as indicated in Fig. 6.10, can be done by limiting their shear capacity to shear buckling strength. However, this approach is uneconomical, as it does not account for the mobilisation of the additional shear capacity as indicated earlier. The shear resistance is improved in the following ways:

i. Increasing in buckling resistance due to reduced c/d ratio;

ii. The web develops tension field action and this resists considerably larger stress than the elastic critical strength of web in shear

Figure 6.13 shows the diagonal tension fields anchored between top and bottom flanges and against transverse stiffeners on either side of the panel with the stiffeners acting as struts and the tension field acting as ties. The plate girder behaves similar to an N-truss Fig.6.14.

The nominal shear strength for webs with intermediate stiffeners can be calculated by this method according to the design provision given in code.

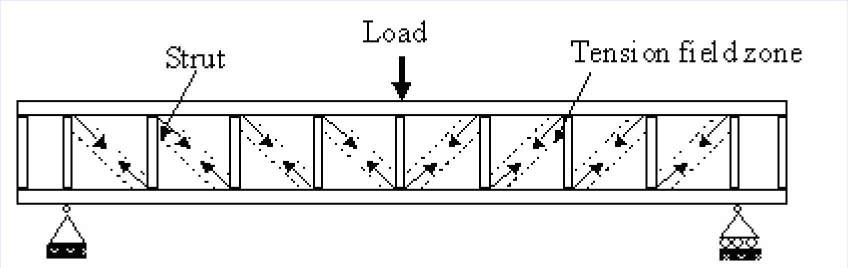


Fig 6.13 Tension field in individual sub-panel

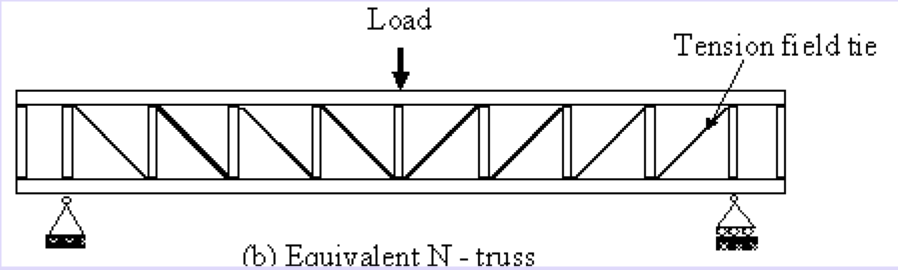


Fig 6.14 Tension field action and the equivalent N-truss

6.3.5 Stiffened web panels

For tension field action to develop in the end panels, adequate anchorage should be provided all around the end panel. The anchor force H_q required to anchor the tension field force is

$$H_q = 1.5 V_{dp} \left(1 - \frac{V_{cr}}{V_{dp}} \right)^{1/2}$$

The end panel, when designed for tension field will impose additional loads on end post; hence, it will become stout (Fig 8.2 of the code). For a simple design, it may be assumed that the capacity of the end panel is restricted to V_{cr}, so that no tension field develops in it (Fig 8.1 of the code). In this case, end panel acts as a beam spanning between the flanges to resist shear and moment caused by H_q and produced by tension field of penultimate panel.

This approach is conservative, as it does not utilise the post-buckling strength of end panel especially where the shear is maximum. This will result in *c/d* value of the end panel spacing to be less than of other panels. The end stiffeners should be designed for compressive forces due to bearing and the moment M_{tf}, due to tension field in the penultimate panel in order to be economical the end panel also may be designed using tension field action. In this case, the bearing stiffeners and end post are designed for a combination due to bearing and a moment equal to 2/3 caused due to tension in the flange $M_{tf,}$ instead of one stout stiffener we can use a double stiffener as shown in Fig. 8.3 of the code. Here, the end post is designed for horizontal shear and moment M_{tf}.

6.3.6 Design of beams and plate girders with solid webs.

The high bending moment and shear forces caused by carrying heavy loads over long spans may exceed the capacity of rolled beam sections. Plate girders can be used in such cases and their proportions can be designed to achieve high strength/weight ratio. In a plate girder, it can be assumed that the flanges resist the bending moment and the web provides resistance to the shear force. For economic design, low flange size and deep webs are provided. This results in webs for which shear failure mode is a consequence of buckling rather than yielding**.**

Minimum web thickness – In general, we may have unstiffened web, a web stiffened by transverse stiffeners, (Fig 6.9) or web stiffened by both transverse and longitudinal stiffeners (Fig 6.10).

By choosing a minimum web thickness t_w, the self-weight is reduced. However, the webs are vulnerable to buckling and hence, stiffened if necessary. The web thickness based on serviceability requirement is recommended in Section **8.6.1.1** of the code.

6.3.6.1 Compression flange buckling requirement

Generally, the thickness of flange plate is not varied along the span of plate girders. For non-composite plate girder the width of flange plates is chosen to be about 0.3 times the depth of the section as a thumb rule. It is also necessary to choose the breadth to thickness ratio of the flange such that the section classification is generally limited to plastic or compact section only. This is to avoid local buckling before reaching the yield stress. In order to avoid buckling of the compression flange into the web, the web thickness shall be based on recommendation given in Section **8.6.1.2** of the code.

6.3.6.2 Flanges

For a plate girder subjected to external loading the minimum bending moment occurs at one section usually, e.g. when the plate girder is simply supported at the ends and subjected to the uniformly distributed load, then, maximum bending moment occurs at the centre. Since the values of bending moment decreases towards the end, the flange area designed to resist the maximum bending moment is not required to other sections. Therefore the flange plate may be curtailed at a distance from the centre of span greater than the distance where the plate is no longer required as the bending moment decreases towards the ends.

Usually, two flange angles at the top and two flange angles at the bottom are provided. These angles extend from one end to the other end of the girder. For a good proportioning, the flange angles must provide an area at least one-third of the total flange area.

Generally, horizontal flange plates are provided to and connected to the outstanding legs of the flange angles. The flange plates provide an additional width to the flange and thus, reduced the tendency of the compression flange to buckle. These plates also contribute considerable moment of inertia for the section of the girder it gives economy as regards the material and cost. At least one flange plate should be run for the entire length of the girder

6.3.6.3 Flange splices

A joint in the flange element provided to increase the length of flange plates is known as flange splice. The flange plate should be avoided as far as possible. Generally, the flange plates can be obtained for full length of the plate girder. In spite of the availability of full length of flange plates, sometimes, it becomes necessary to make flange splices. Flange joints should not be located at the points of main bending moment. The design provisions for flange splices are given in detail in Section **8.6.3.2** of the code.

6.3.6.3.1 Connection of flange to web

Rivets/bolts or weld connecting the flanges angles and the web will be subjected to horizontal shear and sometimes vertical loads which may be applied directly to the flanges for the different cases such as depending upon the directly applied load to the either web or flange and with or without consideration of resistance of the web.

6.3.6.3.2 Splices in the web

A joint in the web plate provided to increase its length is known as web splices. The plates are manufactured upto a limited length. When the maximum manufactured length of the plate is insufficient for full length of the plate girder web splice becomes essential, when the length of plate girder is too long to handle conveniently during transportation and erection. Generally, web splices are not used in buildings; they are mainly used in bridges.

Splices in the web of the plate girder are designed to resist the shear and moment at the spliced section. The splice plates are provided on each side of the web. Groove-welded splice in plate girders develop the full strength of the smaller spliced section. Other types of splices in cross section of plate girders shall develop the strength required by the forces at the point of the splice.

6.3.7 Stiffener design

6.3.7.1 General

These are members provided to protect the web against buckling. The thin but deep web plate is liable to vertical as well as diagonal buckling. The web may be stiffened with vertical as well as longitudinal stiffeners

Stiffeners may be classified as

a) Intermediate transverse web stiffeners

b) Load carrying stiffeners

c) Bearing stiffeners

d) Torsion stiffeners

e) Diagonal stiffeners and

f) Tension stiffeners

The functions of the different types of stiffeners are explained in the Section **8.7.1.1** of the code.

6.3.7.2 Stiff bearing length

The application of heavy concentrated loads to a girder will produce a region of very high stresses in the part of the web directly under the load. One possible effect of this is to cause outwards buckling of this region as if it were a vertical strut with its ends restrained by the beam's flanges. This situation also exists at the supports where the 'load' is now the reaction and the problem is effectively turned upside down. It is usual to interpose a plate between the point load and the beam flange, whereas in the case of reactions acting through a flange, this normally implies the presence of a seating cleat.

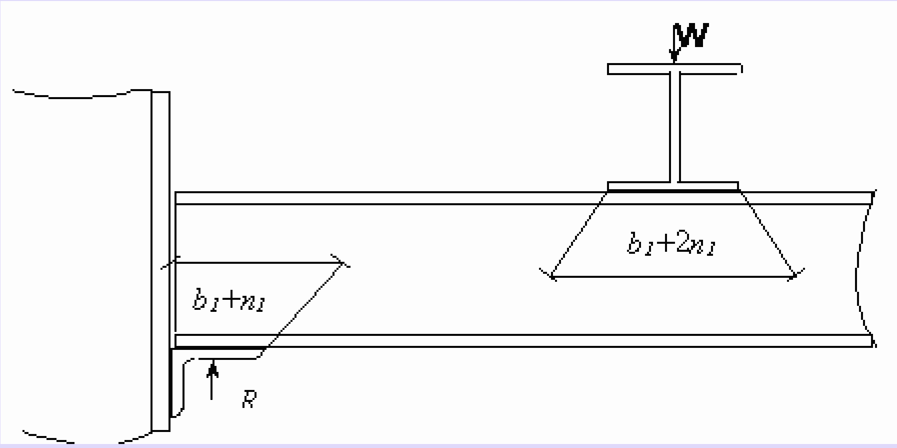


Fig 6.15 Dispersion of concentrated loads and reactions

In both cases, therefore, the load is actually spread out over a finite area by the time it passes into the web as shown in Fig.6.15. It is controlled largely by the dimensions of the plate used to transfer the load, which is itself termed "the stiff length of bearing"

Outstand of web stiffeners and eccentricity of the stiffeners is explained as per Section **8.7.1.2** to **8.7.1.5** of the code.

6.3.7.3 Design of Intermediate transverse web stiffeners

Intermediate transverse stiffeners are provided to prevent out of plane buckling of web at the location of the stiffeners due to the combined effect of bending moment and shear force.

Intermediate transverse stiffeners must be proportioned so as to satisfy two conditions

1) They must be sufficiently stiff not to deform appreciably as the web tends to buckle.

2) They must be sufficiently strong to withstand the shear transmitted by the web.

Since it is quite common to use the same stiffeners for more than one task (for example, the stiffeners provided to increase shear buckling capacity can also be carrying heavy point loads), the above conditions must also, in such cases, include the effect of additional direct loading.

The condition (1) is covered by Section **8.7.2.4** of the code.

The strength requirement is checked by ensuring that the stiffeners acting as a strut is capable of withstanding F_q. The buckling resistance F_q of the stiffeners acting as strut (with a cruciform section as described earlier) should not be less than the difference between the shear actually present adjacent to the stiffeners *V,* and the shear capacity of the (unstiffened) web V_{cr} together with any coexisting reaction or moment. Since the portion of the web immediately adjacent to the stiffeners tends to act with it, this "strut" is assumed to consist also of a length of web of 20 *t* on either side of the stiffeners centre line giving an effective section in the shape of a cruciform. Full details of this strength check are given in Section **8.7.2.5** of the code. If tension field action is being utilized, then the stiffeners bounding the end panel must also be capable of accepting the additional forces associated with anchoring the tension field.

6.3.7.4 Load carrying stiffeners

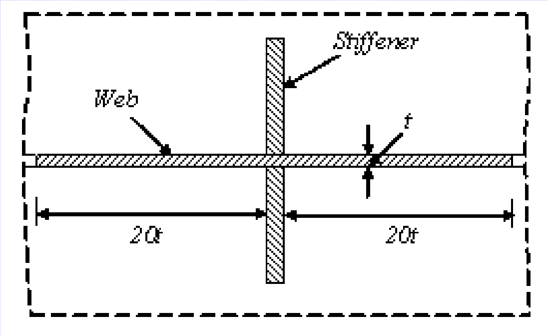


Fig.6.16 Cruciform section of the load carrying stiffener

Whenever there is a risk of the buckling resistance of the web being exceeded, especially owing to concentrated loads, load-carrying stiffeners are provided. The design of load-carrying stiffeners is essentially the same as the design of vertical stiffeners. The load is again assumed to be resisted by strut comprising of the actual stiffeners plus a length of web of 20*t* on either side, giving an effective cruciform section. Providing the loaded flange is laterally restrained, the effective length of the 'strut' may be taken as 0.7*L*. Although no separate stiffener check is necessary, a load-bearing stiffener must be of sufficient size that if the full load were to be applied to them acting independently, i.e., on a cross-section consisting of just the stiffeners, as per Fig 6.16. Then the stress induced should not exceed the design strength by more than 25%. The bearing stress in the stiffeners is checked using the area of that portion of the stiffeners in contact with the flange through which compressive force is transmitted.

6.3.7.5 Bearing stiffeners

Bearing stiffeners are required whenever concentrated loads, which could cause vertical buckling of web of the girder, are applied to either flange. Such situations occur on the bottom flange at the reactions and on the top flange at the point of concentrated loads. Figure 6.17 shows bearing stiffeners consisting of plates welded to the web. They must fit tightly against the loaded flange. There must be sufficient area of contact between the stiffeners and the flange to deliver; the load without exceeding the permissible bearing on either the flange material or the stiffeners must be adequate against buckling and the connection to the web must be sufficient to transmit the load as per Section **8.7.4** of the code.

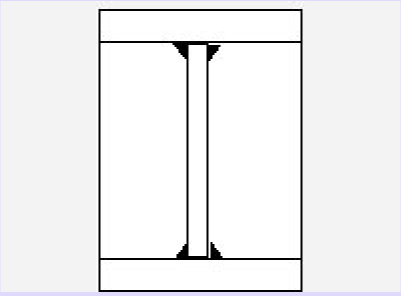


Fig. 6.17 Bearing stiffeners

The bearing stress on the contact area between stiffeners and flanges is analogous to the compressive stress at the junction of web and flange of rolled beams subjected to concentrated load.

Since buckling of bearing stiffeners is analogous to buckling of web at point of concentrated load, the required moment of inertia is not easy to evaluate. The buckled stiffeners may take any forms depending on the manner in which the flanges are restrained. In most cases, the compression flange of the girder will be supported laterally at points of concentrated load by bracing or by beams framing into it, so buckling will approximate the form of an end-fixed column. Even if the flanges are free to rotate, the stiffeners need not be considered as end-hinged columns, because the load concentrated on one end of the stiffeners is resisted by forces distributed along its connection to the web instead of by a force concentrated at the opposite end as in columns.

The connection to the web is merely a matter of providing sufficient welding to transmit the calculated load on the stiffeners as per the Section **8.7.6** of the code.

Design of different types of stiffeners is given in the code from Sections **8.7.4** to **8.7.9**.

6.3.7.6 Connection of web of load carrying and bearing

The web connection of load carrying stiffeners to resist the external load and reactions through flange shall be designed as per the design criteria given in the code.

6.3.7.7 Horizontal stiffeners

Horizontal stiffeners are generally not provided individually. They are used in addition to vertical stiffeners, which are provided close to the support to increase the bearing resistance and to improve the shear capacity.

The location and placing of horizontal stiffeners on the web are based on the location of neutral axis of the girder. Thickness of the web t_w and second moment of area of the stiffeners I_s are as per the conditions and design provisions given in Section **8.7.13** of the code.

6.3.8 Box girders

The design and detailing of box girders shall be such as to give full advantage of its higher load carrying capacity.

Diaphragm shall be used where external vertical as well as transverse forces are to be transmitted from one member to another. The diaphragms and their fastenings shall be proportioned to distribute other force applied to them and in addition, to resist the design transverse force and the resulting shear forces. The design transverse force shall be taken as shared equally between the diaphragms.

When concentrated loads are carried from one beam to the other or distributed between the beams, diaphragms having sufficient stiffeners to distribute the load shall be bolted or welded between the beams. The design of members shall provide for torsion resulting from any unequal distribution of loads.

6.3.9 Purlin and sheeting rails

Purlins attached to the compression flange of a main member would normally be acceptable as providing full torsional restraint; where purlins are attached to tension flange, they should be capable of providing positional restraint to that flange but are unlikely (due to the rather light purlin/rafter connections normally employed) to be capable of preventing twist and bending moment based on the lateral instability of the compression flange.

6.3.10 Bending in a non-principal plane

When deflections are constrained to a non-principal plane by the presence of lateral restraints, the principal axes bending moments are calculated due to the restraint forces as well as the applied forcs by any rational method. The combined effect is verified using the provisions in Section 9. Similarly, when the deflections are unconstrained due to loads acting in a non-principal plane, the principal axes bending moments are arrived by any rational method. The combined effects have to satisfy the requirements of Section 9.

6.4 Summary

Rolled beams and plate girders are important structural members. In this presentation, the limit state method of design of beams as per the IS: 800 revisions have been explained. Laterally supported and unsupported beams have been considered. The phenomenon of lateral torsional buckling has been explained. Shear yielding and shear buckling have been treated. Design of various types of stiffeners for plate girders is explained.

6.5 References

1. David A. Nethercot (2001), Limit states design of structural steelwork, Spon Press, London

2. IS: 800-(2003): Draft Structural Steel Code

3. Teaching Resource for Structural Steel design (2001), INSDAG Publication.

4. Narayanan R., Lawless V, Naji F.J. and Taylor J.C., (1993), Introduction to Concise Eurocde3 (C-EC3)-with worked examples, The Steel Construction Institute.

Problem 1

Design a hand operated overhead crane, which is provided in a shed, whose details are:

Capacity of crane = 50 kN

Longitudinal spacing of column = 6m

Center to center distance of gantry girder = 12m

Wheel spacing = 3m

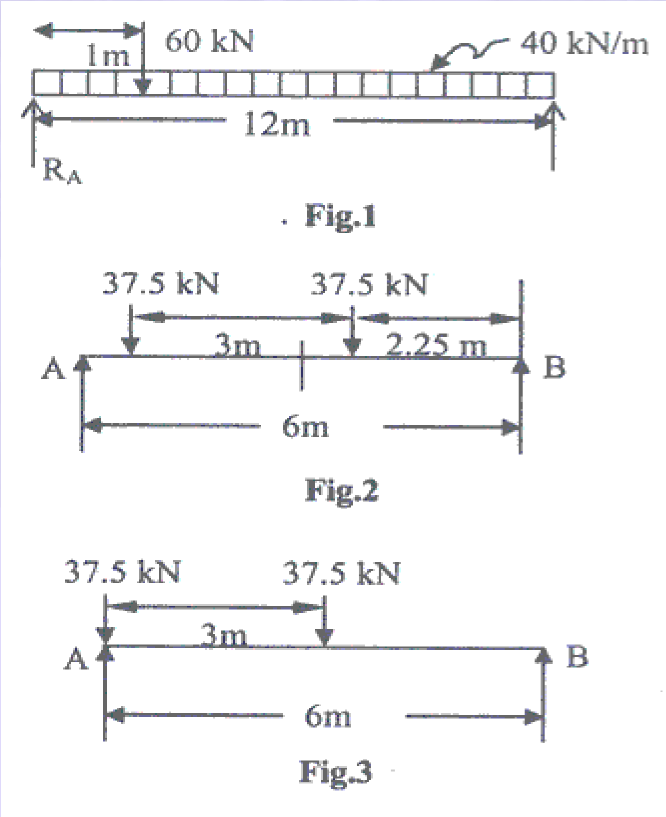
Edge distance = 1m

Weight of crane girder = 40 kN

Weight of trolley car = 10 kN

Design by allowable stress method as per IS: 800 - 1984

To find wheel load (refer fig.1):



R_A = 20 + 60 (11 / 12) = 75 kN

Wheel load = R_A / 2 = 37.5 kN

To find maximum BM in gantry girder (refer fig.2):

R_A = 46.88 kN

R_B = 28.12 kN

Max. BM = 28.12 x 2.25 = 63.27 kN-m

Adding 10% for impact,

M_1 = 1.1 x 63.27 = 69.60 kN-m

Max. BM due to self - weight of girder and rail taking total weight as 1.2 kN/m

$$M_2 = \frac{wl^2}{8} = 5.4 kNm$$

Therefore Total BM, M = 75 kN-m

To find maximum shear force (refer fig.3):

SF = R_A = 59.85 kN.

To find lateral loads:

This is given by 2.5% of (lateral load / number of wheel = 0.025 x 60 / 2 kN = 0.75 kN

Therefore Max BM due to lateral load by proportion is given by, ML = (63.27 / 37.5) x 0.75 = 1.27 kN-m

Design of section

Approximate section modulus Z_c required, (M / σ_{bc}) = 75 x 10^6 / 119 = 636 x $10^3 mm^3$ [for λ = 120, D / T = 25].

Since, the beam is subjected to lateral loads also, higher section is selected.

For, ISMB 450 @ 710.2 N/m,

Z_x = 1350.7 cm^3 , T = 17.3 mm,

$t = 9.4mm$, $I_{yy} = 111.2cm^3$

$r_y = 30.1mm$,

$b_f = 150mm$

To find allowable stresses,

$T/t = 17.4 / 9.4 = 1.85 < 2$

$D/T = 450 / 17.4 = 25.86 \sim 26$

$L/r_y = 6000 / 30.1 = 199.3 \sim 200$

Therefore allowable bending compression about major axis is, $s_{bc'\,x} = 77.6$ N/mm^2

Actual stress in compression side, $s_{b'\,x} = M / Z = 55.5\ N / mm^2$.

The bending moment about Y-axis is transmitted only to the top of flange and the flange is treated as rectangular section. The allowable stress is $s_{bc.\,y}$ = 165 MPa (i.e. 0.66 fy).

Z_y of the flange = $111.2 / 2 = 55.6\ cm^3$

Therefore $s_{by} = M_y / Z_y = 1.27 \times 10^6 / 55.6 \times 10^3$

$= 22.84$ MPa

The admissible design criteria is = $(s_{bx} / s_{bcx}) + (s_{by} / s_{bcy}) = (55.5 / 77.6) + (22.84 / 165) = 0.715 + 0.138 = 0.854 < 1$. Hence , the design is safe. So, ISMB 450 is suitable

Check for shear:

Design shear stress, τ_x $V / (Dt) = 59.85 \times 10^3 / 450 \times 9.4 = 14.15$ MPa. This is less than τ_a $(0.4f_y)$. Hence, the design is o.k.

Check for deflection and longitudinal bending can be done as usual.

Design by limit state method as per IS: 800 draft code

For ISMB 450, properties are given below:

T = 17.4mm, t = 9.4mm, b = 150mm, r_y = 30.1mm, Z_p = 1533.33 cm^3, Z_c = 1350.7 cm^3, Shape factor = 1.15, I_{zz} = 30390.8 cm^4, H^1 = d = 379.2mm

Section classification:

Flange criteria: b / T = 75 / 17.4 = 4.31 < 9.4

No local buckling. Therefore OK

Web criteria: d / t_w = 379.2 / 9.4

= 40.34 < 83.9

No local buckling. Therefore OK

Section plastic.

Shear capacity:

$$F_{vd} = (f_{yw} A_v) / (\sqrt{3}\gamma_{mo})$$

$$= (250 \times 450 \times 9.4) / (\sqrt{3} \times 1.1)$$

$$= 555043 \text{ N} \cong 555 \text{ kN}$$

F_v / F_{vd} = (59.85 x 1.5) / 555 = 0.1634 < 0.6

Check for torsional buckling (CI.8.2.2):

t_f / t_w <= 17.4 / 9.4 = 1.85 < 2.

Therefore β_{LT} = 1.2 for plastic section

M_{cr} = Elastic critical moment

$$M_{cr} = \frac{\beta_{LT}\pi^2 hEI_y}{2(Kl)^2}\left[1 + \frac{1}{20}\left\{\frac{\frac{KL}{r_y}}{\frac{h}{t_f}}\right\}^2\right]^{0.5}$$

$$= \frac{1.2\pi^2 \times 450 \times 2 \times 10^5 \times 834 \times 10^4}{2 \times 6000^2} \times \left[1 + \frac{1}{20}\left\{\frac{\frac{6000}{30.1}}{\frac{450}{17.4}}\right\}^2\right]^{0.5}$$

$$= 246 \times 10^6 \; N\text{-}m$$

$$Now,\ \lambda_{LT} = \sqrt{\frac{\beta_b z_p f_y}{M_{cr}}} = 1.248$$

(β_b = 1 for plastic section)

Therefore $\phi_{LT} = 0.5\ [1 + \alpha_{LT}\ (\lambda_{LT} - 0.2) + \lambda^2_{LT}$

$= 0.5\ [1 + 0.21\ (1.248 - 0.2) + 1.248^2]$

$= 1.389$

$$X_{LT} = \frac{1}{\phi_{LT} + [\phi^2_{LT} - \lambda^2_{LT}]^{0.5}}$$

$$= 1/1/999 = 0.5$$

$$f_{bd} = \frac{X_{LT}\, f_y}{\gamma_{mo}} = (0.5 \times 250)\ /\ 1/1$$

$$= 113.7\ MPa$$

Therefore $M_d = \beta_b .\ z_p .\ f_{bd}$

$= 1 \times 1533.3 \times 10^3 \times 113.7$

$= 1.7434 \times 10^8 = 174.34$ kN-m

Factored longitudinal moment, $M_f = 75 \times 1.5 = 112.5$ kN-m

Factored lateral moment, $M_{fL} = 1.27 \times 1.5 = 1.91$ kN-m

Lateral BM capacity = M_{dL}

$$= \frac{Z_{py}.f_y}{1.10} = \frac{\frac{Z_{ey}}{2}(Shape\,Factor)f_y}{1.10}$$

$$= \frac{\left[\frac{111.2 \times 10^3}{2}\right]1.15 \times 250}{1.10}$$

$$= 14.53 \times 10^6\ N\ or\ 14.53\ kN\text{-}m$$

For Safety,

$$\frac{M_f}{M_d}\Big]_{longi} + \frac{M_f}{M_d}\Big]_{lateral}$$

= (112.5 / 135.9) + (1.91 / 14.53) = 0.96 < 1

Hence, it is safe

Problem 2:

Design a member (beam - column) of length 5.0^M subjected to direct load 6.0^T (DL) and 5.0^T (LL) and bending moments of M_{zz} {3.6^{TM} (DL) + 2.5^{TM} (LL)} and M_{yy} {0.55^{TM} (DL) + 0.34^{TM} (LL)} at top and M_{zz} {5.0^{TM} (DL) + 3.4^{TM} (LL)} and M_{yy} {0.6^{TM} (DL) + 0.36^{TM} (LL)} at bottom.

LSM {Section 9.0 of draft IS: 800 (LSM version)}

Factored Load,

N = 6.0 x 1.50 + 5.0 x 1.5 = 16.5^T (Refer Table 5.1 for load factors)

Factored Moments:

At top, $M_z = 9.15^{TM}$, $M_y = 1.335^{TM}$

At bottom, $M_z = 12.60^{TM}$, $M_y = -1.44^{TM}$

Section used = MB500.

For MB500:

$A = 110.7\ cm^2$; $r_{yy} = 3.52$ cm,

$I_{zz} = 48161.0\ cm^4$; $Z_{zz} = 1809\ cm^3$

$Z_{\ yy} = 152.2\ cm^3$; $Z_{pz} = 2074.7\ m^3$;

$Z_{py} = 302.9\ cm^3$

d = 465.6 mm, $t_w = 10.29$ mm

$r_{zz} = 20.21$ cm, b = 180 mm;

$I_{yy} = 1369.8\ cm^3$

(i) Sectional Classifications (Refer Fig: 3.1 and Table - 3.1 of draft code):

$b / t_f = 90 / 17.2 = 5.23 < 8.92$, therefore the section is plastic.

$d / t_w = 465.6 / 10.2 = 45.65 < 47$, therefore the section is semi-compact for direct load and plastic for bending moment.

(ii) Check for resistance of the section against material failure due to the combined effects of the loading (Clause-9.3.1):

N_d = Axial strength = A. f_y / γ_{mo}

$= 110.7 \times 2500 / 1.10 \times 10^{-3} = 251.68^T$

$N = 16.5^T$

$n = N / N_d = 16.5 / 251.68 = 0.066 < 0.2$

Therefore $M_{ndz} = 1.1 M_{dz} (1-n) < M_{dz}$

Therefore $M_{ndz} = 1.1\{1.0 * 2074.7 * 2500 / 1.10\} (1 - 0.066) * 10^{-5} < M_{dz}$

Therefore $M_{ndz} = 48.44^{T\text{-}m} > M_{dz}\ (=47.15^{T\text{-}m})$

Therefore $M_{ndz} = M_{dz} = 47.15^{T\text{-}m}$

For, $n <= 0.2$ $M_{ndy} = M_{dy}$

$M_{ndy} = (1.0 \times 302.9 \times 2500) / (1.10 \times 100000) = 6.88^{T\text{-}m}$

($\beta_b = 1.0$ for calculation of M_{dz} and M_{dy} as per clause 8.2.1.2)

Therefore $(M_y / M_{ndy})^{\alpha 1} + (M_z / M_{ndz})^{\alpha 2} = (1.44 / 6.88)^1 + (12.60 / 47.15)^2 = 0.281 <= 1.0$

{α_1 = 5n but >= 1, therefore α_1 = 5 x 0.066 = 0.33 = 1

α_2 = 2 (As per Table 9.1)}

Alternatively,

$(N / N_d) + (M_z / M_{dz}) + (M_y / M_{dy})$ = { $(16.5 \times 10^3 \times 1.10) / (110.7 \times 2500)$ + $(12.60 \times 10^5 \times 1.10) / (302.9$
$\times 2500) = 0.54 <= 1.0$

(iii) Check for resistance of the member for combined effects of buckling (Clause- 9.3.2):

(a) Determination P_{dz}, P_{dy} and P_d (Clause 7.12)

$KL_y = KL_z = 0.85 \times 500 = 425$mm

$(KL_z / r_z) = 425 / 20.21 = 21.03$

$(KL_y / r_y) = 425 / 3.52 = 120.7$

Therefore, non- dimensional slenderness ratios, $\lambda_z = 0.237$ and $\lambda_y = 1.359$.

For major axis buckling curve **'a'** is applicable (Refer Table 7.2).

From Table 7.4a,

f_{cdz} = 225.4 MPa

$P_{dz} = 225.4 \times 10 \times 110.7 \times 10^{-3} = 249.65^T$

For minor axis buckling curve 'b' is applied (Refer Table 7.2)

From Table 7.4b,

f_{cdy} = 90.80 MPa

$P_{dy} = 90.80 \times 10 \times 110.7 \times 10^{-3} = 100.60^T$

Therefore, $P_d = P_{dy} = 100.60^T$

(b) Determination of M_{dz} (Clause 9.3.2.2 and Clause 8.2.2).

Elastic critical moment is given by (clause 8.2.21).

$M_{cr} = [(\beta_{LT}\,\pi^2 EI_y h) / \{2(KL)^2\}]\ [1 + 1/20\ \{(KL / r_y) / (h / t_f)\}^2]^{0.5}$

$=[(1.20 \times \pi^2 \times 2 \times 10^5 \times 1369.8 \times 10^4 \times 500) / (2 \times 4250^2)]\ [1 + 1/20\ \{(120.70) / (500 / 17.2)\}^2]^{0.5}$

$= 6.129 \times 10^8$ N-mm

$$\lambda_{LT} = \sqrt{(\beta_b Z_p f_y / M_{cr})}$$
$$= \sqrt{(1.0 \times 2074.70 \times 10^3 \times 250 / 6.129 \times 10^8}$$
$$= 0.92$$
$$\phi_{LT} = 0.5\,[1 + \alpha_{LT}\ (\lambda_{LT} - 0.2) + \lambda_{LT}{}^2]$$
$$= 0.5\,[1 + 0.21(0.92 - 0.2) + 0.92^2]$$
$$= 0.999$$
$$X_{LT} = 1 / [\phi_{LT} + \{\phi_{LT}{}^2 - \lambda_{LT}{}^2\}^{0.5}]$$
$$= 0.72 \leq 1.0$$
$$f_{bd} = X_{LT} f_y / \gamma_m = 0.72\,x\,250 / 1.1 = 168.3\,MPa$$
$$M_{dz} = \beta_b . Z_{pz} . f_{bd}$$
$$= 1.0 \times 2074.7 \times 1638 \times 10^{-5}$$
$$= 33.98^{T\text{-}m}$$
$$\left[\beta_{LT} = 1.20,\ \beta_b = 1.0, \alpha_{LT} = 0.21\right]$$

(c) Determination of M_{dy} (Clause 9.3.2.2)

$M_{dy} = \beta_b\ Z_{py}.\ f_y / \gamma_m$

$= 1.0 \times 302.90 \times 2500 / 1.1 \times 10^{-5} = 6.88^{T\text{-}m}$

(d) Determination of C_z (Clause 9.3.2.2) From Table - 9.2,

$$\psi_z = 9.15 / 12.6 = 0.726$$
$$\therefore \beta_{mz} = 1.8 - 0.7 \times 0.726 = 1.292$$
$$\therefore \mu_{fz} = \lambda_z (2\beta_{mz} - 4) + (Z_{pz} - Z_{ez}) / Z_{ez} \leq 0.9$$

= 0.237 (2 x 1.292 - 4) + 0.1469

= -0.188

For torsional buckling,

$$\mu_{LT} = 0.15\lambda_y \beta_{MLT} - 0.15 \leq 0.90$$

$$Since,\ \beta_{MZ} = \beta_{MLT},$$

$$\therefore \mu_{LT} = 0.15 \times 1.292 \times 1.359 - 0.15 = 0.113$$

$$Since\ \mu_z \text{ is larger of } \mu_{lz} \text{ and } \mu_{LT}, \mu_z = 0.113$$

$$\therefore C_z = 1 - (\mu_z.P)/P_{dz}$$

$$= 1 - 0.113 \times 16.5 / 249.65 = 0.993 \leq 1.50$$

(e) Determination of C_y (Clause 9.3.2.2)

$$\psi_y = 1.335/(-1.44) = -0.927$$

$$\therefore \beta_{my} = 1.8 - 0.7 \times (-0.927) = 2.449$$

$$\therefore \mu_y = \lambda_y(2\beta_{MY} - 4) + (Z_{py} - Z_{ey})/Z_{ey} \leq 0.9$$

$$= 1.359(2 \times 2.449 - 4) + 0.99 = 2.22 \approx 0.90$$

$$\therefore C_y = 1 - (\mu_y.P)/P_{dy}$$

$$= 1 - 0.90 \times 16.5/100.6 = 0.85$$

$$\therefore P/P_d + C_Z M_z/M_{dz} + C_y M_y/M_{dy}$$

$$= 16.50/100.60 + 0.993 \times 12.60/33.98 + 0.85 \times 1.44/6.88$$

$$= 0.164 + 0.368 + 0.178 = 0.71 \leq 1.0$$

Therefore, section is safe. Interaction value is less in LSM than WSM.

Design of typical beam

Example-1

The beam (ISMB 400) in Fig.1 is designed considering it is fully restrained laterally.

1. WSM (clause 6.2 of IS:800 - 1984):

Bending moment, $M=18.75^{Tm}$

For ISMB 450, $Z=1350.7\ cm^3$

Therefore

$s_{bc(cal)} = 18.75 * 10^5 / 1350.7$

$= 1388.17\ kg/cm^2 < 1650\ Kg/cm^2$

Therefore percentage strength attained is 0.8413 or 84.13%s

2. LSM (clause 8.2 of draft IS:800):

Factored load = $1.5 * 2.0 + 1.5 * 4.0 = 9.0^T$

Factored bending moment = $9.0 * 5.0^2 / 8$

$= 28.125^{Tm}$

Factored shear force = $22.50^T = Fv$

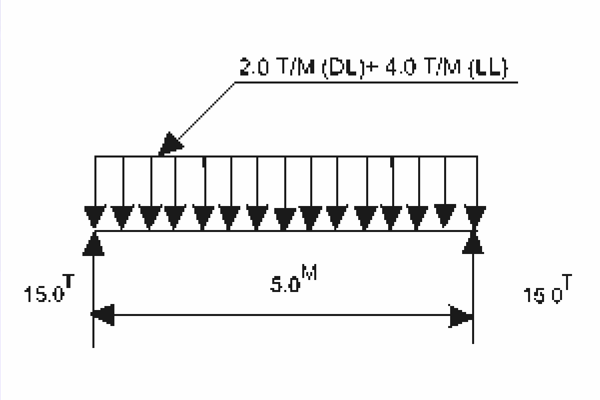


Fig.1

For, ISMB450,

$D = 450$ mm $B = 150$ mm

$t_w = 9.4$ mm $T=17.4$ mm

$I_{xx} = 30390.8$ cm^4 $Iyy = 834.0$ cm

$r_{yy} = 3.01$ cm $h_1 = 379.2$ cm = d

i) Refering Table 3.1 of the code for section classification, we get :

Flange criterion = (B / 2) / T = (150 / 2) /17.4

= 4.31 < 9.4.

Web criteria = $d / t_w = 379.2 / 9.4$

= 40.34 < 83.9.

Therefore, section is plastic.

ii) Shear capacity (refer clause 8.4 of draft code) :

$F_{vd} = (f_{yw} .A_v)/ (\sqrt{3} g_{mo}) = (f_{yw} .ht_w)/(\sqrt{3} g_{mo})$

$= (2500 * 45.0 * 0.94)/ (\sqrt{3} * 1.10 * 1000)$

= 55.50T

$F_v / F_{vd} = 22.50 /55.5 = 0.405 < 0.6.$

Therefore , shear force does not govern permissible moment capacity (refer clause 8.2.1.2 of the draft code).

iii) Since the section is ' plastic' ,

$M_d = (Z_p .f_y) / g_{mo}$

where , Z_p = Plastic Modulus = 1533.36 cm^3

(refer Appendix-I of draft code).

Therefore , $M_d = (1533.36*2500) /1.10 * 10^5$

$= 34.85^{Tm}$

Percentage strength attained for LSM is (28.125 / 34.85) = 0.807 or 80.7 % which is less than 84.3 % in the case of WSM .

iv) Now , let us check for <u>web buckling :</u>

K_v = 5.35 (for transverse stiffeners only at supports as per clause 8.4.2.2 of draft code).

The elastic critical shear stress of the web is given by :

$\tau_{cr.e} = (k_v .\pi^2 .E) / \{ 12 (1 - \mu^2)(d / t_w)^2 \}$

$= (5.35 \pi^2 E) / \{ 12 (1 - 0.3^2)(379.2 / 9.4)^2 \}$

$= 594.264$ N / mm^2 .

Now as per clause 8.4.2.2 of draft code ,

$\lambda_w = \sqrt{\ } \{ f_{yw} / (\sqrt{3}\, \tau_{cr.e})\}^{0.5}$

$= \{250 / (\sqrt{3} * 594.264)\}^{0.5} = 0.4928 < 0.8$.

where , λ_w is a non-dimensional web slenderness ratio for shear buckling for stress varying from greater than 0.8 to less than 1.25 .

Therefore , τ_b (shear stress corresponding to buckling) is given as :

$\tau_b = f_{yw} / \sqrt{3} = 250 / \sqrt{3} = 144.34$ N / mm^2 .

$V_d = d.t_w. \tau_b / \gamma_{mo}$

$= 37.92 * 0.94 * 1443.34 * 10^{-3} / 1.10$

$= 46.77^{Ts}$

As $V_D > F_v$ ($= 22.5^T$), the section is safe in shear.

For checking of deflection :

$s = (5 * 60 * 500^4) / (384 * 2.1 * 10^6 * 30390.8)$

= 0.76 cm = 7.6 mm = L / 658

Hence O.K.

Example - 2

The beam (ISMB 500) shown in Fig2 is carrying point load as shown , is to be designed. The beam is considered to be restrained laterally.

1 . <u>In WSM (clause 6.2 of IS : 800 - 1984) :</u>

Bending Moment , M = (5 + 20) * 3 / 4

= 26.25^{Tm}

For, ISMB 500 : Z = 1808.7 cm^3

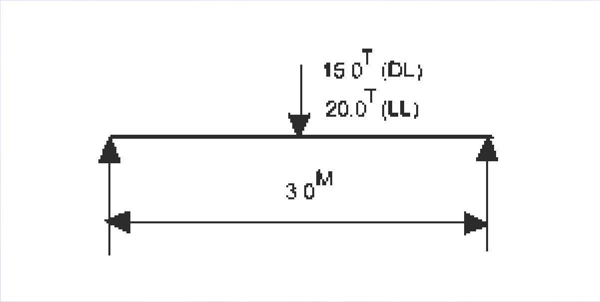


Fig .2

So , $\sigma_{bc(cal)}$ = 26.25 * 10^5 / 1808.7 Kg / cm^2

= 1451.318 Kg / m^2 .

Therefore , percentage strength attained is 0.88 or 88 %.

2 . <u>In LSM (clause 8.2 of draft IS : 800):</u>

Factored load = 1.5 * 15 + 1.5 * 20 = 52.50^T

Therefore,

Factored bending moment = 52.5 * 3.0 / 4

= 39.375^{Tm}

Factored shear force, F_v = 52.50 / 2 = 26.25^T

i) For ,<u>section classification</u> of this ISMB500 , (refer table 3.1 of the code) , we get :

$D = 500$ mm $\quad d = 424.1$ mm

$r_1 = 17$mm $\quad r_y = 3.52$ cm

$Z_p = 20.25.74\ cm^3$ $\quad T = 17.2$ mm

$b = 180$ mm $\quad t_w = 10.2$ mm .

Flange criterion , $(b / 2) / T = (180 / 2) / 17.2$

$= 5.23 < 9.4$

Web criterion , $d / t_w = 424.1 /10.2$

$= 41.57 < 83.9$ (O.K)

Therefore the section is plastic.

ii) <u>Shear capacity</u> (clause 8.4):

$F_{vd} = (f_{yw} .A_v)/ (\sqrt{3} \gamma_{mo})$

$= (2500 * 50 * 1.02)/ (\sqrt{3} * 1.10 * 1000)$

$= 66.92^T$

$F_v = 26.25^T$

Since $F_v / F_{vd} < 0.6$. shear force does not govern Bending Strength

iii) For <u>moment check :</u>

Since the section is plastic ,

$M_d = (Z_p .f_y) / \gamma_{mo}$

$= (2025.74 * 2500) /1.10 * 10^5$

$= 46.04^{Tm}$

Therefore , Percentage strength attained is (39.375 / 46.04) = 0.855 or 85.5 % which is less than 88 % in case of WSM

iv) Check for <u>web buckling :</u>

K_v = 5.35 (for transverse stiffeners only at supports as per clause 8.4.2.2 of draft code).

$\tau_{cr.e} = (k_v .\pi^2 .E) \ / \{ 12 (1 - \mu^2)(d / t_w)^2 \}$

$= (5.35 \pi^2 E) / \{ 12 (1 - 0.3^2)(424.1 / 10.2)^2 \}$

$= 559.40 \text{ N / mm}^2$.

Now as per clause 8.4.2.2 ,

$\lambda_w = \sqrt{\ } \{ f_{yw} / (\sqrt{3} \tau_{cr.e})\}^{0.5}$

$= \{250 / (\sqrt{3} * 559.40)\}^{0.5} = 0.508 < 0.8$.

Therefore ,

$\tau_b = f_{yw} / \sqrt{3} = 250 / \sqrt{3} = 144.34 \text{ N / mm}^2$.

$V_D = d.t_w. \tau_b / \gamma_{mo}$

$= 42.41 * 1.02 * 1443.4 * 10^{-3} / 1.10$

$= 56.76^T$

Since $V_D > F_v$ ($= 26.25^T$), the section is safe against shear.

For checking of deflection :

$\sigma = (35000 * 300^3) / (48 * 2.1 * 10^6 * 45218.3)$

= 0.207 cm = 2.07 mm = L / 1446

Hence O.K.

Example - 3

The beam , ISMB500 as shown in Fig.3 is to be designed considering no restraint along the span against lateral buckling .

i) In <u>WSM (clause 6.2, 6.2.2, 6.2.3 and 6.2.4 and 6.2.4.1 of IS : 800 - 1984) :</u>

Bending moment, $M = 2.1 * 6^2 / 8 = 9.45^{Tm}$

For, ISMB500 :

$L = 600$ cm $\quad r_y = 3.52$ cm

$T = 17.2$ mm $\quad Z = 1808.7\ cm^3$

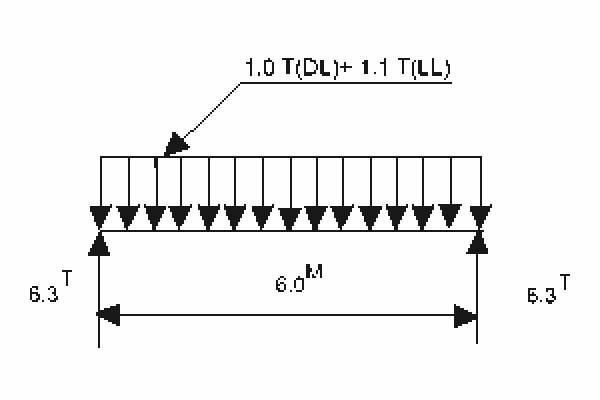


Fig.3

Therefore,

$Y = 26.5 * 10^5 / (600 / 3.52)^2 = 91.2$

$X = 91.21 \sqrt{[1 + (1 / 20)\{(600 * 1.72) / (3.52 * 50)\}^2]}$

$= 150.40$

and $f_{cb} = k_1 (X + k_2 y) C_2 / C_1 = X$

(since $C_2 = C_1$, $k_1 = 1$ and $k_2 = 0$)

Now ,

$\sigma_{bc(perm)} = (0.66\ f_{cb}.f_y) / \{f_{cb}^n + f_y^n\}^{1/n}$

$= 746\ Kg / cm^2$.

and $\sigma_{bc(cal)} = 9.45 * 10^5 / 1808.7$

$= 522.5$ Kg / cm^2.

Therefore , percentage strength attained is (522.5 / 746) = 0.7 or 70 %.

ii) In <u>LSM (clause 8.2 of draft IS : 800) :</u>

For MB 500 :

D = 500 mm T = 17.2 mm

B = 180 mm t = 10.2 mm

$Z_p = 2025.74$ cm^3 $r_{yy} = 3.52$ cm

$H_1 = d = 424.1$ mm

$I_{zz} = 45218.3$ cm^4 , $I_{yy} = 1369.8$ cm^4

iii) <u>Classification of section (ref . Table 3.1 of the code):</u>

b / T = 90 / 17.2 = 5.2 < 9.4 , hence O.K.

d / t = 424.1 / 10.2 = 41.6 < 83.90 (O.K)

Therefore , the section is Plastic .

iv) Check for <u>torsional buckling (clause 8.2.2) :</u>

t_f / t_w for ISMB 500 = 17.2 / 10.2 = 1.69 $\leq$ 2.0

Therefore , $\beta_{LT} = 1.20$,for plastic and compact aestions.

M_{cr} = Elastic critical moment given as :

$M_{cr} = \{ (\beta_{LT} \pi^2 .EI_y) / (KL)^2 \}[1 + (1 / 20) \{KL / r_y) / (h / t_f) \}^2]^{0.5} (h/ 2)$

(ref . clause 8.2.2.1)

$= (1.20 \pi^2 * 2 * 10^6 * 1369.8 / 600^2)[1 + (/ 20)\{ (600 / 3.52) / (50 / 1.72)\}^2]^{0.5} (50 / 2)$

= 371556.3 Kg-cm .

Now , $\lambda_{LT} = \sqrt{} (\beta_b .Z_p .f_y / M_{cr}) = 1.1675$

(since $\beta_b = 1.0$ for plastic and compact sections)

Therefore ,

$\phi_{LT} = 0.5 [1 + \alpha_{LT} (\lambda_{LT} - 0.2) + \lambda_{LT}^2]$

$= 0.5 [1 + 0.2 (1.1675 - 0.2) + 1.1675^2]$

$= 1.283$ ($\alpha_{LT} = 0.21$ for rolled section).

Therefore $X_{LT} = 1 / [\phi_{LT} + \{ \phi_{LT}^2 - \lambda_{LT}^2 \}^{0.5}]$

$= 1 / [1.283 + \{ 1.283^2 - 1.1675^2 \}^{0.5}]$

$= 0.55$

$M_d = 1.0 * 0.55 * 2025.74 * 2500 / 1.10 * 10^{-5}$

$= 25.32^{Tm}$

Now actual moment is obtained as follows:

Factored load = $1.0 * 1.5 + 1.1 * 1.5 = 3.15^{Tm}$

Factored moment = $14.175^{Tm} < M_d$

Percentage strength attained is (14.75 / 25.32) = 0.56 or 56 % which is less than 70 % in case of WSM.

Hence , ***the beam is safe both in LSM and WSM design but percentage strength attained is comparatively less in LSM for the same section***

Problem:

The girder showed in Fig. E1 is fuly restrained against lateral buckling throughtout its sapan. The span is 36 m and carries two concentrated loads as show in Fig. E1. Design a plate girder.

Yield stress of steel, f_y = 250 N/mm^2

Material factor for steel, γ_m = 1.15

Dead Load factor, γfd = 1.50

Imposed load factor, γfl = 1.50

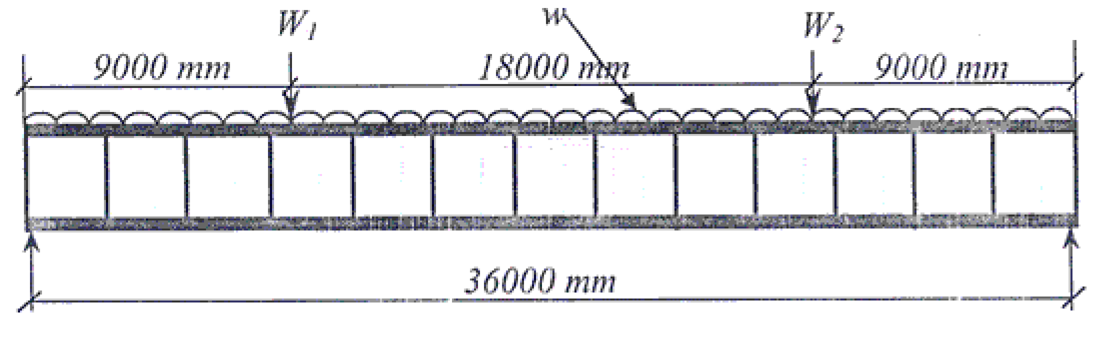


Fig. E1 Example plate girder

1.0 Loading

Dead load:

Uniformly distributed load, wd = 18 kN/m

Concentrated load, W_{1d} = 180 KN

Concentrated load, W_{2d} = 180 KN

Live load:

Uniformly distributed load, W = 35 kN/m

Concentrated load, W_{1l} = 400 kN

Concentrated load, W_{2l} = 400 kN

Factored Loads

$w' = w_d * \gamma_{fd} + w_l * \gamma f_l = 18 * 1.5 + 35 * 1.5 = 79.5$ kN/m

$W'_1 = W_{1d} * \gamma fd + W_{1l} * \gamma f_l = 180 * 1.5 + 400 * 1.5 = 870$ kN

$W'_2 = W_{2d} * \gamma fd + W_{2l} * \gamma f_l = 180 * 1.5 + 400 * 1.5 = 870$ kN

2.0 Bending moment and shear force

	Bending moment (kN-m)	Shear force (kN)
UDL effect	$\frac{w\ell^2}{8} = \frac{79.5*36*36}{8} = 12879$	$\frac{w\ell}{2} = 1431$
Concentrated load effect	$\frac{w\ell}{4} = 870*9 = 7830$	w=870
Total	20709	2301

The desigh shear forces and bending moments are shown inf Fig. E2.

3.0 Initial sizing of plate girder

Depth of the plate girder:

The recommended span/depth ratio for simply supported girder varies between 12 for short span and wo for long span girder. Let us consider depth of the girder as 2600 mm.

$$\frac{\ell}{d} = \frac{36000}{2600} = 1\ 38$$

Depth of 2600 mm is acceptable.

(For drawing the bending moment and shear force diagrams, factored loads are considered)

Fig.E2 Bending moment and shear force diagrams

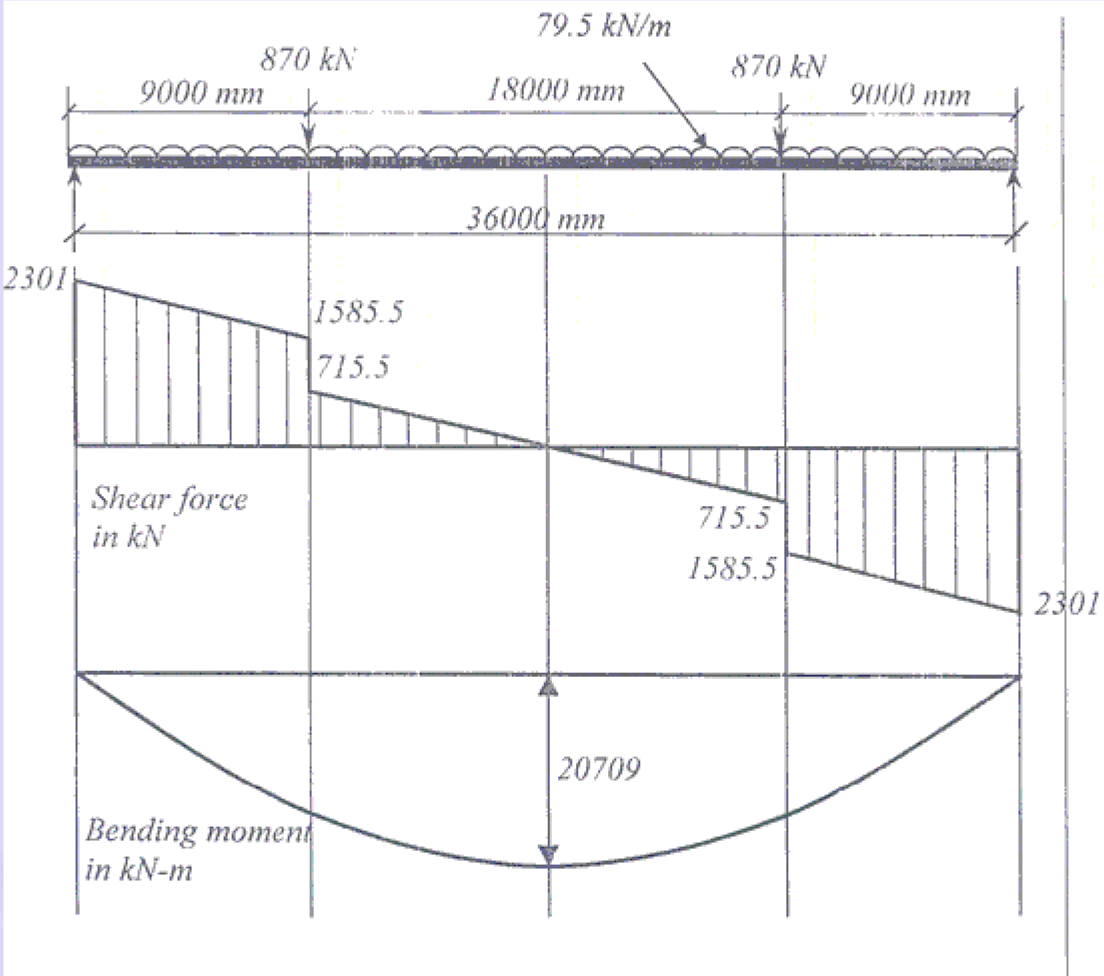


Fig. E2 Bending moment and shear force diagrams

Flange:

P_y = 250/1.15 = 217.4 N/mm ^{2}

Single flange area,

$$A_f = \frac{M_{max}}{d_{Py}} = \frac{20709*10^6}{2600*217.4} = 366375.5 mm^2$$

By thumb rule, the flange width is assumed as 0.3 times the depth of the section.

Try 780 X 50 mm, giving an area = 39000 mm$^{2.}$

Web:

Minimum web thickness for plate girder in builidings usually varies between 10 mm to 20 mm. Here, thickness is assumed as 16mm.

Hence, web size is 2600 X 16 mm

4.0 Section classification

Flange:

$$\varepsilon = \left\{\frac{250}{f_y}\right\}^{\frac{1}{2}} = \left\{\frac{250}{250}\right\}^{\frac{1}{2}} = 1.0$$

$$b = \frac{B-t}{2} = \frac{780-16}{2} = 382$$

$$\frac{b}{T} = \frac{382}{50} = 7.64 < 9\varepsilon$$

Hence, Flange is COMPACT SECTION.

Web:

$$\frac{d}{t} = \frac{2600}{250} = 162.5 > 67\varepsilon$$

Hence, the web is checked for shear buckling.

5.0 Checks

Check for serviceability:

$$\frac{d}{250} = \frac{2600}{250} = 10.4\text{mm} < t$$

$$\text{Since}, t > \frac{d}{250}$$

Web is adequate for serviceability.

Check for flange buckling in to web:

Assuming stiffener spacing, a > 1.5 d

$$t \geq \frac{d}{294}\left(\frac{P_{yf}}{250}\right)^{\frac{1}{2}} = \frac{2600}{294} x \left(\frac{217.4}{250}\right)^{\frac{1}{2}} = 8.2 \text{ mm}$$

Since, t (=16 mm) > 8.2 mm, the web is adequate to avoid flange buckling into the web.

Check for moment carrying capacity of the flanges:

The moment is assumed to be resisted by flanges alone and the web resists shear only.

Distance between centroid fo flanges, h_s = d + T = 2600 + 50 = 2650 mm

A_f = B * T = 780 * 50 = 39000 mm^2

M_c = P_{yf} * A_f * h_s = 217.4 * 39000 * 2650 * 10-6 = 222468.3 kN-m

> 20709 kN-m

Hence, the section in adequate for carrying moment and web is designed for shear.

6.0 Web design

The stiffeners are spaced as shown in Fig.E5. Three different spacing values 2500, 3250 and 3600 mm are adopted for trail as shown in fig. E5.

End panel (AB) design:

d=2600 mm

t = 16 mm

Maximum shear stress in the panel is

$$f_v = \frac{F_{VA}}{dt} = \frac{2301*10^3}{2600*16} = 55.3\ \text{N/mm}^2$$
$$\frac{a}{d} = \frac{2500}{2600} = 0.96$$
$$\frac{d}{t} = \frac{2600}{16} = 162\ 5$$

Calcualtion of critical stress,

$$\begin{aligned} q_e(\text{when } a/d \leq 1) &= \left[0.75 + 1/\ (a/d)^2\right]\left[1000/(d/t)\right]^2 \\ &= \left[0.75 + 1/(0.96)^2\right]\left[1000/(162.5)\right]^2 \\ &= 69.5\ \text{N/mm}^2 \end{aligned}$$

Slenderness parameter,

$$\begin{aligned} \lambda_w &= \left[0.6\left(f_{yw}/\gamma_m\right)/\ qe\right]^{\frac{1}{2}} \\ &= \left[0.6(250/1.15)/69.5\right]^{\frac{1}{2}} \\ &= 1.37 > 1.25 \end{aligned}$$

Hence, Critical shear strength ($q_{cr} = q_e$) = 69.5 N/mm^2

Since, $f_v < q_{cr}$ (55.3 < 69.5)

Tension field action need not be utilised for design.

Checks for the end panel AB:

End panel AB should also be checked as a beam (Spanning between the flanges of the girder) capable of resisting a shear force R_{tf} and a moment M_{tf}due to anochor forces.

(In the following calculations boundary stiffeners are omitted for simplicity)

Check for shear capacity of the end panel:

$$H_q = 0.75dt\ P_y\left[1-\frac{q_{cr}}{0.6P_y}\right]^{\frac{1}{2}}$$

$$q_{cr} = 69\ 5\ \text{N/mm}^2$$

$$H_q = 0\ 75*2600*16*250/1.15\left[1-\frac{69.5}{0.6*(250/1.15)}\right]^{\frac{1}{2}} = 4636\ \text{kN.}$$

$$R_{tf} = \frac{H_q}{2} = \frac{4636}{2} = 2318\ \text{kN}$$

$$A_v = t.a = 16*2500 = 4000\ \text{mm}^2$$

$$P_v = 0.6P_{yw}A_v = 0\ 6*(250/1.15)*40000/1000 = 5217\ \text{kN}$$

Since, $R_{tf} < P_v$, the end panel can carry the shear force.

Check for moment capacity of end panel AB:

$$M_{tf} = \frac{H_q d}{10}$$

$$= \frac{4636*2600}{10}*10^{-3} = 1205\ \text{kN-m}$$

$$y = \frac{a}{2} = \frac{2500}{2} = 1250$$

$$I = \frac{1}{12}ta^3 = \frac{1}{12}*16*2500^3 = 2083*10^7\ \text{mm}^4$$

$$M_q = \frac{1}{y}py = \frac{2083*10^7}{1250}*(250/1.15)*10^{-6} = 3623\ \text{kN-m}$$

Since, $M_{tf} < M_q$ $\quad (1205 < 3623)$

The end panel can carry the bending moment.

Design of panel BC:

Panel BC will be designed using tension field action

d = 2600 mm ; t = 16 mm

$$f_v = \frac{F_{VB}}{dt} = \frac{2102.3*10^3}{2600*16} = 50.5\ N/mm^2$$

$$\frac{a}{d} = \frac{3250}{2600} = 1.25$$

$$\frac{d}{t} = \frac{2600}{16} = 162.5$$

$$P_y = \frac{250}{1.15} = 217.4 N/mm^2$$

Calculation of basic shear strength, q_{b}:

$$\text{Elastic critical stress, } q_e \text{ (when } a/d > 1) = \left[1.0 + 0\ 75/(a/d)^2\right]\left[1000/(d/t)\right]^2$$

$$= \left[1.0 + 0.75/(1.25)^2\right]\left[1000/(162.5)\right]^2$$

$$= 56.0\ N/mm^2$$

$$\text{Slendernessparameter, } \lambda_n = \left[0.6\left(f_{yw}/\gamma_m\right)/q_e\right]^{\frac{1}{2}}$$

$$= \left[0.6(250/1\ 15)/56\ 0\right]^{\frac{1}{2}}$$

$$= 2.33 > 1\ 25$$

$$\text{Hence, Critical shear stength } = 56.0\ N/mm^2\ (q_{cr} = q_e)$$

$$\phi_t = \frac{1.5 q_{cr}}{\sqrt{1 + \left(\frac{a}{d}\right)^2}} = \frac{1.5*56.0}{\sqrt{1+(1.25)^2}} = 52\ 5$$

$$y_b = \left(P_{yw}{}^2 - 3q_{cr}{}^2 \_ \phi_t{}^2\right)^{\frac{1}{2}} - \phi_t = \left(217.4^2 - 3*56\ 0^2 + 52\ 5^2\right)^{\frac{1}{2}} - 52.5 = 149$$

$$q_b = q_{cr} + \frac{y_b}{2\left[\frac{a}{c} + \sqrt{1 + \left(\frac{a}{d}\right)^2}\right]} = 56.0 + \frac{149}{2\left[1\ 25 + \sqrt{1+(1\ 25)^2}\right]} = 82.1 N/mm^2$$

$$\text{Since, } q_b > f_v \qquad (82.1 > 50.5)$$

Panel BC is safe against shear buckling.

7.0 Design of stiffeners

Load bearing stiffener at A:

Design should be made for compression force due to bearing and moment.

Design force due to bearing, F_b = 2301 kN

Force (F_m) due to moment M_{tf}, is

$$F_m = \frac{M_{tf}}{a} = \frac{1205}{2500} * 10^3 = 482 \text{ kN}$$

Total compression = $F_c = F_b + F_m$ = 2301 + 482 = 2783 kN

Area of Stiffener in contact with the flange, A:

Area (A) should be greater than

$$\frac{0.8\, F_c}{P_{ys}}$$

$$\frac{0.8\, F_c}{P_{ys}} = \frac{0.8 * 2783}{217.4} * 10^3 = 10241 \text{ mm}^2$$

Try stiffener of 2 flats of size 270 x 25 mm thick

Allow 15 mm to cope for web/flange weld

A = 255 * 25 * 2 = 12750 mm^2 > 10241 mm^2

Bearing check is ok.

Check for outstand:

Outstand from face of web should not be greater than

$$20\, t_s \varepsilon$$

$$\varepsilon = \left\{\frac{250}{f_y}\right\}^{\frac{1}{2}} = \left\{\frac{250}{250}\right\}^{\frac{1}{2}} = 1.0$$

$$\text{Outstand } b_s = 250 \text{ mm} < 20\, t_s \varepsilon (= 20 * 25 * 1.0 = 500)$$

$$b_s = 250 \text{ mm} < 13.7\, t_s \varepsilon (= 13.7 * 25 * 1.0 = 342.5)$$

Hence, outstand criteria is satisfied.

Check stiffener for buckling:

The effective stiffener section is shown in Fig. E3

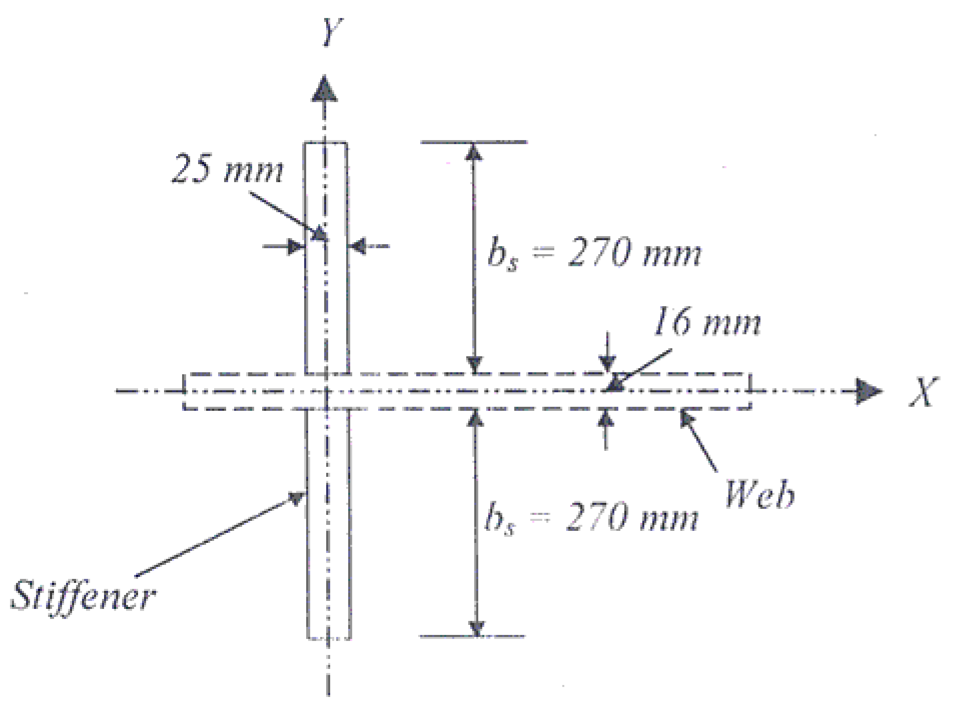


Fig. E3 End bearing stiffener

The buckling resistance due to web is neglected here for the sake of simplicity.

$$I_x = \frac{25*556^3}{12} - \frac{1}{12}*25*16^3 = 35807*10^4 \text{ mm}^4$$

$$A_e = \text{Effective area} = 270*25*2 = 13500 \text{ mm}^2$$

$$r_x = \left[\frac{I_x}{A_e}\right]^{\frac{1}{2}} = \left[\frac{35807*10^4}{13500}\right]^{\frac{1}{2}} = 162.8 \text{ mm}$$

Flange is restrained against rotation in the plane of stiffener, then

l_e = 0.71 = 0.7 * 2600 = 1820

$$\lambda = \frac{l_e}{r_x} = \frac{1820}{162.8} = 11.2$$

$$\text{For } f_y = 250 \text{ N/mm}^2 \text{ and } \lambda = 11.2$$

$$\sigma_c = 250 \text{N/mm}^2$$

from table(30 of chapter on axially compressed columns

Buckling resistance of stiffener is

$$P_c = \sigma_c A_e / \gamma_m = 250 * 1350/1.15 = 2935 \text{ kN}$$

$$\text{Since } F_c < P_c \; (2783 < 2935)$$

Therefore, stiffener provided is safe against buckling.

Check stiffener A as a bearing stiffener:

Local capacity of the web:

Assume, stiff bearing length b_1=0

n_2 = 2.5 * 50 * 2 = 250 BS 5950 : Part -1, Clause 4.5.3

$P_{crip} = (b_1 + n_2)\, tP_{yw}$

$= (0 + 250) * 16 * (250/1.15) * 10^{-3} = 870$ kN

Bearing stiffener is designed for F_A

$F_A = F_x = P_{crip}$ = 2783 - 870 = 1931 kN

Bearning capacity of stiffener alone

$P_A = P_{ys} * A$ = (50/1.15) * 13500/1000 = 2935 kN

Since, $F_A < P_A$ (1931 < 2935)

The designed stiffener is OK in bearing.

Stiffener A - Adopt 2 flats 270 mm X 25 mm thick

Design of intermediate stiffener at B:

Stiffener at B is the most critical one and will be chosen for the design.

Minimum Stiffness

$$I_s \geq 0.75dt^3 \text{ for } a \geq d\sqrt{2}$$

$$I_s \geq \frac{0.75dt^3}{a^3} \text{ for } a < d\sqrt{2}$$

$$d\sqrt{2} = \sqrt{2} * 2600 = 3677 \text{ mm}$$

$$a < d\sqrt{2} \qquad (3250 < 3677)$$

Conservatively ' t ' is taken as actual web thickness and minimum ' a ' is used.

$$\frac{1.5d^3t^3}{a^2} = \frac{1.5 * 2600^3 * 16^3}{3250^2} = 1022 * 10^4 \text{ mm}^4$$

Try intermediate stiffener of 2 flats 120 mm X 14 mm

$$(I_s)_{Provided} = \frac{14 * 256^3}{12} - \frac{14 * 16^3}{12} = 1957 * 10^4 \text{ mm}^4$$

$$I_s > \frac{1.5d^3t^3}{a^2}, \text{the section statisfied minimum stiffeness requirement}$$

Check for outstand:

Outstand of the stiffener $\leq 13.7\, t_s \varepsilon$

$13.7 t_s \varepsilon = 13.7 * 14 * 1.0 = 192$ mm

Outstand = 120 mm (120 < 192)

Hence, outstand criteria is satisfied.

Buckling check:

Stiffener force, $F_q = V - V_s$

Where V = Total shear force

$V_s = V_{cr}$ of the web

a / d = 3600 / 2600 = 1.38

d / t = 2600 / 16 = 162.5

$$\text{Elastic critical stress, } q_e \text{ (when } a/d > 1) = \left[1.0 + 0.75/(a/d)^2\right]\left[1000/(d/t)\right]^2$$

$$= \left[1.0 + 0.75/(1.38)^2\right]\left[1000/(162.5)\right]^2$$

$$= 52.8 \text{ N/mm}^2$$

$$\text{Slenderness parameter, } \lambda_w = \left[0.6\left(f_{yw}/\gamma m\right)/qe\right]^{\frac{1}{2}}$$

$$= \left[0.6(250/1.15)/52.8\right]^{\frac{1}{2}}$$

$$= 2.47 > 1.25$$

Hence, Critical shear strength = 52.8 N/mm^2 ($q_{cr} = q_e$)

$V_{cr} = q_{cr}dt = 52.8 * 2600 * 16 * 10^{-3} = 2196$ kN

Buckling resistance of intermediate stiffener at B:

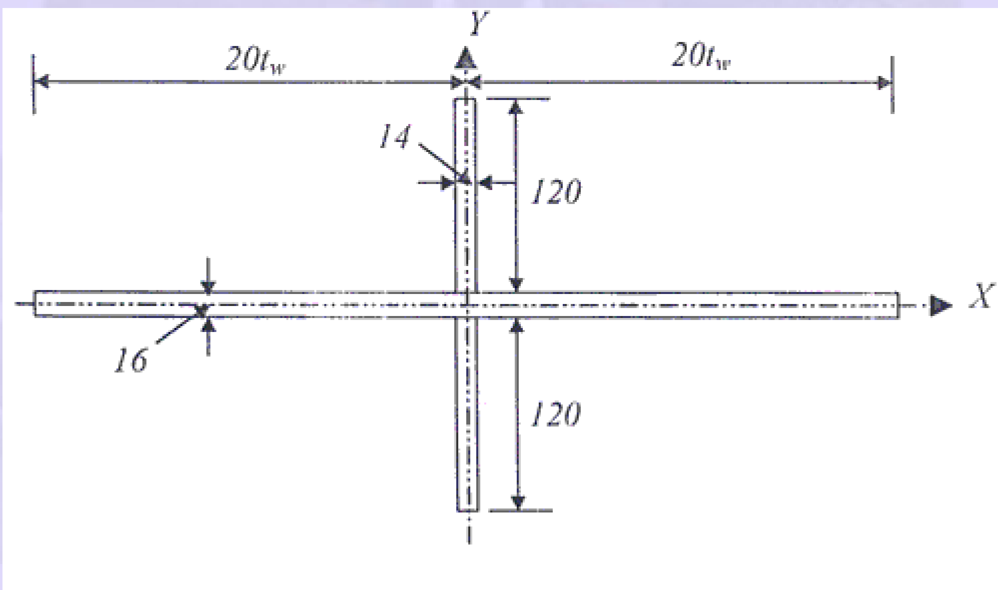


Fig.E4 Effective section

$$20\ t_w = 20 * 16 = 3230\ \text{mm}$$

$$I_x = \frac{1}{12} * 14 * 256^3 + \frac{640 * 16^3}{12} - \frac{14 * 16^3}{12} = 1979 * 10^4\ \text{mm}^4$$

$$A = 240*14 + 640*16 = 13600\ \text{mm}^2$$

$$r_x = \left[\frac{1979 * 10^4}{13600}\right]^{\frac{1}{2}} = 38.1$$

$$l_e = 0.7 * 2600 = 1820$$

$$\lambda = \frac{l_e}{r_x} = \frac{1820}{38.1} = 48.0$$

For $f_y = 250\ \text{N/mm}^2$ and $\lambda = 48.0$

From table3 of chapter on axially compressed columns,

$$\sigma_c = 213.2\ \text{N/mm}^2$$

Buckling resistance = (213.2/1.15) * 13600 * 10^{-3} = 2521 kN

Shear force at B, V_B = 2301-{(2301 - 1585.5)*(2500/9000)] = 2102 kN

Stiffener force, Fq = [21201 - 2196] < 0

and

Fq < Buckling resistance

Hence, intermediate stiffener is adequate

Intermediate stiffener at B - Adopt 2 flats 120 mm X 14 mm

Intermediate stiffener at E (Stiffener subjected to external load):

Minimum stiffness calculation:

$$a = 3600 \qquad d\sqrt{2} = 3677$$

$$a < d\sqrt{2} \qquad (3600 < 3677)$$

$$I_s \geq \frac{1.5 d^3 t^3}{a^2} = \frac{1.5 * 2600^3 * 16^3}{3600^2} = 833 * 10^4$$

Try intermediate stiffener 2 flats 100 mm X 12 mm thick

$$(I_s)_{Provided} = 1007 * 10^4 \text{ mm}^4$$

$$(I_s)_{Provided} > I_s \qquad [1007 * 10^4 > 833 * 10^4]$$

Hence, OK

Buckling Check:

$$\frac{F_q - F_x}{P_q} + \frac{F_x}{P_x} + \frac{M_s}{M_{ys}} \le 1$$

$$F_q = V - V_s \qquad V = 1585.5 \text{ kN}$$

$$V_s = V_{cr} = q_{cr} dt = 52.8 * 2600 * 16 * 10^{-3} = 2196 \text{ kN}$$

$$F_q \text{ is negative and } F_q - F_x = 0$$

$$M_s = 0$$

$$F_x = 870 \text{ kN}$$

Buckling resistance of load carrying stiffener at D:

(Calculation is similar to stiffener at B)

$$20 t_w = 20 * 16 = 320 \text{ mm}$$

$$I_x = \frac{1}{12} * 12 * 216^3 + \frac{640 * 16^3}{12} - \frac{12 * 16^3}{12} = 1029 * 10^4 \text{ mm}^4$$

$$A = 200 * 12 + 640 * 16 = 12640 \text{ mm}^2$$

$$r_x = \left[\frac{1029 * 10^4}{12640}\right]^{\frac{1}{2}} = 28.5$$

$$l_e = 0.7 * 2600 = 1820$$

$$\lambda = \frac{l_e}{r_x} = \frac{1820}{28.5} = 63.9$$

$$\text{For } f_y = 250 \text{ N/mm}^2 \text{ and } \lambda = 63.9$$

From table3 of chapter on axially comperssed columns,

$$\sigma_c = 180 \text{ N/mm}^2$$

Buckling resistance, P_x (180/1.15) * 2640 * 10^{-3} = 1978 kN.

F_x/P_x = 870/1978 = 0.44 <1.0

Hence, Stiffener at D is OK against buckling

Stiffener at D - Adopt flats 100 mm X 12 mm thick

Web check between stiffeners:

$f_{ed} \le P_{ed}$

$f_{ed} = w^1 / t = 79.5 / 16 = 4.97\ \text{N/mm}^2$

When compression flange is restrained against rotation relative to the web

$$P_{ed} = \left[2.75 + \frac{2}{(a/d)^2}\right]\frac{E}{(d/t)^2} = \left[2.75 + \frac{2}{\left(\frac{3600}{2600}\right)^2}\right]\frac{200000}{\left(\frac{2600}{16}\right)^2}$$

$$= \frac{3.79*20000}{26406} = 28.7\ \text{N/mm}^2$$

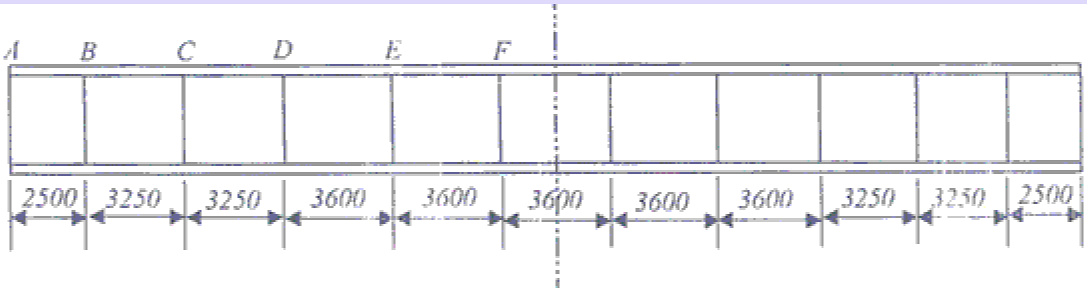
Since,

$f_{ed} \le P_{ed}$

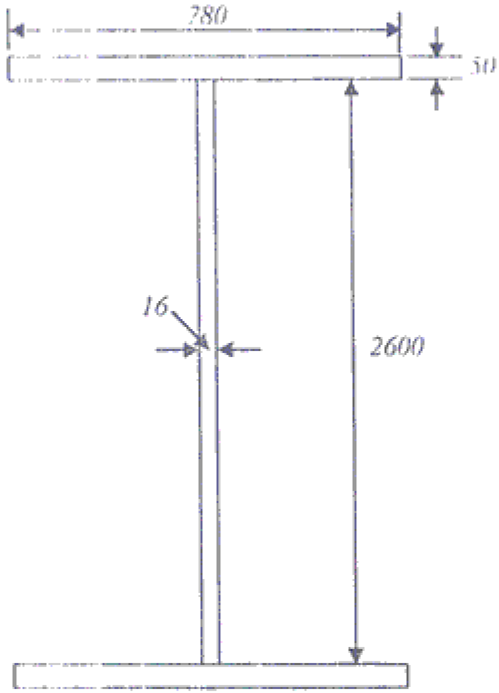
[4.97 <28.7], the web is OK for all panels.

8.0 FINAL GIRDER

(All dimensions are in mm)



(a) Longitudinal section of plate girder



(b) Flange and web proportions

Fig. E5 Final girder

7. BEAM COLUMNS

7.1 Introduction

The Indian steel code is now in the process of revision as specification-based design gives way to performance-based design. An expert committee mainly comprising eminent academics from IIT Madras, Anna University Chennai, SERC Madras and INSDAG Kolkata was constituted to revise IS: 800 in LSM version. The Limit State Method (referred to as LSM below) is recognized, as one of the most rational methods toward realization of performance-based design, but to date there are no steel-intensive buildings in India that have been designed using LSM. We considered that, because building collapse is caused by excessive deformation, the ultimate state should be evaluated from the deformation criteria. The proposed design procedure evaluates the ultimate limit state on the basis of the deformation capacity of structural members.

The magnification factors, used to confirm suitable flexural mechanisms, severely affect the overall probability of failure, and should be determined so that the overall probability of failure does not exceed specific allowable limits.

In practice, the structural members are generally subjected to various types of combination of stress resultants. Depending upon external actions over the members in structural framing system, the combined forces may be broadly categorized as i) Combined Shear and Bending, ii) Combined Axial Tension and Bending and iii) Combined Axial Compression and Bending.

Normally, the design of an individual member in a frame is done, by separating it from the frame and dealing with it as an isolated substructure. The end conditions of the member should then comply with its deformation conditions, in the spatial frame, in a conservative way, e.g. by assuming a nominally pinned end condition, and the internal action effects, at the ends of the members, should be considered by applying equivalent external end moments and end forces. Before proceeding for any analysis, classification of these members shall have to be satisfied in accordance with clause no. 3.7 and all related sub-clauses under section 3 of IS: 800 – LSM version.

For all practical purposes, we can equate the third case with the case of Beam-columns. Beam-columns are defined as members subject to combined bending and compression. In principle, all members in moment resistant framed structures (where joints are considered as rigid) are actually beam-columns, with the particular cases of beams (F = 0) and columns (M = 0) simply being the two extremes. Depending upon the exact way in which the applied loading is transferred into the member, the form of support provided and the member's cross-sectional shape, different forms of response will be possible.

The simplest of these involves bending applied about one principal axis only, with the member responding by bending solely in the plane of the applied moment.

Recently, IS: 800, the Indian Standard Code of Practice for General Construction in Steel is in the process of revision and an entirely new concept of limit state method of design has been adopted in line with other international codes of practice such as BS, EURO, and AISC. Additional Sections and features have been included to make the code a state-of-the-art one and for efficient & effective use of structural steel. Attempt has been made in the revised code to throw some light into the provisions for members subjected to forces, which are combined in nature.

7.2 Concept of limit state design of beam columns

Steel structures are important in a variety of land-based applications, including industrial (such as factory sheds, box girder cranes, process plants, power and chemical plants etc.), infrastructural (Lattice girder bridges, box girder bridges, flyovers, institutional buildings, shopping mall etc.) and residential sector. The basic strength members in steel structures include support members (such as rolled steel sections, hollow circular tubes, square and rectangular hollow sections, built-up sections, plate girders etc.), plates, stiffened panels/grillages and box girders. During their lifetime, the structures constructed using these members are subjected to various types of loading which is for the most part operational, but may in some cases be extreme or even accidental.

Steel-plated structures are likely to be subjected to various types of loads and deformations arising from service requirements that may range from the routine to the extreme or accidental. The mission of the structural designer is to design a structure that can withstand such demands throughout its expected lifetime.

The structural design criteria used for the *Serviceability Limit State Design* (hereafter termed as **SLS**) design of steel-plated structures are normally based on the limits of deflections or vibration for normal use. In reality, excessive deformation of a structure may also be indicative of excessive vibration or noise, and so, certain interrelationships may exist among the design criteria being defined and used separately for convenience.

The SLS criteria are normally defined by the operator of a structure, or by established practice, the primary aim being efficient and economical in-service performance without excessive routine maintenance or down-time. The acceptable limits necessarily depend on the type, mission and arrangement of structures. Further, in defining such limits, other disciplines such as machinery designers must also be consulted.

The structural design criteria to prevent the *Ultimate Limit State Design* (hereafter termed as ULS) are based on plastic collapse or ultimate strength. The simplified ULS design of many types of structures has in the past tended to rely on estimates of the buckling strength of components, usually from their elastic buckling strength adjusted by a simple plasticity correction. This is represented by point A in Figure 7.1. In such a design scheme based on strength at point A, the structural designer does not use detailed information on the post-buckling behavior of component members and their interactions. The true ultimate strength represented by point B in Figure 7.1 may be higher although one can never be sure of this since the actual ultimate strength is not being directly evaluated.

In any event, as long as the strength level associated with point B remains unknown (as it is with traditional allowable stress design or linear elastic design methods), it is difficult to determine the real safety margin. Hence, more recently, the design of structures such as offshore platforms and land-based structures such as steel bridges has tended to be based on the ultimate strength.

The safety margin of structures can be evaluated by a comparison of ultimate strength with the extreme applied loads (load effects) as depicted in Figure 7.1. To obtain a safe and economic structure, the ultimate load-carrying capacity as well as the design load must be assessed accurately. The structural designer may even desire to estimate the ultimate strength not only for the intact structure, but also for structures with existing or premised damage, in order to assess their damage tolerance and survivability.

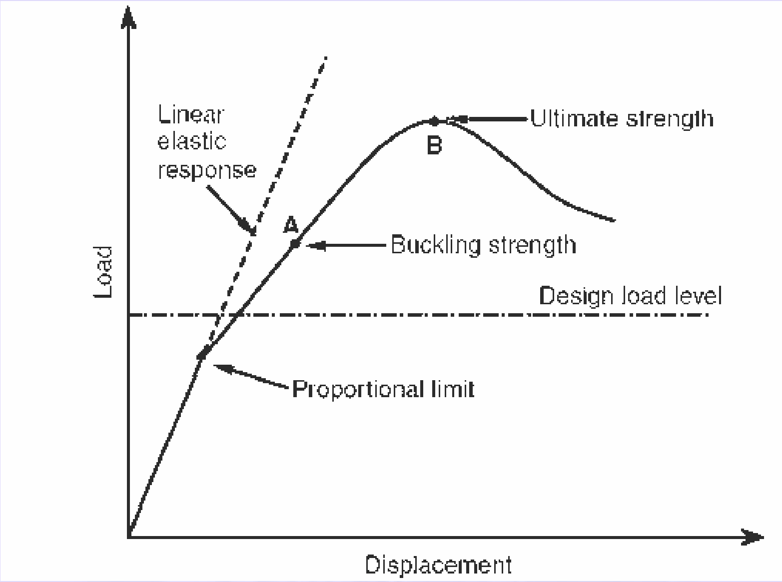


Fig 7.1 Structural design considerations based on the ultimate limit state

In the structural design process, **"analysis"** usually means the determination of the stress resultants, which the individual structural members must be capable to resist. **"Design"** can mean the development of the structural layout, or arrangement of members, but it usually means the selection of sizes of members to resist the imposed forces and bending moments. Three methods of analysis are available, i.e. elastic analysis, plastic analysis and advanced analysis. Limit state design is a design method in which the performance of a structure is checked against various limiting conditions at appropriate load levels. The limiting conditions to be checked in structural steel design are ultimate limit state and serviceability limit state.Limit state theory includes principles from the elastic and plastic theories and incorporates other relevant factors to give as realistic a basis for design as possible.

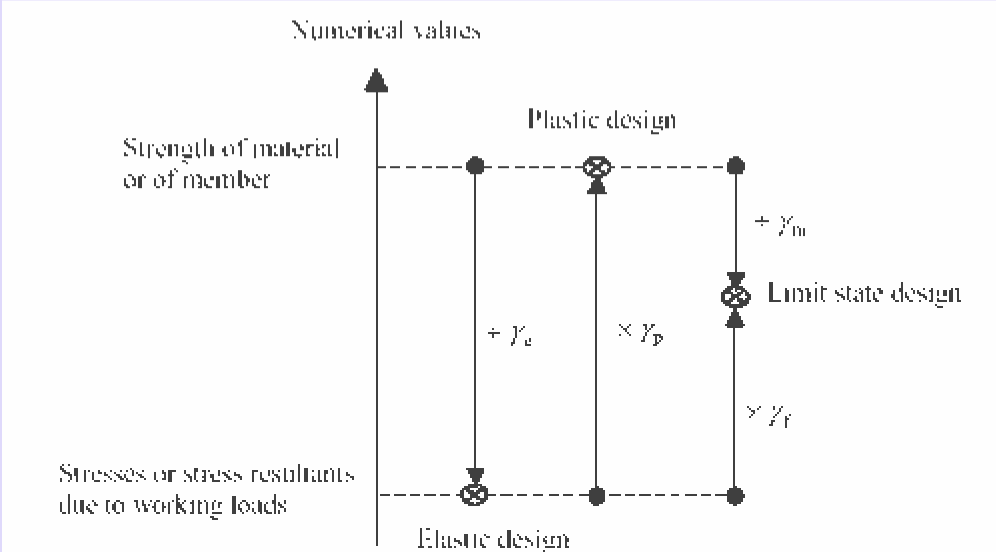


Fig. 7.2 Level for different design methods at which calculations are conducted (Commentary on BS5950 1 2000)

Ultimate Limit State Design of Steel Structures reviews and describes both fundamentals and practical design procedures in this field. Designs should ensure that the structure does not become unfit / unserviceable for the use for which it is intended to. The state at which the unfitness occurs is called a limit state.

Figure 7.2 shows how limit-state design employs separate factors γ_f, which reflects the combination of variability of loading γ_l, material strength γ_m and structural performance γ_p. In the elastic design approach, the design stress is achieved by scaling down the strength of material or member using a factor of safety γ_e as indicated in Figure 7.2, while the plastic design compares actual structural member stresses with the effects of factored-up loading by using a load factor of γ_p.

Special features of limit state design method are:

- Serviceability and the ultimate limit state design of steel structural systems and their components.
- Due importance has been provided to all probable and possible design conditions that could cause failure or make the structure unfit for its intended use.

- The basis for design is entirely dependent on actual behaviour of materials in structures and the performance of real structures, established by tests and long-term observations

- The main intention is to adopt probability theory and related statistical methods in the design.

- It is possible to take into account a number of limit states depending upon the particular instance

- This method is more general in comparison to the working stress method. In this method, different safety factors can be applied to different limit states, which is more rational and practical than applying one common factor (load factor) as in the plastic design method.

- This concept of design is appropriate for the design of structures since any development in the knowledge base for the structural behaviour, loading and materials can be readily implemented.

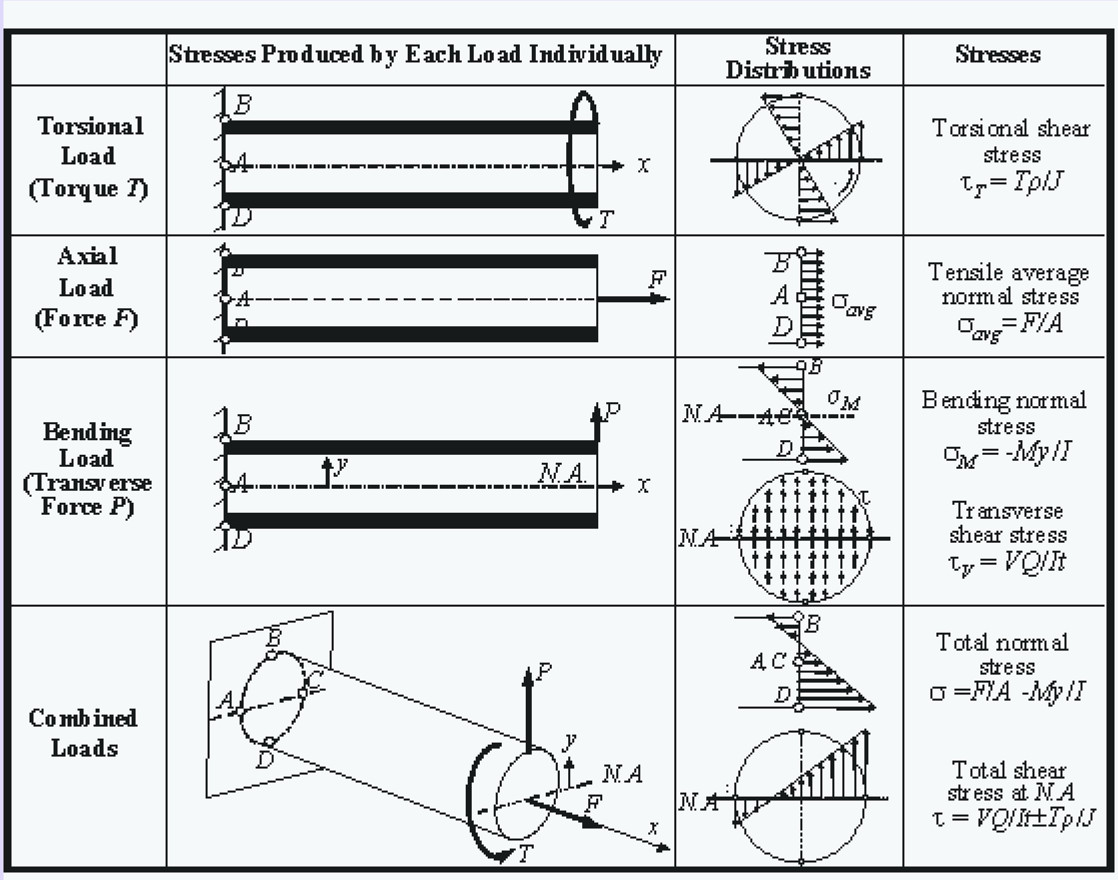
7.3 Design of members subjected to combined forces

7.3.1 General

In the previous chapters of Draft IS: 800 – LSM version, we have stipulated the codal provisions for determining the stress distribution in a member subjected to different types of stress resultants such as axial tensile force (Section 6), axial compressive force (Section 7) and bending moment along with transverse shear force (Section 8). Most often, the cross section of a member is subjected to several of these loadings *simultaneously*. As we shall see presently, we may combine the knowledge that we have acquired in the previous sections. As long as the relationship between stress and the loads is ***linear*** and the geometry of the member would ***not undergo significant change*** when the loads are applied, the principle of superposition can be applied. Here, as shown in Table 7.1, one typical case of combination due to tensile force *F*, torque *T* and transverse load *P* has been diagrammatically discussed.

In addition to the pure bending case, beams are often subjected to transverse loads which generate both bending moments M(x) and shear forces V(x) along the beam. The bending moments cause bending normal stresses s to arise through the depth of the beam, and the shear forces cause transverse shear-stress distribution through the beam cross section as shown in Fig. 7.3.

Table 7.1 Superposition of individual loads (a case study for solid circular shaft)



	Stresses Produced by Each Load Individually	Stress Distributions	Stresses
Torsional Load (Torque T)			Torsional shear stress $\tau_T = T\rho/J$
Axial Load (Force F)			Tensile average normal stress $\sigma_{avg} = F/A$
Bending Load (Transverse Force P)			Bending normal stress $\sigma_M = -My/I$ Transverse shear stress $\tau_y = VQ/It$
Combined Loads			Total normal stress $\sigma = F/A - My/I$ Total shear stress at N.A $\tau = VQ/It \pm T\rho/J$

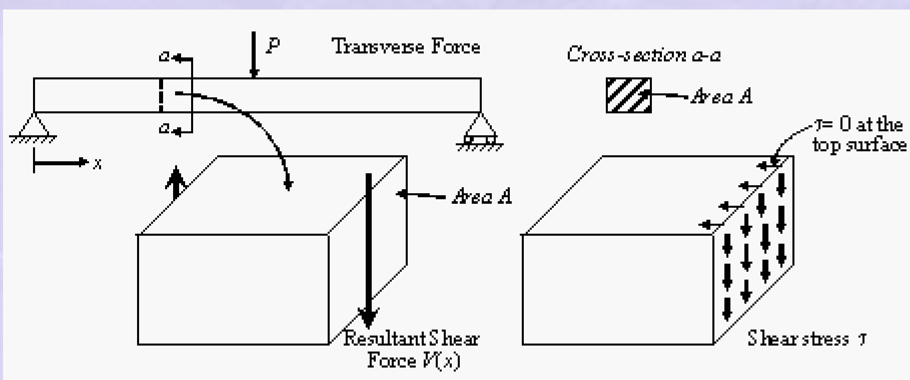


Fig.7.3 Beam with transverse shear force showing the transverse shear stress developed by it

7.3.2 General procedure for combined loading

- Identify the relevant equations for the problem and use the equations as a check list for the quantities that must be calculated.

- Calculate the relevant geometric properties (A, I_{yy} , I_{zz} , J) of the cross-section containing the points where stresses have to be found.

- At points where shear stress due to bending is to be found, draw a line perpendicular to the center-line through the point and calculate the first moments of the area (Q_y , Q_z) between free surface and the drawn line. Record the s-direction from the free surface towards the point where stress is being calculated.

- Make an imaginary cut through the cross-section and draw the free body diagram. On the free body diagram draw the internal forces and moments as per our sign conventions if subscripts are to be used in determining the direction of stress components. Using equilibrium equations to calculate the internal forces and moments.

- Using the equations identified, calculate the individual stress components due to each loading. Draw the torsional shear stress $\tau_{x\theta}$ and bending shear stress τ_{xs} on a stress cube using subscripts or by inspection. By examining the shear stresses in x, y, z coordinate system obtain τ_{xy} and τ_{xz} with proper sign.

- Superpose the stress components to obtain the total stress components at a point.

- Show the calculated stresses on a stress cube.

- Interpret the stresses shown on the stress cube in the x, y, z coordinate system before processing these stresses for the purpose of stress or strain transformation.

7.3.3 Design of member subjected to combined shear and bending:

In general, it has been observed that for structures, which are subjected to combined shear and bending the occurrence of high shear force is seldom. Here, high shear force has been designated as that shear force, which is more than 50 percent of the shear strength of the section. For structures where the factored value of applied shear force is less than or equal to 50 percent of the shear strength of the section no reduction in moment capacity of the section is required (refer clause 8.4 of section 8 of draft IS: 800 – LSM version) i.e. the moment capacity may be taken as M_d (refer clause 8.2 of section 8 and clause 9.2.1 of section 9 of draft IS: 800 – LSM version). If the factored value of actual shear force is more than 50 percent of the shear strength of the section, the section shall be checked and moment capacity, M_{dx}, shall be reduced depending upon classification of section (refer clause 9.2.2 of section 9 of draft IS: 800 – LSM version). This is done to take care of the increased resultant vector stress generated due to vector addition of stress due to high shear and bending moment at that particular section. The corresponding bending moment capacity is reduced by incorporating a factor 'β' $\beta = \left(2V/V_e - 1\right)^2$, which in turn depends upon the ratio of actual value of high shear and shear strength of that particular section. In no case, the maximum value of moment capacity shall exceed $1.2 Z_e f_y / \gamma_{m0}$, where, Z_e is elastic section modulus of the whole section γ_{m0}, is the partial safety factor against yield stress and buckling and f_y is the characteristic yield stress (250 N/mm^2). For semi-compact sections, this reduction in moment capacity is not required to be exercised as the section will be predominantly governed by the elastic moment capacity i.e. $Z_e f_y / \gamma_{m0}$, where the terms Z_e, γ_{m0} and f_y are as defined previously.

7.3.4 Design of member subjected to combined bending and axial force:

7.3.4.1 General

As with combined bending moment and axial force in the elastic range, the plane of zero strain moves from the centroid so that the ultimate stress distribution appears as follows:

- To compute the modified plastic moment, Mp', the stress blocks are divided into three components.

- The outer pair is equal and opposite stress blocks which provide the moment.

- The inner block acts in one direction, the area being balanced around the original neutral axis, defined by d_N. In the cases illustrated the inner block has a constant width, b or t_w, so that

$$M'_p = 2A_p\,\bar{y}\,\sigma_y;\;\; N = bd_N\,\sigma_y\,(\text{rectangle});\, N = t_w\,d_N\sigma_y\,(\text{I}-\text{section})$$

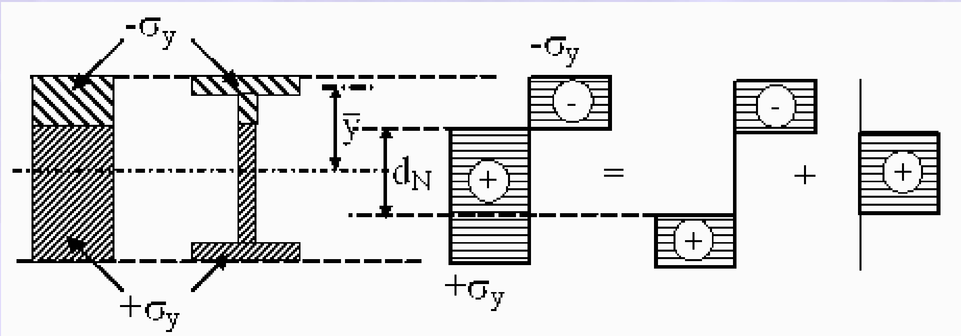


Fig.7.4 Stress block diagram for combined bending and axial force with reduced moment carrying capacity for I-section

We can draw interaction diagrams relating N and M for initial yield and ultimate capacity as follows:

When the web and the flange of an I-beam have different yield strengths it is possible to calculate the full plastic moment or the combined axial force and bending moment capacity taking into account the different yield strengths.

$$M_p = 2\sum_i A_i \, \bar{y}_i \, \sigma_{yi}$$

for all i segments with different yield stresses.

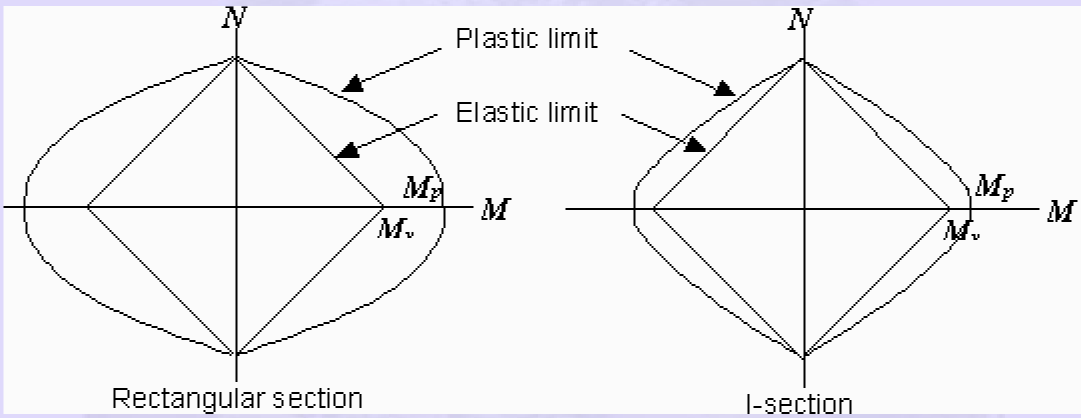


Fig.7.5 Interaction diagram of bending moment and axial tension or compression

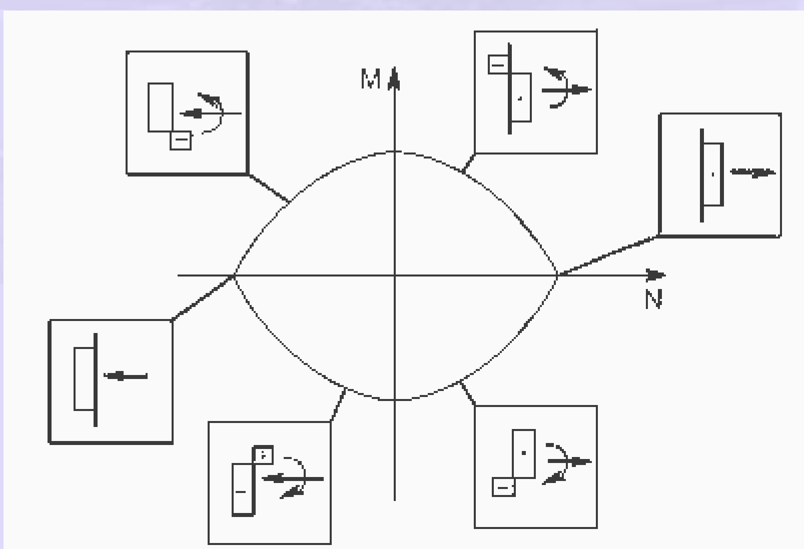


Fig.7.6 Plastic limit envelope with stress distributions for combined bending (M) and axial force (N)

7.3.4.2 Members subjected to combined bending & axial forces

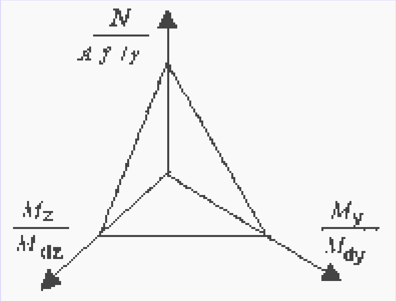


Fig. 7.7 Interaction surface of the ultimate strength under combined biaxial bending moment and axial tensile force

Any member subjected to bending moment and normal tension force should be checked for lateral torsional buckling and capacity to withstand the combined effects of axial load and moment at the points of greatest bending and axial loads. Figure 7.4 illustrates the type of three-dimensional interaction surface that controls the ultimate strength of steel members under combined biaxial bending and axial force. Each axis represents a single load component of normal force N, bending about the y and z axes of the section (M_y or M_z) and each plane corresponds to the interaction of two components.

7.3.4.2.1 Local capacity check (Section 9 Draft IS: 800 – LSM version)

For *Plastic and Compact sections*, the design of members subjected to combined axial load and bending moment shall satisfy the following interaction relationship:

$$\left(\frac{M_y}{M_{ndy}}\right)^{u1} + \left(\frac{M_z}{M_{ndz}}\right)^{u2} \leq 1.0$$

Where

M_y, M_z = factored applied moments about the minor and major axis of the cross section, respectively

M_{ndy}, M_{ndz} = design reduced flexural strength under combined axial force and the respective uniaxial moment acting alone, **(Cl. 9.3.1.2 of Draft IS: 800 – LSM version)**

N = factored applied axial force (Tension T, or Compression F)

N_d = design strength in tension (T_d) as obtained from **(section 6 Draft IS: 800 – LSM version)** or in compression and $N_d = A_g f_y / \gamma_{m0}$

A_g = gross area of the cross section

n = N / N_d

α_1, α_2 = constants as given in **Table 7.2** of new IS: 800 and shown below:

Table 7.2 Constants α_1 and α_2 *(Section 9.3.1.1of Draft IS: 800 – LSM version)*

Section	α_1	α_2
I and Channel	$5n \geq 1$	2
Circular tubes	2	2
Rectangular tubes	$1.66/(1-1.13n^2) \leq 6$	$1.66/(1-1.13n^2) \leq 6$
Solid rectangle	$1.73+1.8n^3$	$1.73+1.8n^3$

A typical interaction diagram for I beam-column segment with combined biaxial bending and compressive force is as shown in figure 7.8

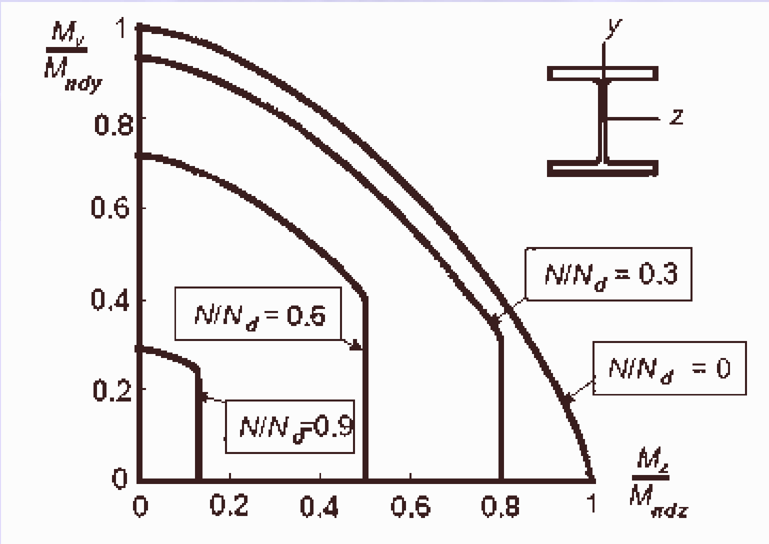


Fig. 7.8 Interaction curves for I beam-column segment in bi-axial bending and compression

The above interaction formulae, $\left(\frac{M_y}{M_{ndy}}\right)^{u1} + \left(\frac{M_z}{M_{ndz}}\right)^{u2} \leq 1.0$ is a function of α_1 and α_2 and the values of α_1 and α_2 are in turn functions of the ratio $n = N / N_d$ (refer Table 7.2 above), where, N and N_d are factored applied axial force and design strength in tension or compression as defined earlier. It can be observed from the table 7.2 that for I-section or channel section, in case, the ratio $n = N / N_d$ is equal to 0.2, the value of α_1 becomes 1 and for $n = N / N_d > 0.2$, the value of α_1 is more than 1. As the value of α_1 increase above 1, the value of the component $\left(\frac{M_y}{M_{ndy}}\right)^{u1}$ gets reduced since the ratio $\left(\frac{M_y}{M_{ndy}}\right)^{u1}$ is always less than 1. The values of M_{ndy} and M_{ndz} are also proportionately reduced to accommodate the value of axial tension or compression depending upon type of sections. The ratio $n = N / N_d$ is also directly related in reducing the bending strength, M_{ndy} and M_{ndz}

The code stipulates **(as per clause 9.3.1.2 of Draft IS: 800 – LSM version)** that for plastic and compact sections without bolts holes, the following approximations may be used while calculating / deriving the values of design reduced flexural strength M_{ndz} and M_{ndy} under combined axial force and the respective uniaxial moment acting alone i.e. M_{ndz} and M_{ndy} acting alone. The values of reduced flexural strength of the section, (either M_{ndz} or M_{ndy}) is directly related to the geometry of a particular section. We will now discuss how these values are changing depending upon geometry of a particular section:

i) Plates (Section 9 of Draft IS: 800 – LSM version)

For rolled steel plates irrespective of their thickness the value of reduced flexural strength can be derived from the equation:– $M_{nd} = M_d\left(1 - n_2\right)$. Here, the equation for reduced flexural strength is again a function of the ratio $n = N / N_d$, where, *N* and N_d are factored applied axial force and design strength in tension or compression as defined earlier. For smaller values of the ratio *n*, the reduction in flexural strength is not significant since the reduction in flexural strength is directly proportional to square of the ratio *n*. It is obvious from the equation that as the ratio tends towards the value 1, the amount of reduction in flexural strength increases and for extreme case, when the ratio is equal to 1, the value of reduced flexural strength is zero i.e. for this particular case no flexural or bending strength is available within the plate section. This situation also satisfies the Condition, $\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0$

ii) Welded I or H sections (Section 9 of Draft IS: 800 – LSM version)

For welded I or H sections, the reduced flexural strength about the major axis can be derived from the equation: – $M_{ndz} = M_{dz}(1-n)/(1-0.5a) \le M_{dz}$ and about the minor axis: – $M_{ndy} = M_{dy}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] \le M_{dy}$ where, $n = N / N_d$ and

$a = (A - 2bt) / A / \le 0.5$

Here the reduction in flexural strength for major axis is linearly and directly proportional to the ratio *n* and inversely proportional to the factor *a*, which is a reduction factor for cross sectional area ratio. It is pertinent to note that for a particular sectional area *A*, as the width and/or thickness of the flange of I or H section increases, the factor *a* reduces which in turn increases the value of M_{ndz}.

For minor axis, the reduction in flexural strength is non-linearly proportional to both the factors *n* and *a*, but as the value of the factor *n* increases considering other factor remaining unchanged, the value of M_{ndy} decreases, conversely as the value of the factor increases *a*, the value of M_{ndy} increases. For a particular case, when the numerical value of factor *n* is equal to 1 and the numerical value of the factor is *a* 0.5, the numerical value of M_{ndy} becomes zero. It can be observed that the factor *a* being the area ratio, it takes into account the effect of flange width and flange thickness. As the value of b or t_f increases, the value of the factor *a* reduces which in turn reduces further the value of design reduced flexural strength M_{ndy}.

iii) Standard I or H sections (Section 9 of Draft IS: 800 – LSM version)

For standard I or H sections, the reduced flexural strength about the major axis can be derived from the equation: – $M_{ndz} = 1.11 M_{dz}(1-n) \leq M_{dz}$ and about the minor axis :–for $n \leq 0.2$, $M_{ndz} = M_{ndy}$ and for, $n \leq 0.2$ where, $M_{ndy} = 1.56 M_{dy}(1-n)(n+0.6)$

Unlike welded I or H sections, Here we do not find the factor *a*, but reduction in flexural strength for major axis is linearly and directly proportional to the ratio *n*. It is pertinent to note that for all cases, as the factor increases *n*, further reduction in reduced flexural strength of the member takes place. For a particular case, when the factor *n* becomes 1, the value of M_{ndz} reduces to zero.

For minor axis, no reduction in flexural strength takes place till the ratio $n = N/N_d$ is restricted to 0.2. When the value of *n* is more than 0.2, the reduction in flexural strength for minor axis is linearly and directly proportional to the ratio *n*. For a value of *n* = 1, the value of M_{ndy} reduces to zero.

iv) Rectangular Hollow sections and Welded Box sections (Section 9 of Draft IS: 800 – LSM version)

When the section is symmetric about both axis and without bolt holes, the reduced flexural strength about the major axis can be derived from the equation: –

$$M_{ndz} = M_{dz}(1-n)/(1-0.5a_w) \leq M_{dz}$$

and about the minor axis:–

$$M_{ndy} = M_{dy}(1-n)/(1-0.5a_f) \leq M_{dy}.$$

As indicated in above equations, for rectangular hollow sections and welded box sections, the reduction in flexural strength for both the axes, takes place in line with that of M_{ndz} for welded I or H sections as described earlier in ***ii)*** above. The only variation is, the factor *a* is replaced either by a_w for M_{ndz} or by a_f for M_{ndy} .

v) Circular Hollow Tubes without Bolt Holes (Section 9 of Draft IS: 800 – LSM version)

The reduced flexural strength about both the axes can be derived from the equation:– $M_{nd} = 1.04\ M_d \left(1 - n^{1.7}\right) \le M_d$. For smaller values of the ratio *n*, the reduction in flexural strength is not significant since the reduction in flexural strength is directly proportional to the power of 1.7 for the ratio *n*. When the ratio *n* is equal to 1, the value of reduced flexural strength is zero i.e. for this particular case no flexural or bending strength is available for the circular section.

Usually, the points of greatest bending and axial loads are either at the middle or ends of members under consideration. Hence, the member can also be checked, conservatively, as follows:

$$\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0$$

where *N*, is the factored applied axial load in member under consideration, $N_d \left(A_g f_y / \gamma_{m0}\right)$ is the strength in tension as obtained from section **6,** M_z and M_y are the applied moment about the major and minor axes at critical region, M_{dz} and M_{dy} are the moment capacity about the major and minor axes in the absence of axial load i.e. when acting alone and A_g is the gross area of cross-section. This shows that in point of time, the summation of ratios of various components of axial forces and bending moments (including bi-axial bending moments) will cross the limiting value of 1.

For *Semi-compact section*s, when there is no high shear force **(as per 9.2.1 of Draft IS: 800 – LSM version)** semi-compact section design is satisfactory under combined axial force and bending, if the maximum longitudinal stress under combined axial force and bending f_x ,satisfies the following criteria.

$$f_x \le f_y / \gamma_{m0}$$

For cross section without holes, the above criteria reduces to

$$\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \le 1.0$$

Where N_d, M_{dy}, M_{dz} are as defined earlier

7.3.4.2.2 Overall member strength check (section 9 of Draft IS : 800 - LSM version)

Members subjected to combined axial force and bending moment shall be checked for overall buckling failure considering the entire span of the member. This essentially takes care of lateral torsional buckling.

a) For Bending moment and Axial Tension, the member should be checked for lateral torsional buckling to satisfy overall stability of the member under reduced effective moment M_{eff} due to tension and bending. The reduced effective moment M_{eff} , can be calculated as per the equation $M_{eff} = \left[M - \psi T Z_{ec} / A\right] \le M_d$ but in no case shall exceed the bending strength due to lateral torsional buckling M_d **(as per 8.22 of Draft IS :800 - LSM version)**. Here M,T are factored applied moment and tension respectively, A is the area of cross section, Z_{ec} elastic section modulus of the section with respect to extreme compression fibre and the factor ψ is equal to 0.8 when tension and bending moments are varying independently or otherwise equal to 1. For extreme case, when the factor $\psi T Z_{ec} / A$ is equal to M, M_{eff} reduces to zero.

b) For Bending moment and Axial Compression , when the member is subjected to combined axial compression and biaxial bending, the section should be checked to satisfy the generalized interaction relationship as per the equation $\frac{P}{P_d} + \frac{K_y M_y}{M_{dy}} + \frac{K_z M_z}{M_{dz}} \le 1.0$. Here K_y, K_z are the moment amplification factor about minor and major axis respectively $\left(K_z = 1 - \frac{\mu_y P}{P_{dz}}\right)$ and $K_y = 1 - \frac{\mu_y P}{P_{dy}}$ where the factor μ_z and μ_y are dependent on equivalent uniform moment factor, b obtained from **Table 9.2 of Draft IS : 800 - LSM version**, according to the shape of the bending moment diagram between lateral bracing points in the appropriate plane of bending and non-dimensional slenderness ratio, l), P is the applied factored axial compression, M_y, M_z are the applied factor bending moments about minor and major axis of the member, respectively and P_d, M_{dy}, M_{dz} are the design strength under axial compression , bending about minor and major axis respectively , as governed by overall buckling criteria. The design compression strength, P_d , is the smallest of the minor axis (P_{dy}) and major axis (P_{dz}) buckling strength as obtained from **7.12 of Draft IS :800 - LSM version** and the design bending strength (M_{dz}) about major axis is equal to (M_d),where (M_d) is the design flexural strength about minor axis given by section **8.2.1 of Draft IS : 800 - LSM version**, when lateral torsional buckling is not significant and by section **8.2.2 of Draft IS: 800 - LSM version**, where lateral torsional buckling governs. For design Bending Strength about minor axis, $M_{dy} = M_d$ where, M_d is the design flexural strength about minor axis calculated using plastic section modulus for plastic and compact sections and elastic section modulus for semi-compact sections

c) The factors are as defined below.

μ_z is the larger of μ_{LT} and μ_{fz} as given below.

$$\mu_{LT} = 0.15\lambda_y \beta_{MLT} - 0.15 \le 0.90$$

$$\mu_{fz} = \lambda_z (2\beta_{Mz} - 4) + \left[\frac{Z_Z - Z_{eZ}}{Z_{eZ}}\right] \le 0.90$$

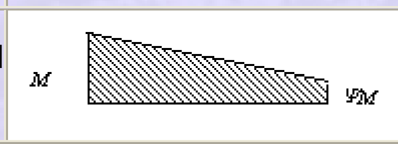
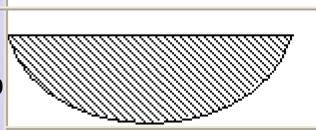
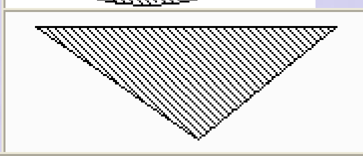
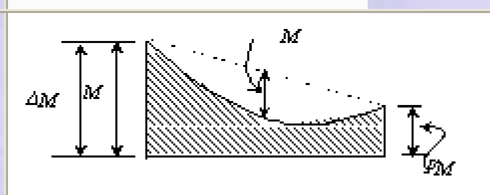
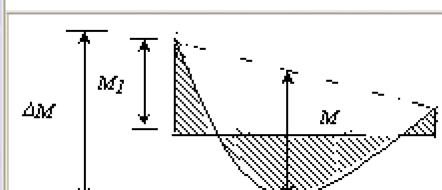
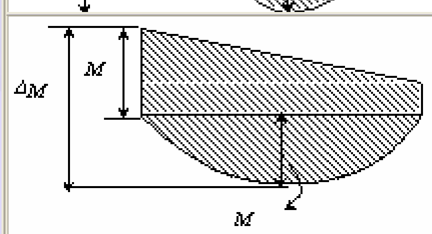
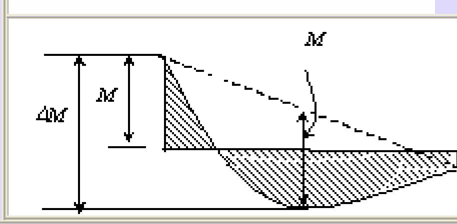
$$\mu_y = \lambda_y (2\beta_{My} - 4) + \left[\frac{Z_y - Z_{ey}}{Z_{ey}}\right] \le 0.90$$

$\beta_{My}, \beta_{Mz}, \beta_{MLT}$ = equivalent uniform moment factor obtained from **Table7.3 of Draft IS: 800-LSM version**, according to the shape of the bending moment diagram between lateral bracing points in the appropriate plane of bending

λ_y, λ_z = non-dimensional slenderness ratio **(7.1.2 of Draft IS 800- LSM version)** about the respective axis.

Table 7.3 OF Draft IS: 800-LSM Version, Equivalent uniform moment factor

(Section 9.3.2.2.1 of draft IS: 800-LSM version)

Particulars	BMD	b_m
Due to end moments		1.8-0.7y
Moment due to lateral loads		1.3
		1.4
moment due to lateral loads and end moments	   	1.8-0.7y+M_Q / Δ_M(0.74-0.5) $M_Q = \lvert M_{max} \rvert$ due to lateral load alone $\Delta_M = \lvert M_{max} \rvert$ (same curvature) $\Delta_M = \lvert M_{max} \rvert + \lvert M_{min} \rvert$ (reverse curvature)

7.4 Summary

This lecture note constitutes the limit state method of design for members subjected to combined stress resultants as per stipulations of the draft IS: 800 – LSM version where the draft code has dealt mainly with i) combined shear and bending ii) combined axial tension and bending and iii) combined axial compression and bending. These are explained in detail. The effect of reduction in bending moment capacity has been explained. The effect of lateral torsional buckling has also been explained. The concept of overall strength determination has been explained. The effect of relevant factors including equivalent uniform moment factor has been discussed.

7.5 References

1. Draft IS: 800 (2003), Code of Practice for General Construction in Steel

2. David A. Nethercot (2001), Limit states design of structural steelwork, Spon Press, London.

3. Teaching Resource for Structural Steel design (2001), INSDAG Publication.

4. Commentary on BS5950 1 – 2000, Commentary on Eurocde3 Part 1.1, Simplified Design to BS 5400, The Steel Construction Institute

Examples

Problem: 1

A non – sway intermediate column in a building frame with flexible joints is 4.0 m high and it is ISHB 300 @ 588 N/m steel section. Check the adequacy of the section when the column is subjected to following load:

Factored axial load = 500 kN

Factored moments:

	M_x	M_y
Bottom	+ 7.0 kN –m	- 1.0 kN - m
Top	+ 15.0 kN – m	+ 0.75 kN – m

[f_y = 250 N/mm^2 ; E = 2* 10^5 N/mm^2]

Assume effective length of the column as 3.4 m along both the axes.

Cross-section properties:

Flange thickness	=	T	=	10.6 mm
Clear depth between flanges	=	d	=	300 – (10.6 * 2)
			=	278.8 mm
Thickness of web	=	t	=	7.6 mm
Flange width	=	2b	=	250 mm
		b	=	125 mm
Area of cross-section	=	A_g	=	7485 mm^2

τ_x = 129.5 mm

τ_y = 54.1 mm

I_x = $12545.2*10^4$ mm^4

I_y = $2193.6*10^4$ mm^4

Z_x = $836.3*10^3$ mm^3

Z_y = $175.5*10^3$ mm^3

Z_{px} = $953.4*10^3$ mm^3

Z_{py} = $200.1*10^3$ mm^3

(i) Type of section:

$$\frac{b}{T} = \frac{125}{10.6} = 11.8 < 13.65\in$$

$$\frac{d}{t} = \frac{278.8}{7.6} = 36.7 < 40.95\in$$

$$\text{where, } \in = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Hence, cross- section is " SEMI-COMPACT" (Class 3)

(ii) Check for resistance of cross-section to the combined effects for yielding:

f_{yd} = f_y/γ_a = 250/1.15

= 217.4 N/mm^2

A_g = 7485 mm^2

Z_x = $836.3*10^3$ mm^3

Z_y = $175.5*10^3$ mm^3

F_c = 500 kN

M_x = 15 kN-m

M_y = 1.0 kN-m

The interaction equation is:

$$\frac{F_c}{A_g\, f_y\, d} + \frac{M_x}{Z_x\, f_y\, d} + \frac{M_y}{Z_y\, f_y\, d} \le 1$$

$$\frac{500\,x\,10^3}{7845\,x\,217.4} + \frac{15\,x\,10^6}{836.3\,x\,217.4} + \frac{1\,x\,10^3}{175.5\,x\,10^3\,x\,217.4}$$

$$= 0.307 + 0.083 + 0.026 = 0.416 < 1.0$$

Hence, section is O.K. against combined effects

(iii) Check for resistance of cross-section to the combined effects for buckling:

Slenderness ratios:

Effective length of the column = 3.4 m

λ_x = 3400/129.5 = 26.3

λ_y = 3400/54.1 = 62.8

$\lambda_1 = \pi(E/f_y)^{1/2} = \pi(200000/250)^{1/2}$

= 88.9

Non-dimensional slenderness ratios:

$$\bar{\lambda} = \frac{\lambda}{\lambda_1}$$

$$\bar{\lambda}_x = \frac{26.3}{88.9} = 0.296$$

$$\bar{\lambda}_y = \frac{62.8}{88.9} = 0.706$$

Calculation of χ :

Imperfection factors:

$\alpha_x = 0.21$; $\alpha_y = 0.34$

ϕ - values:

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

$\phi_x = 0.5[1 + 0.21(0.296 – 0.2) + (0.296)^2] = 0.554$

$\phi_y = 0.5[1 + 0.34(0.706 – 0.2) + (0.706)^2] = 1.006$

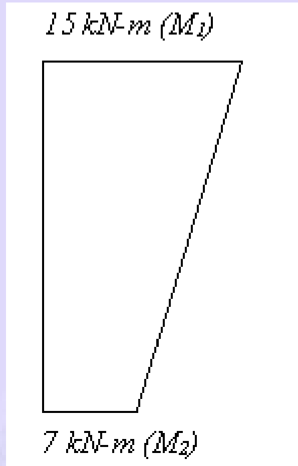
χ - values: $$\chi = \frac{\chi_1}{\phi + \left(\phi^2 - \bar{\lambda}^2\right)^{\frac{1}{2}}} \leq 1.0$$

$\chi_x = 1/[0.554 + (0.554^2 – 0.296^2)^{1/2}] = 0.978$

$\chi_y = 1/[1.006 + (1.006^2 – 0.706^2)^{1/2}] = 0.580$

The interaction equation is

$$\frac{F_c}{f_d} + \frac{k_x M_x}{M_{ux}} + \frac{k_y M_y}{M_{uy}} \leq 1$$



$\psi_x = M_2/M_1 = 7/15 = 0.467$

$\beta_{Mx} = 1.8 - 0.7\psi = 1.8 - 0.7 \times 0.467$

$= 1.473$

$\mu_x = \bar{\lambda}_x (2\beta_{Mx} - 4) = 0.296\ (2 \times 1.473 - 4)$

$= -0.312$

$$k_x = 1 - \frac{\mu_x F_c}{P_{cx}} = 1 - \frac{\mu_x F_c}{\chi_x A f_y} = 1 - \frac{(-0.312) \times 500 \times 10^3}{0.978 \times 7485 \times 250} = 1.085$$

$\psi_y = 0.75/(-1.0) = -0.75$

$\beta_{My} = 1.8 - 0.7\psi$

$= 1.8 + 0.7 \times 0.75 = 2.325$

$\mu_y = \bar{\lambda}_y (2\beta_{My} - 4)$

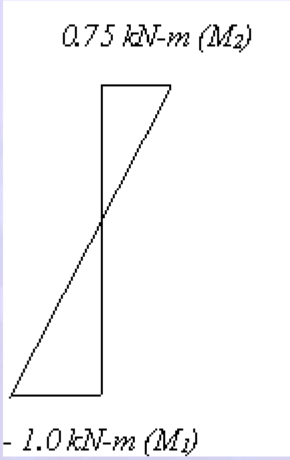
$= 0.706\ (2 \times 2.325 - 4) = 0.459$

$$k_y = 1 - \frac{\mu_y F_c}{P_{cy}} = 1 - \frac{\mu_y F_c}{\chi_y A f_y} = 1 - \frac{0.459 \times 500 \times 10^3}{0.58 \times 7485 \times 250} = 0.788$$

Note: $F_{cl} = \chi_{min} A_g f_{yd}$

$M_{ux} = Z_x f_{yd}$

$M_{uy} = Z_y f_{yd}$



Substituting the interaction equation,

$$\frac{500 \times 10^3}{7845 \times 217.4 \times 0.58} + \frac{15 \times 10^6 \times 1.085}{836.3 \times 10^3 \times 217.4} + \frac{1 \times 10^6 \times 0.788}{175.5 \times 10^3 \times 217.4}$$

$$= 0.530 + 0.089 + 0.021 = 0.640 < 1.0$$

Hence, section is O.K. against combined effects for buckling.