6017

BOARD DIPLOMA EXAMINATION, (C–16)

SEPTEMBER/OCTOBER - 2020

DCE—FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 hours]   [*Total Marks* : 80
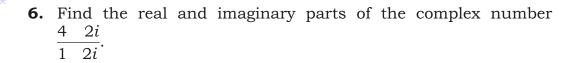
PART—A   3×10=30

Instructions : (1) Answer **all** questions.

              (2) Each question carries **three** marks.

1. Resolve $\dfrac{1}{(x-5)(x-7)}$ into partial fractions.

2. If $A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$ then find $A^2 - 3A$.

3. Using Laplace's expansion, find the value of $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$.

4. Prove that

$$\frac{\cos 11^{\circ} + \sin 11^{\circ}}{\cos 11^{\circ} - \sin 11^{\circ}} = \tan 56^{\circ}$$

5. Prove that

$$\frac{1 + \cos 2A}{\sin 2A} = \cot A$$

6. Find the real and imaginary parts of the complex number $\dfrac{4+2i}{1-2i}$.

7. Find the perpendicular distance of the point $(3, -2)$ to the line $3x + 4y + 10 = 0$.

8. Find the point of intersection of the lines $2x + 4y = 6$ and $x + 4y = 3$.

9. Evaluate $\underset{x \to 2}{\text{Lt}} \dfrac{x^2 - 4}{x - 2}$.

10. Find the derivative of $3\tan x + 4\log x - 7x^2$ with respect to x.

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PART—B 10×5=50

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Instructions : (1) Answer *any* **five** questions.

               (2) Each question carries **ten** marks.

11. *(a)* Prove that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

(b) Using matrix inversion method, solve the equations

$$2x + y + z = 4,\ x + y + z = 1,\ x + 3y + 2z = 2$$

12. *(a)* Prove that

$$\frac{\cos 3A + \cos A}{\sin 3A + \sin A} = \cot 2A$$

(b) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then prove that $x + y + z = xyz$.

13. *(a)* Solve $\sin\theta + \cos\theta = \sqrt{2}$.

(b) Solve the triangle ABC with $a = 1,\ b = 2,\ c = \sqrt{3}$.

14. *(a)* Find the equation of the circle with $(-5, 1)$ and $(3, -7)$ as the end points of a diameter.

(b) Find the centre, foci, vertices, eccentricity, length of latus rectum and equations of the directrices of the ellipse $9x^2 + 25y^2 = 225$.

15. *(a)* Find $\dfrac{dy}{dx}$, if $y = \sin^{-1}(3x - 4x^3)$.

(b) Find $\dfrac{dy}{dx}$, if $x^y = y^x$.

16. *(a)* If $y = x^{x^{x^{... \text{ to } \infty}}}$, then prove that

$$\frac{dy}{dx} = \frac{y^2}{x(1 - y \log x)}$$

(b) If $y = \log(e^x + e^y)$, then show that

$$\frac{z}{x} + \frac{z}{y} = 1$$

17. *(a)* Find the angles between the curves $y^2 = 4x$ and $x^2 = 4y$.

(b) A particle is moving along a straight line according to the law $s = 2t^3 - 3t^2 - 15t + 18$ (t in sec). Find its velocity, when its acceleration is zero.

18. *(a)* The sum of two numbers is 24. Find the numbers when the sum of their squares is a minimum.

(b) The radius of a spherical balloon is increased by 2%. Find the approximate percentage increase in its surface area.

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