6052

BOARD DIPLOMA EXAMINATION, (C–16)
OCTOBER—2020
DME—FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 hours] [*Total Marks* : 80

PART—A 3×10=30

Instructions **:** (1) Answer **all** questions.

(2) Each question carries **three** marks.

1. Resolve $\dfrac{2}{(x+3)(x+4)}$ into partial fractions.

2. If $A=\begin{bmatrix}1 & -3\\ 2 & 1\end{bmatrix}$, then find $A^2-3A+2I$ where I, is a unit matrix of order 2.

3. Evaluate $\begin{vmatrix}1 & 0 & -2\\ 3 & -1 & 2\\ 4 & 5 & 6\end{vmatrix}$, using Laplace expansion.

4. Prove that, $\dfrac{\cos 19^\circ - \sin 19^\circ}{\cos 19^\circ + \sin 19^\circ} = \tan 26^\circ$.

5. Prove that, $\dfrac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta} = \tan\left(\dfrac{\theta}{2}\right)$.

6. Find the modulus amplitude form of the complex number $-1-i$.

7. Find the equation of the line which makes intercepts -4 with x-axis and 1 with y-axis.

8. Find the equation of the straight line passing through the point $(-4, 3)$ and perpendicular to the line $3x+y-31=0$.

9. Evaluate $\lim_{x\to 5}\left(\frac{x^2-25}{x^3-125}\right)$.

10. Find the derivative of $\tan x \log x$ with respect to x.

PART—B

10×5=50

Instructions **:** (1) Answer *any* **five** questions.

(2) Each question carries **ten** marks.

11. *(a)* Solve the equations $x+2y+3z=6$, $3x-2y+z=2$ and $4x+2y+z=7$ by Crammer's Rule.

(b) Find the adjoint of the matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

12. *(a)* Prove that, $\sin A+\sin(120^\circ+A)-\sin(120^\circ-A)=0$.

(b) Prove that, $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\frac{\pi}{4}$.

13. *(a)* Solve the equation $(2\cos\theta+1)(\cos\theta-1)=0$.

(b) In a ΔABC, prove that $(b+c)\sin\left(\frac{A}{2}\right)=a\cos\left(\frac{B-C}{2}\right)$.

14. *(a)* Find the equation of the circle whose center is at the point $(-3, 2)$ and radius is 4 units.

(b) Find the equation of the rectangular hyperbola whose focus is at the point $(1, 2)$ and directrix is the line $3x + 4y - 5 = 0$.

15. *(a)* Find $\frac{dy}{dx}$, if $y = \sin^{-1}(3x - 4x^3)$.

(b) Find $\frac{dy}{dx}$, if $x^3 + y^3 = 6xy$.

16. *(a)* Find $\frac{d^2y}{dx^2}$, if $x = 36(\theta - \sin\theta)$, $y = 36(1 - \cos\theta)$.

(b) If $u(x, y) = \frac{xy}{x + y}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$.

17. *(a)* Find the equations of tangent and normal to the curve $y = x^3 - 3x^2 - x + 5$, at the point $(1, 2)$.

(b) The displacement s of a particle is given at any time t by the relation $s = 2t^3 - 3t^2 + 15t + 18$. Find its velocity when the acceleration is 0.

18. *(a)* Find the maximum and minimum values of $f(x) = x^3 - 6x^2 + 9x + 1$.

(b) The side of a square plate is increased by 0·1%. Find the approximate percentage increase in its area.

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