4455

BOARD DIPLOMA EXAMINATION, (C–14)
JUNE—2019

DCE—FOURTH SEMESTER EXAMINATION

ENGINEERING MATHEMATICS—III

Time : 3 hours] [*Total Marks* : 80

PART—A

3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

(3) Answers should be brief and straight to the point and shall not exceed *five* simple sentences.

1. Solve $\dfrac{d^2 x}{dt^2} + 5\dfrac{dx}{dt} + 6x = 0$.

2. Solve $(D^3 + D^2 + 4D + 4)y = 0$, where $D \equiv \dfrac{d}{dx}$.

3. Find the particular integral of $(D^2 + 5D + 6)y = e^x$, where $D \equiv \dfrac{d}{dx}$.

4. Find the Laplace transform of $t^3 + 5\cos t$.

5. Find the Laplace transform of $t^3 e^{-3t}$.

6. Find the inverse Laplace transform of $\dfrac{s^2 - 3s + 4}{s^3}$.

7. Find the inverse Laplace transform of $\dfrac{s+2}{(s+1)(s-2)}$.

8. Define Fourier series of a function $f(x)$ in the interval $(0, 2\pi)$

9. Find the value of a_0 in the Fourier cosine series of $f(x) = 1$ in the interval $(0, 1)$.

10. A bag contains 9 balls of which 4 are red, 3 are blue and 2 are yellow. The balls are similar in shape and size. A ball is drawn at random from the bag. Find the probability that the ball will be either red or blue.

<div align="center">PART—B</div> 10×5=50

Instructions : (1) Answer *any* **five** questions.

(2) Each question carries **ten** marks.

(3) Answers should be comprehensive and the criterion for valuation is the content but not the length of the answer.

11. *(a)* Solve $(D^2 + 36)y = \sin^2 x$, where $D \equiv \dfrac{d}{dx}$.

(b) Solve $(D^2 - D - 2)y = 3e^{2x}$, where $D \equiv \dfrac{d}{dx}$.

12. Solve $(D - 2)^2 y = 8(e^{2x} + \sin 2x + x^2)$, where $D \equiv \dfrac{d}{dx}$.

13. *(a)* Find the Laplace transform of $\dfrac{1 - e^t}{t}$.

(b) If $L\left\{\dfrac{\sin t}{t}\right\} = \tan^{-1}\dfrac{1}{s}$, find $L\left\{e^t \dfrac{\sin 3t}{t}\right\}$

14. *(a)* Show that $L^{-1}\left\{\dfrac{1}{s(s^2 + a^2)}\right\} = \dfrac{1 - \cos at}{a^2}$.

(b) Using Laplace transform method, solve $y'' + y = t$, if $y(0) = 1$ and $y'(0) = 0$.

15. Expand $f(x) = x - x^2$, $-\pi < x < \pi$ in a Fourier series and hence deduce that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \cdots = \dfrac{\pi^2}{12}$.

16. Obtain the Fourier series for $f(x) = \dfrac{\pi - x}{2}$ in $0 \leq x \leq 2$.

17. Two students A and B appeared in an examination. The probability that A will qualify the examination is 0·05, B will qualify the examination is 0·10 and that both A and B will qualify the examination is 0·02. Find the probability that *(a)* both A and B will not qualify the examination, *(b)* at least one of them will not qualify the examination and *(c)* only one of them will qualify the examination.

18. *(a)* If A and B are independent events with $P(A) = 0·2$ and $P(B) = 0·5$, then find *(i)* $P(B/A)$, *(ii)* $P(A/B)$ and *(iii)* $P(A \cap B)$.

(b) In a certain college, 25% of the boys and 10% of the girls are studying Mathematics. The girls constitute 60% of the student strength. If a student at random is found studying Mathematics, find the probability that the student is a girl.

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