



C16-A/AA/BM/CH/CHST/AEI/MNG/
MET/TT/IT/PCT—102

6002

BOARD DIPLOMA EXAMINATION, (C-16)
MARCH/APRIL—2018
FIRST YEAR (COMMON) EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 hours]

[Total Marks : 80

PART - A

3×10=30

- Instructions** : (1) Answer **all** questions.
(2) Each question carries **three** marks.
(3) Answers should be brief and straight to the point and shall not exceed *five* simple sentences.

1. Resolve

$$\frac{3x}{(x-2)(x-1)}$$

into partial fractions.

2. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, then show that $A^2 - 4A - 7I = O$, where I is the identity matrix and O is the null matrix.

3. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & 6 & x \end{pmatrix}$ and $\det A = 45$, then find the value of x .

4. If $A = B = 45^\circ$, then show that $(1 + \tan A)(1 + \tan B) = 2$.

*

5. If $x = \frac{1}{2\cos\theta}$, then show that $x^2 = \frac{1}{2\cos 2\theta}$.

6. Express the complex number $\sqrt{3} + i$ in modulus-amplitude form.

7. Find the distance from the origin to the straight line $3x + 4y + 5 = 0$.

8. Find the equation of the straight line passing through the point $(3, -4)$ and parallel to the line $3x - y - 31 = 0$.

9. Evaluate :

$$\lim_{x \rightarrow 0} \frac{3x - \sin 3x}{x^3}$$

10. Find the derivative of $\log[\log(\log x)]$ with respect to x .

PART - B

10×5=50

Instructions : (1) Answer any **five** questions.

(2) Each question carries **ten** marks.

(3) Answers should be comprehensive and the criterion for valuation is the content but not the length of the answer.

11. (a) If $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then find A^{-1} .

(b) Solve the equations $3x + y + z = 3$, $2x + 2y + 5z = 1$ and $x + 3y + 4z = 2$ by Cramer's rule.

*

12. (a) Prove that

$$\frac{\sin 8A}{\cos 8A} - \frac{\sin 6A}{\cos 6A} = \tan 7A$$

(b) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then prove that $xy + yz + zx = 1$.

- * 13. (a) Solve $\cos^{-1} \sqrt{3} \sin^{-1} 1$
 (b) In a $\triangle ABC$, if $A = 60^\circ$, then prove that

$$\frac{c}{a} \frac{b}{c} \frac{a}{b} = 1$$
14. (a) Find the centre and radius of the circle $x^2 + y^2 - 6x - 4y - 12 = 0$.
 (b) Find the equation of the parabola whose focus is the point $(3, -4)$ and directrix is the line $x - y - 5 = 0$.
15. (a) Differentiate $e^{\tan^{-1} x}$ with respect to $\tan^{-1} x$.
 (b) If $y = \sqrt{\log x} \sqrt{\log x} \sqrt{\log x} \dots$, then find $\frac{dy}{dx}$.
16. (a) If $x = a(\cos t - t \sin t)$ and $y = a(\sin t + t \cos t)$, then find $\frac{d^2y}{dx^2}$.
 (b) If $u = \tan^{-1} \frac{x^3 + y^3}{x + y}$, then prove that

$$x \frac{u}{x} + y \frac{u}{y} = \sin 2u$$
17. (a) Find the lengths of the tangent, normal, sub-tangent and subnormal to the curve $y = x^3 - 2x^2 + 4$ at the point $(2, 4)$.
 (b) A circular metal plate expands by heat so that its radius is increasing at the rate 0.02 cm/sec. At what rate its area is increasing when the radius is 20 cm?
- * 18. (a) A right circular cylinder is inscribed in a sphere of radius R . Show that the volume of the cylinder is maximum when its height is $\frac{2R}{\sqrt{3}}$.
 (b) If the radius of a spherical balloon is increased by 0.1% , find the approximate percentage increase in its volume.
